EECS 240
Analog Integrated Circuits

Lecture 5: Electronic Noise

Ali M. Niknejad and Bernhard E. Boser

Department of Electrical Engineering and Computer Sciences

Electronic Noise

- Why is this important?

- Signal-to-noise ratio
  - Signal Power $P_{sig} \sim (V_{DD})^2$
  - Noise Power $P_{noise} \sim k_B T/C$
  - $SNR = P_{sig} / P_{noise}$

- Technology Scaling
  - V_{DD} goes down → SNR down
  - Or C up → Power up

- Low Power means understanding noise

Types of "Noise"

- Interference ("human" made, not "fundamental")
  - Signal coupling
    - Capacitive
    - Inductive
    - Substrate
    - bond wires
  - Supply noise

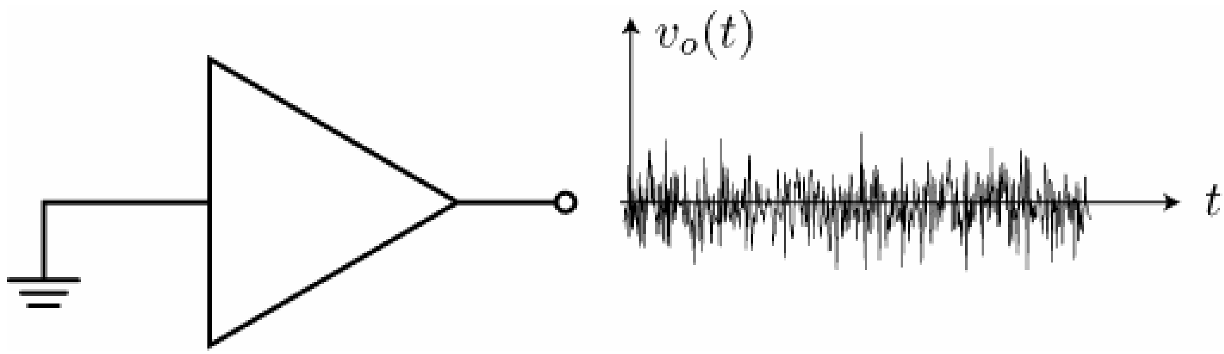
  → Solutions:

    → Fully differential circuits

    → Layout techniques

- **<u>Device noise</u>**
  - Caused by discreteness of charge
  - "fundamental" – **thermal noise**
  - "manufacturing process related" – flicker noise

Noise in Amplifiers



- All electronic amplifiers generate noise. This noise originates from the random thermal motion of carriers and the discreteness of charge.
- Noise signals are random and must be treated by statistical means. Even though we cannot predict the actual noise waveform, we can predict the statistics such as the mean (average) and variance.

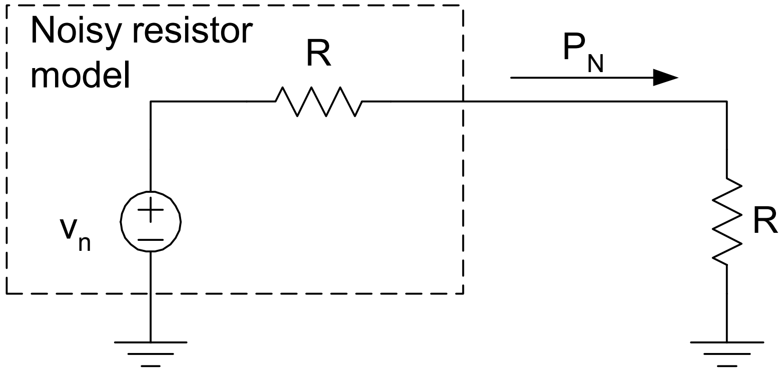
Thermal Noise of a Resistor

- Origin: Brownian Motion
  - Thermally agitated particles
  - E.g. ink in water, electrons in a conductor
- Random → use statistics to describe
- Available noise power:

$$P_N = k_B T B$$

  - Noise power in bandwidth B delivered to a matched load
  - Example: B = 1Hz → $P_N = 4 \times 10^{-21}$W = -174 dBm
  - Reference: J.B. Johnson, "Thermal Agitation of Electricity in conductors," Phys. Rev., pp. 97-109, July 1928.

Resistor Noise Model



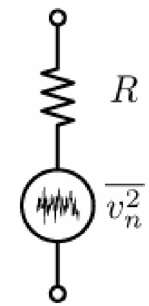
$$P_N = k_B T B = \frac{\overline{v_n^2}}{4R}$$

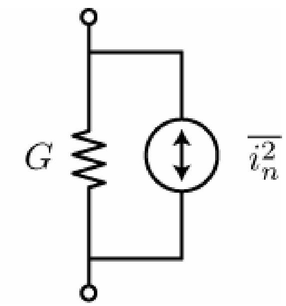
Mean square noise voltage:

$$\overline{v_n^2} = 4 k_B T R B$$

Thermal Noise

- Present in all dissipative elements (resistors)
- Independent of DC current flow
- Random fluctuations of $v(t)$ or $i(t)$
  - Instantaneous noise is unpredictable
  - Result of many random, superimposed collisions
  - Relaxation time constant $\tau \sim 0.17$ps
  - Consequences:
    - Zero mean
    - Gaussian amplitude distribution (pdf)
    - Power spectral density "white" up to about $1/\tau = 2\pi \times 1000$ GHz
  - $k_B T = 4 \times 10^{-21}$ J   ($T = 290\text{K} = 16.9^{o}\text{C}$)





$$\overline{v_n^2} = 4k_B TRB$$

$$\overline{i_n^2} = \frac{4k_B TB}{R}$$

- Example:

  R = 1kΩ,   B = 1MHz → <u>4μV rms</u>   or   <u>4nA rms</u>   or   <u>4nV/rt-Hz</u>

Noise of Passive Networks

- Capacitors, Inductors
- Noise calculations
  - Instantaneous voltages add
  - Power spectral densities add
  - RMS voltages do <u>NOT</u> add
- Example: R_1+R_2 in series
- Generalization to arbitrary RLC networks

Noise in Diodes

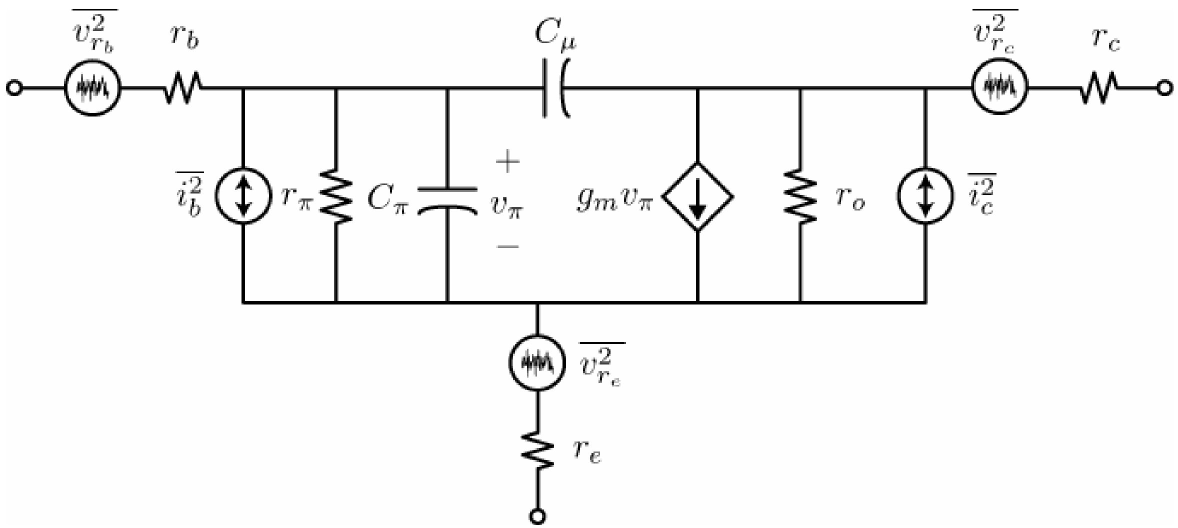
- Shot noise
  - Zero mean
  - Gaussian pdf
  - Power spectral density
  - Proportional to current
  - Independent of temperature

$$\overline{i_n^2} = 2qI_D\Delta f$$

- Example:
  I_D = 1mA, B = 1MHz → <u>17nA rms</u>

- Shot noise versus thermal noise ($r_d = V_t/I_D$)
  Thermal equilibrium

BJT Noise



$$\overline{v_b^2} = 4k_B T r_b \Delta f$$

$$\overline{i_b^2} = 2qI_B \Delta f + K_1 I_B^\alpha \frac{\Delta f}{f}$$

$$\overline{i_c^2} = 2qI_C \Delta f$$

- The collector and base shot noise are partially correlated.
- The base resistance contributes thermal noise.
- Note that r_o does not contribute noise. It's not a *physical* resistor.

FET Noise

- In addition to the extrinsic physical resistances in a FET (r_g, r_s, r_d), the channel resistance also contributes thermal noise.
- The channel conductance is calculated by

$$G_{ds} = \frac{I_{ds}}{V_{dsat}} = \mu C_{ox} \frac{W}{L} \frac{1}{2} \frac{V_{dsat}^2}{V_{dsat}} = g_m$$

- The noise injection is a distributed process. The net sum of the noise injection gives (note γ)

$$\overline{i_d^2} = 4kT\gamma g_m \Delta f$$

More Fundamental Expression

$$\overline{i_d^2} = 4kT\frac{\mu}{L^2}|Q_i|\Delta f$$

- A more fundamental equation is derived [Tsividis] results in the above equation
- In nonsaturation with $V_{ds} = 0$, the device is a resistor so the thermal noise is given by

$$\overline{i_d^2} = 4kT\frac{W}{L}\mu C_{ox}(V_{GS} - V_T)\Delta f$$

Strong Inversion Noise

- In saturation, the drain current is given by

$$\overline{i_d^2} = 4kT\frac{2}{3}\frac{W}{L}\mu C_{ox}(V_{GS} - V_T)\Delta f$$

- For a long channel model, you can substitute g_m for the above factor. In practice the form involving inversion charge is more accurate and used by SPICE/BSIM.

Weak Inversion

- The origin of noise in weak inversion is shot noise. So the result should be $\sim 2qI_{DS}$.
- But using the expression for inversion charge in weak inversion we get the same result! (similar to a diode)

$$Q_i = WL\frac{Q_{i0}+Q_{iL}}{2} = \frac{L^2}{2\mu\frac{kT}{q}} I'_{DS}\left(1+e^{-qV_{DS}/kT}\right)$$

$$\overline{i_d^2} = 2qI'_{DS}\left(1+e^{-qV_{DS}/kT}\right)\Delta f$$

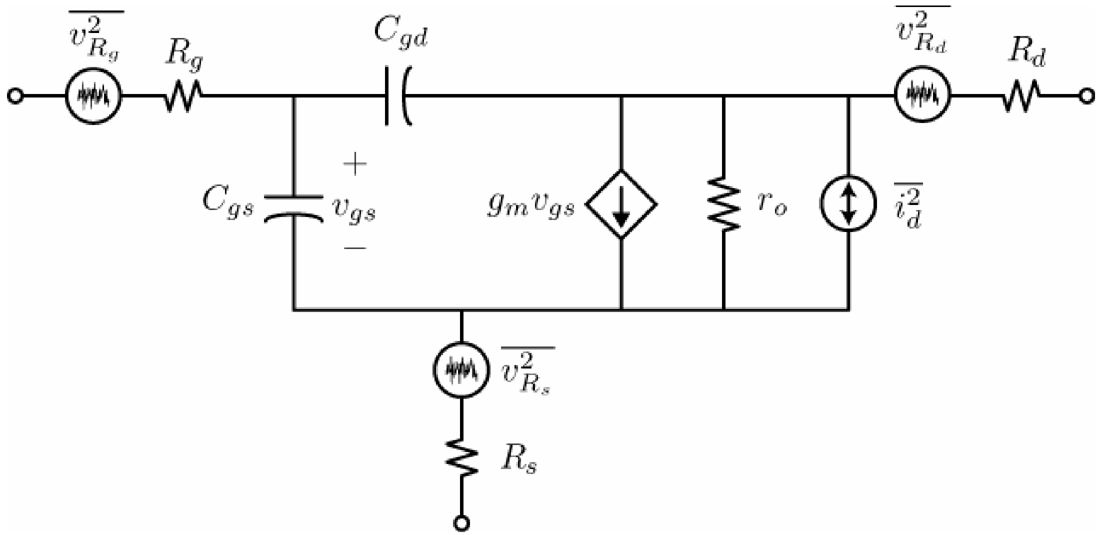
Thermal Noise for Short Channels

- Thermal noise (strong inversion)
  - Drain current (use g_{ds0} not g_m)
    $\gamma = 2/3$ for small fields (long L)
    can be 1-2 or even larger for short L

$$\overline{i_d^2} = 4kT\gamma g_{ds0}\Delta f$$
$$= 4kT\frac{\gamma}{\alpha} g_m \Delta f$$

- Since the expression with gm is convenient for input referred noise calculations, it is often used. The expression with g_{ds0} is more accurate and should be used. The factor α captures the drop in g_m for a short channel device.
- Gate induced noise (142/242 topic)
- Gate current (leakage → shot noise)
- No noise from r_o (not physical resistor)
- Extrinsic noise sources (drain/source/gate resistance)

FET Noise Model



- The resistance of the substrate also generates thermal noise. In most circuits we will be concerned with the noise due to the channel and the input gate noise due to Rg

1/f Noise

- Flicker noise
  - $K_{f,NMOS} = 2.0 \times 10^{-29}$ AF
    $K_{f,PMOS} = 3.5 \times 10^{-30}$ AF

$$\overline{i_{1/f}^2} = \frac{K_f I_D}{L^2 C_{ox}} \frac{\Delta f}{f}$$

  - Strongly process dependent (also model)
- Example: $I_D = 10\mu A$, $L = 1\mu m$,
  - $C_{ox} = 5.3 fF/\mu m^2$, $f_{hi} = 1MHz$

$f_{lo} = 1Hz$ → 722pA rms
$f_{lo} = 1/year$ → 1082pA rms

$$\overline{i_{1/f,total}^2} = \int_{f_{lo}}^{f_{hi}} \frac{K_f I_D}{L^2 C_{ox}} \frac{df}{f} = \frac{K_f I_D}{L^2 C_{ox}} \ln \frac{f_{hi}}{f_{lo}}$$

1/f Noise Corner Frequency

- Definition (MOS)

$$\frac{K_f I_D}{L^2 C_{ox}} \frac{\Delta f}{f_{co}} = 4k_B T_r \gamma g_m \Delta f$$

$$f_{co} = \frac{K_f I_D}{L^2 C_{ox}} \frac{1}{4k_B T_r \gamma g_m}$$

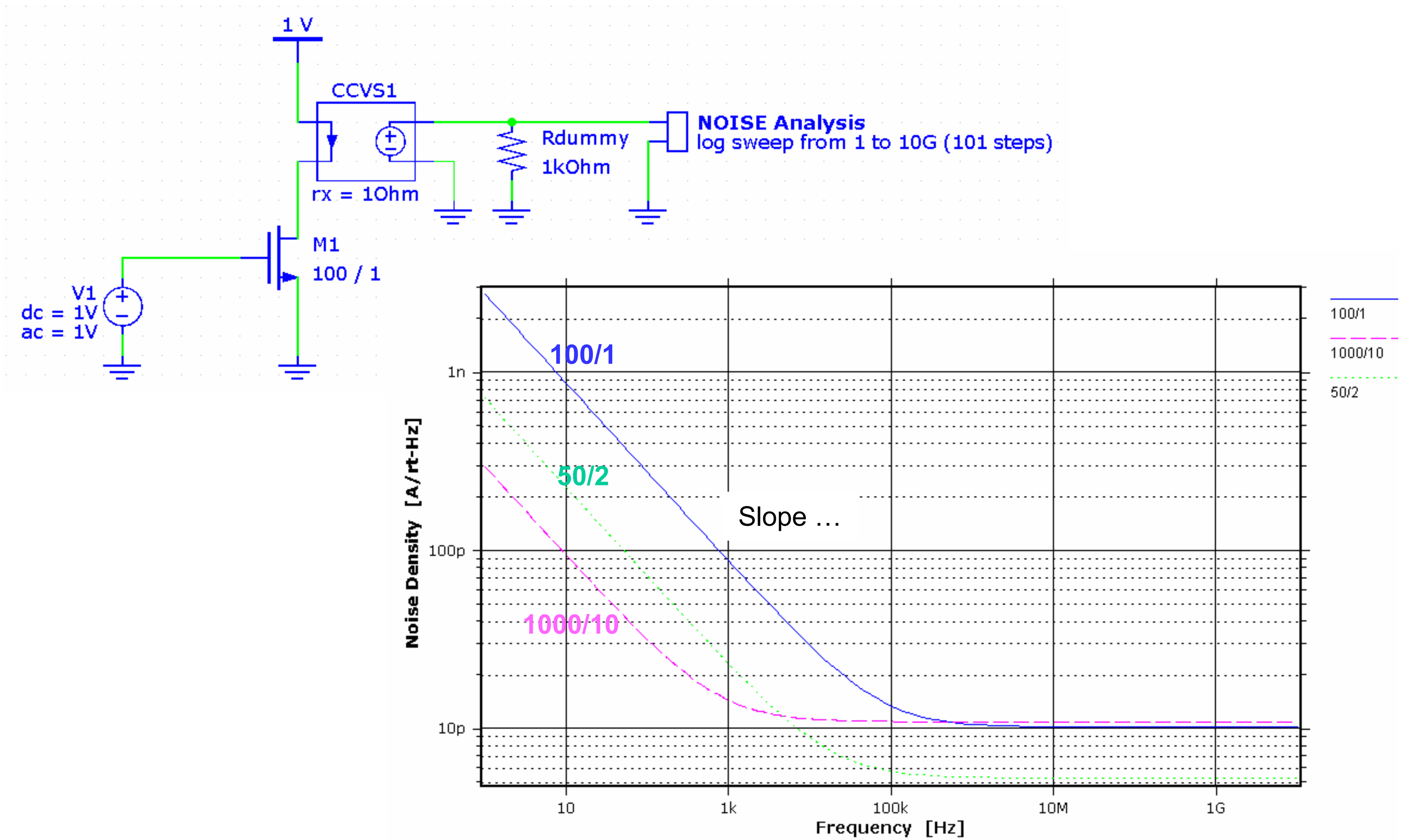
$$= \frac{K_f}{4k_B T_r \gamma C_{ox}} \frac{1}{L^2} \frac{1}{g_m / I_D}$$

$$= \frac{K_f}{8k_B T_r \gamma C_{ox}} \frac{V^*}{L^2}$$

- Example:
  - V* = 200mV, γ = 1

		NMOS	*PMOS*
L = 0.35μm	→	192kHz	34kHz
L = 1.00μm	→	24kHz	4kHz

SPICE Noise Analysis



Noise Calculations

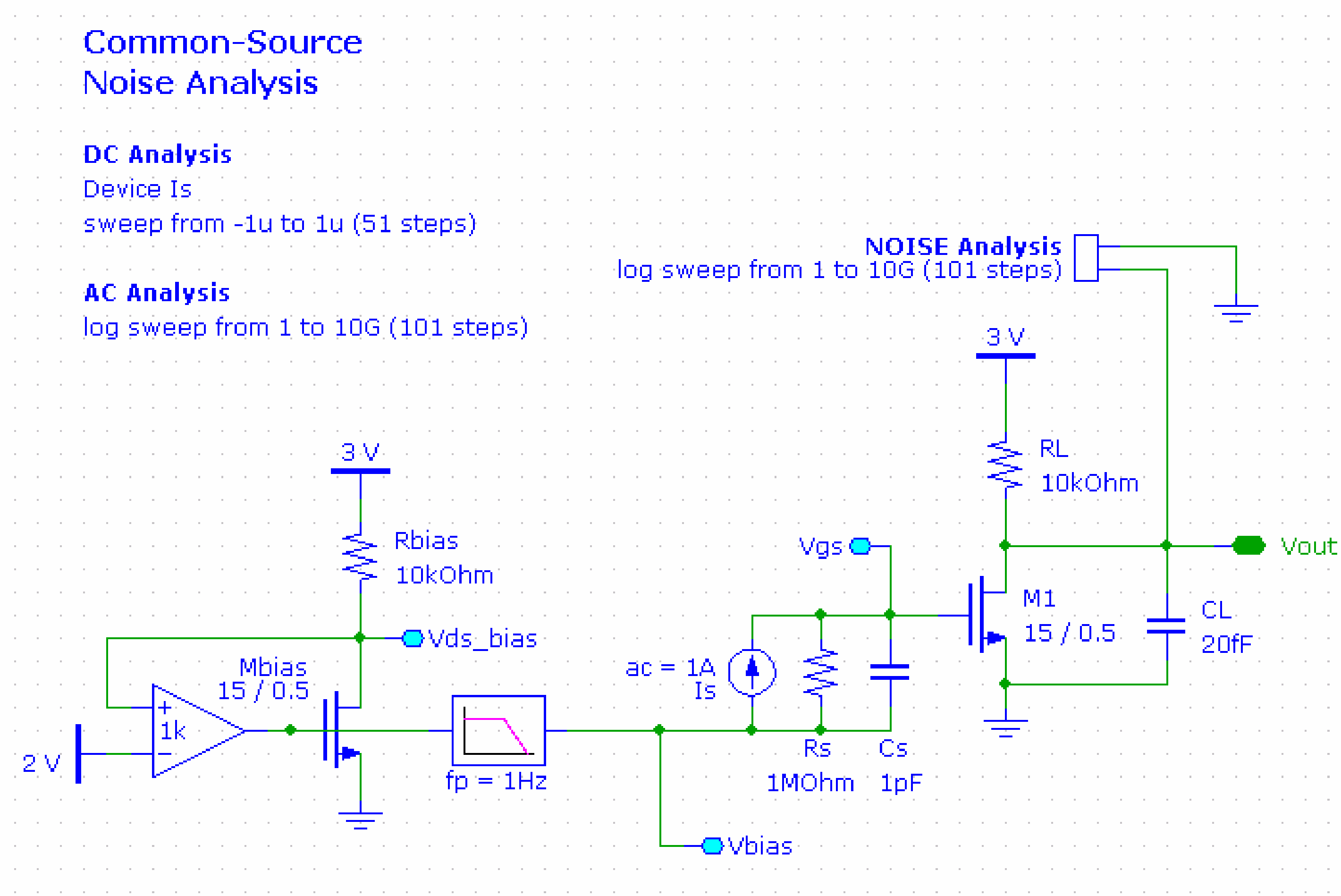
- Output spectral noise density
- Method:
    1) Small-signal model
    2) All inputs = 0 (linear superposition)
    3) Pick output v_o or i_o
    4) For each noise source v_x, i_x
       Calculate $H_x(s) = v_o(s) / v_x(s)$ (… i_o, i_x)
    5) Total noise at output is

$$\overline{v_{on,T}^2(f)} = \sum_x \left|H_x(s)\right|_{s=2\pi jf}^2 \overline{v_x^2(f)}$$

simpler notation: $\overline{v_{on,T}^2(f)} = S_n(f)$

- Tedious but simple …

Example: Common Source



Simulation Result