1. We need to find the maximum value of resistance R so tha the current mismatch is less than 1%. First consider the transistors biased in strong-inversion:

$$VGS2 = VGS1 - I0 \cdot R = VGS1 - Iref \cdot R$$

$$\Delta VGS = -Iref \cdot R$$

$$\Delta I = gm \cdot \Delta VGS = gm \cdot Iref \cdot R$$

$$\frac{\Delta I}{Iref} = \frac{2 \cdot Iref}{Vod} \cdot R$$

$$R < \frac{Vod \cdot 0.01}{2 \cdot Iref} \qquad \frac{.5 \cdot .01}{2 \cdot 100 \cdot 10^{-6}} = 25$$

In weak inversion the solution is similar except the transistor gm is larger resulting in a more stringent requirement

$$gm = \frac{Iref}{n \cdot Vt}$$

$$\frac{\Delta I}{Iref} = gm \cdot R = \frac{Iref}{n \cdot Vt} \cdot R$$

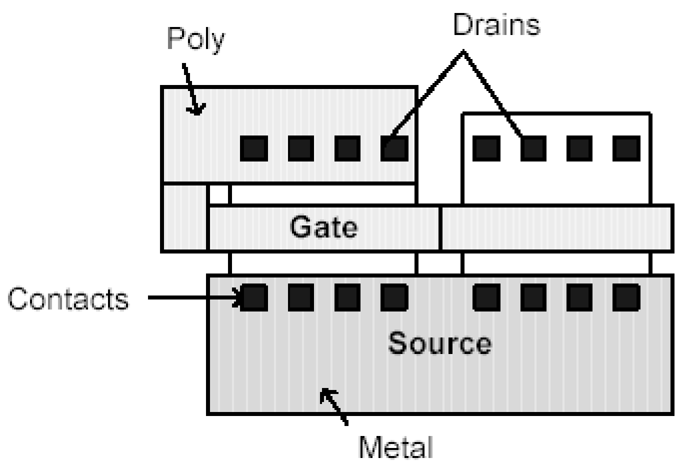
$$R < \frac{n \cdot Vt \cdot 0.01}{Iref} \qquad \frac{1.5 \cdot 25 \cdot 10^{-3} \cdot .01}{100 \cdot 10^{-6}} = 3.75$$

First note that the resistances were quite small and one can easily obtain values in this range due to interconnect such as contacts to diffusion, metal to metal vias, poly lines, metal lines .....

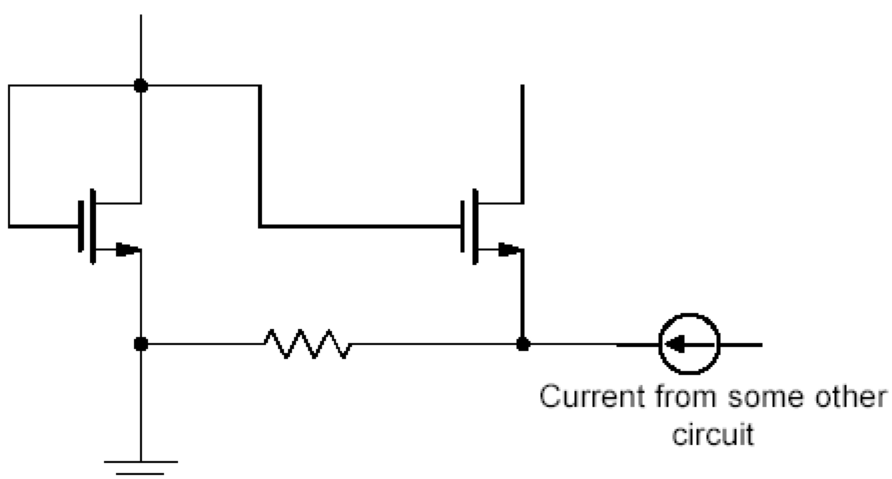
(1) Keep devices close together, and next to each other if possible. Layout the devices with the same orientation (do not swap source/drain) and match the interconnect resistances.

(2) Reduce the resistance between the devices as much as possible. Do this by increasing the number of diffusion contacts and increasing the metal width that connects them. When using vias, always use several parallel vias instead of a single via contact. In addition to lowering the resistance, this helps with reliability since a single via is more likely to fail.

(3) Avoid other circuits sharing the same ground.



What happens to the matching in this case?



Problem 2) Resistor matching problem. Since W variations is the only source of mismatch, to get a yield of 89% requires a k=1.6 (assume Gaussian statistics).

$$R1 = Rsq \cdot \frac{L}{W + \frac{\Delta W}{2}} = Rsq \cdot \frac{L}{W} \cdot \left(1 - \frac{1}{2} \cdot \frac{\Delta W}{W}\right)$$

$$R2 = Rsq \cdot \frac{L}{W - \frac{\Delta W}{2}} = Rsq \cdot \frac{L}{W} \cdot \left(1 + \frac{1}{2} \cdot \frac{\Delta W}{W}\right)$$

$$\frac{\Delta R}{R} = \frac{\Delta W}{W} \qquad W = \frac{\Delta W}{\frac{\Delta R}{R}} \qquad \frac{20 \cdot 10^{-9}}{\left(\frac{0.2 \cdot 10^{-2}}{1.6}\right)} = 1.6 \times 10^{-5}$$

$$L = \frac{R}{Rsq} \cdot W \qquad \frac{10^3}{100} \cdot 16 = 160$$

Probelm 3) To find the transconductance of M1 note that the currents of the M1 and M2 are forced to be equal due to the current mirror, so

$$\frac{\Delta Vgs}{R} = I \qquad \Delta Vgs = Vgs1 - Vgs2 = VT1 + \sqrt{\frac{2 \cdot I}{\mu \cdot Cox \cdot \left(\frac{W1}{L1}\right)}} - VT2 - \sqrt{\frac{2 \cdot I}{\mu \cdot Cox \cdot \left(\frac{W2}{L2}\right)}}$$

$$\Delta Vgs = \Delta VT + \sqrt{\frac{2I}{\mu \cdot Cox \cdot \left(\frac{W1}{L1}\right)}} \cdot \left(1 - \sqrt{m}\right)$$

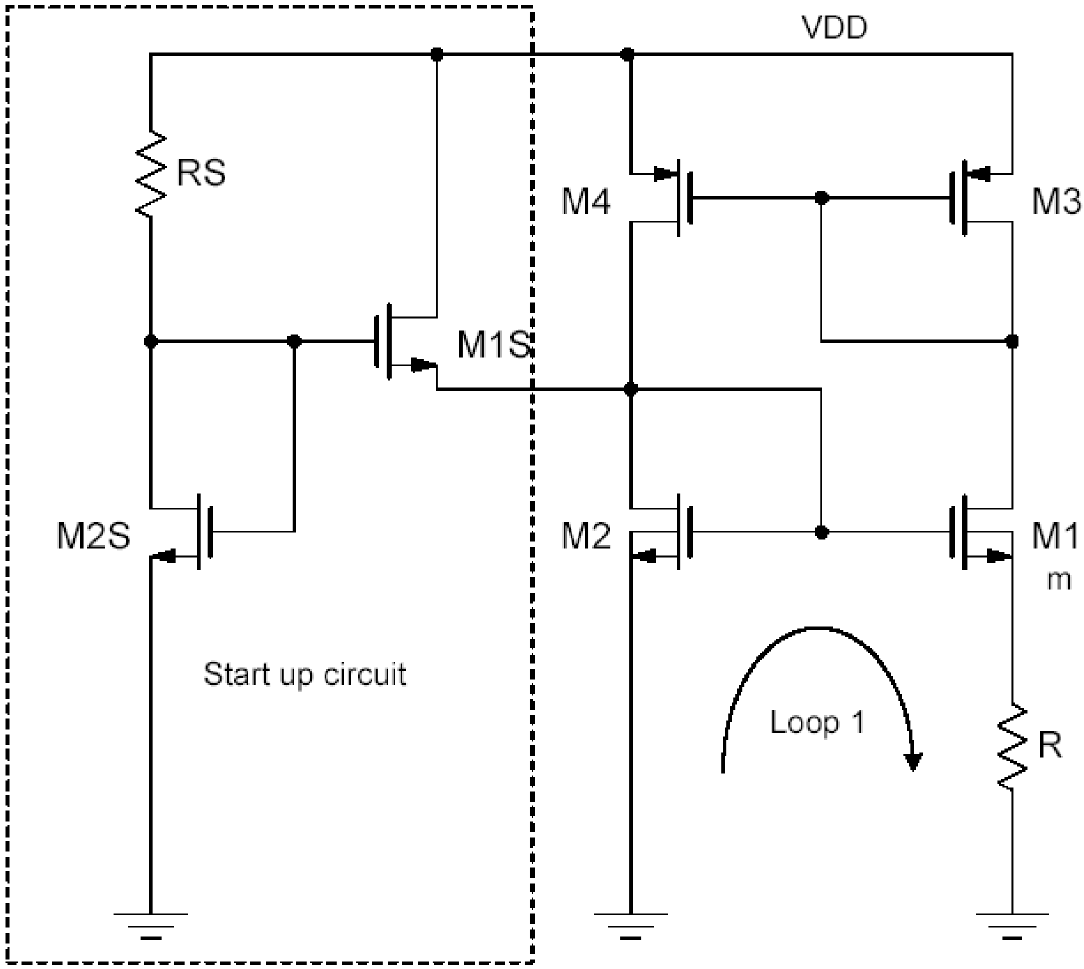
Assume that the body effect is negligible so that the threshold voltages are equal

$$\Delta Vgs = I \cdot R = Vod \cdot \left(1 - \sqrt{m}\right)$$

$$Vod = \frac{I \cdot R}{1 - \sqrt{m}} \qquad gm = \frac{2I}{Vod} = \frac{2 \cdot I}{I \cdot R} \cdot \left(1 - \sqrt{m}\right) = \frac{2\left(1 - \sqrt{m}\right)}{R}$$

gm of the transistor is equal to the conductance of R times a constant. Make m=1/4 for gm = 1/R. Note that this gm is held constant over process and temperature if R is constant (say external).

Startup circut: This one is very simple but the tradeoff is that it may be hard to turn off M1S completely, especially for low VDD.



Size M2S and RS such that Vgs(M2S) < Vgs(M2). Make M1S weak or it's (W/L)<<1

Problem 4) Calculate the 3dB frequency and total output noise for a differential pair in unity gain feedback configuration. To meet swing specifications, keep the solution as a function of Vod1 and Vod2.

$$f3db = fu = \frac{gm1}{2\cdot\pi\cdot CL} \qquad CL = \frac{gm1}{2\cdot\pi\cdot f3db} = \frac{ISS}{2\cdot\pi\cdot f3db\cdot Vod1}$$

$$ISS = 2\cdot\pi\cdot f3db\cdot Vod1\cdot CL$$

$$io^2 = 2\cdot(id1 + id2) = 2\cdot 4k\cdot T\cdot\gamma\cdot\left(\frac{1}{gm1} + \frac{1}{gm2}\right) = 2\cdot 4k\cdot T\cdot\gamma\cdot\frac{1}{gm1}\cdot\left(1 + \frac{Vod1}{Vod2}\right)$$

$$io^2 = 4\cdot k\cdot T\cdot\gamma\cdot RN$$

$$Rreal = \frac{1}{gm1} \qquad \frac{RN}{Rreal} = 2\cdot\gamma\cdot\left(1 + \frac{Vod1}{Vod2}\right)$$

$$vo^2 = \frac{k\cdot T}{CL}\cdot 2\cdot\gamma\cdot\left(1 + \frac{Vod1}{Vod2}\right)$$

$$CL = \frac{k\cdot T}{vo^2}\cdot 2\cdot\gamma\cdot\left(1 + \frac{Vod1}{Vod2}\right)$$

$$ISS = 2\cdot\pi\cdot f3db\cdot Vod1\cdot\frac{k\cdot T}{vo^2}\cdot 2\cdot\gamma\cdot\left(1 + \frac{Vod1}{Vod2}\right)$$