Smith Chart† notes

1. A graphical method of solution for T-line calculations (and avoid calculations with complex numbers and exponentials)

2. A graphical method for passive network calculations
   a) matching
   b) impedance transformation

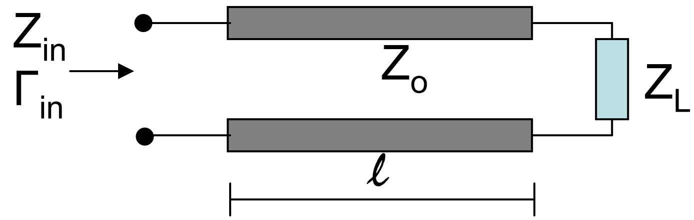
3. A graphical method for amplifier and oscillator design
   a) bilinear transformation (circles mapping into circles)
   b) preservation of matching theorem
   c) signal flow analysis of circuits

†Philip H. Smith from Bell Labs. "Transmission-line calculator," *Electronics*, vol. 12, p.29, January 1939. and "An improved transmission line calculator," *Electronics*, vol. 17, p. 130, January 1944.
"Electronic Applications of the Smith Chart: in waveguide, circuit and component analysis," P.H. Smith, Krieger Publishing Co., 1983

Preliminaries

a) 1-to-1 correspondence between Z and Γ

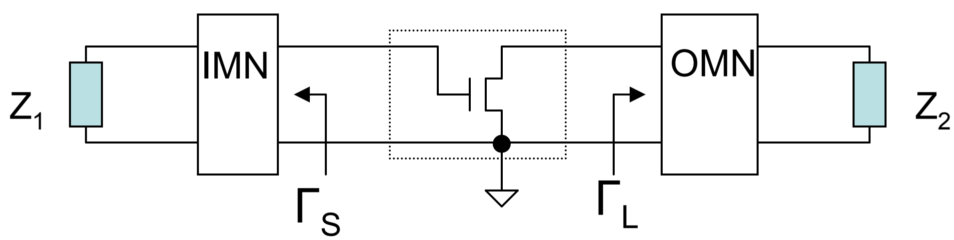
$Z=(1+\Gamma)/(1-\Gamma)$
$\Gamma=(Z-1)(Z+1)$



b) T.Line problems involve analysis of how Γ (or Z) varies along the line with distance from load or generator

c) Amplifiers and oscillator designs follow the procedure:

- get
  - (i) transistor data [S] (and Noise parameters)
  - (ii) required specifications for LNA or Oscillator
- Find Γ_S and Γ_L to attend specifications
- Realize Γ_S and Γ_L from known source and load impedances



Bilinear Transformations

In a complex plane, a circle with center at (x_o,y_o) and radius R is described by

$$(x-x_o)^2 + (y-y_o)^2 - R^2 = 0$$

or

Now, let $Z = x+jy$, $Z_o = x_o+jy_o$. The circle equation can be written as

$$|Z-Z_o|^2 - R^2=0$$

← compact

or

$$ZZ^*-ZZ_o^*-Z^*Z_o+(Z_oZ_o^*-R^2) = 0$$

← more useful for inspection

by inspection we will want to find Z_o and R in new expressions

Recall circle equation: $ZZ^* - ZZ_o^* - Z^*Z_o + (Z_oZ_o^* - R^2) = 0$

Consider the bilinear transformation:

$W=(AZ+B)/(CZ+D)$

A, B, C, D are complex constants. *W* and *Z* are complex variables.

(i) This transformation will map circles in the *Z*-plane into circles in the *W*-plane.

(i) Straight lines are limiting cases.

Case 1) Let $|W|^2 = \rho^2$ or $WW^* - \rho^2 = 0$; a circle centered at the origin.

Using the transformation, we get:

$(AZ+B)/(CZ+D)\,(A^*Z^*+B^*)/(C^*Z^*+D^*) - \rho^2 = 0$

Expanding:

$ZZ^*(AA^* - \rho^2CC^*) - Z(\rho^2CD^* - AB^*) - Z^*(\rho^2C^*D - A^*B) + BB^* - \rho^2DD^* = 0$

By comparison, this is a circle with center (coefficient of "$-Z^*$"):

$Z_o = (\rho^2C^*D - A^*B)/(AA^* - \rho^2CC^*) = (\rho^2C^*D - A^*B)/(|A|^2 - \rho^2|C|^2)$

Radius is found from constant term that is *"(ZoZo*-R2)":*

$R^2 = Z_oZ_o^* - (|B|^2 - \rho^2|D|^2)/(|A|^2 - \rho^2|C|^2) \Rightarrow R = \rho(|AD-BC|)/(|A|^2 - \rho^2|C|^2)$

Case 2) Let $|W-W_o|^2 = \rho^2$; a circle centered at W_o.

Then:

$W-W_o=(AZ+B)/(CZ+D)-W_o=[(A-CW_o)Z+(B-DW_o)]/(CZ+D)$

We make $A'=A-CW_o$; $B'=B-DW_o$, and reuse the equations of case 1.
Thus making Z also describe a circle.

Many amplifier design relations involves bilinear transformations.
Example: input coefficient of reflection (Γ_{in}) as a function of the load (Γ_L)

$$\Gamma_{in}=(\Delta\Gamma_L-S_{11})/(S_{22}\Gamma_L-1)$$

$$\Delta=S_{11}S_{22}-S_{12}S_{21}$$

Smith chart "r" and "x" circles

Given a lossless T.line with characteristic impedance $Z_o=R_o$, terminated by an impedance Z_L, we write

$\Gamma = \Gamma_r+j\Gamma_i = (Z_L-R_o)/(Z_L+R_o)$

Which after normalization by Ro, leads to

$\Gamma = \Gamma_r+j\Gamma_i = (z_L-1)/(z_L+1)$
where $z_L=Z_L/Ro$

Therefore:
$z_L = r+jx = (1+\Gamma)/(1-\Gamma) = [(1+\Gamma_r) + j\Gamma_i]/[(1-\Gamma_r) - j\Gamma_i]$
And, after multiplying numerator and denominator by complex conjugate of denominator, we reach

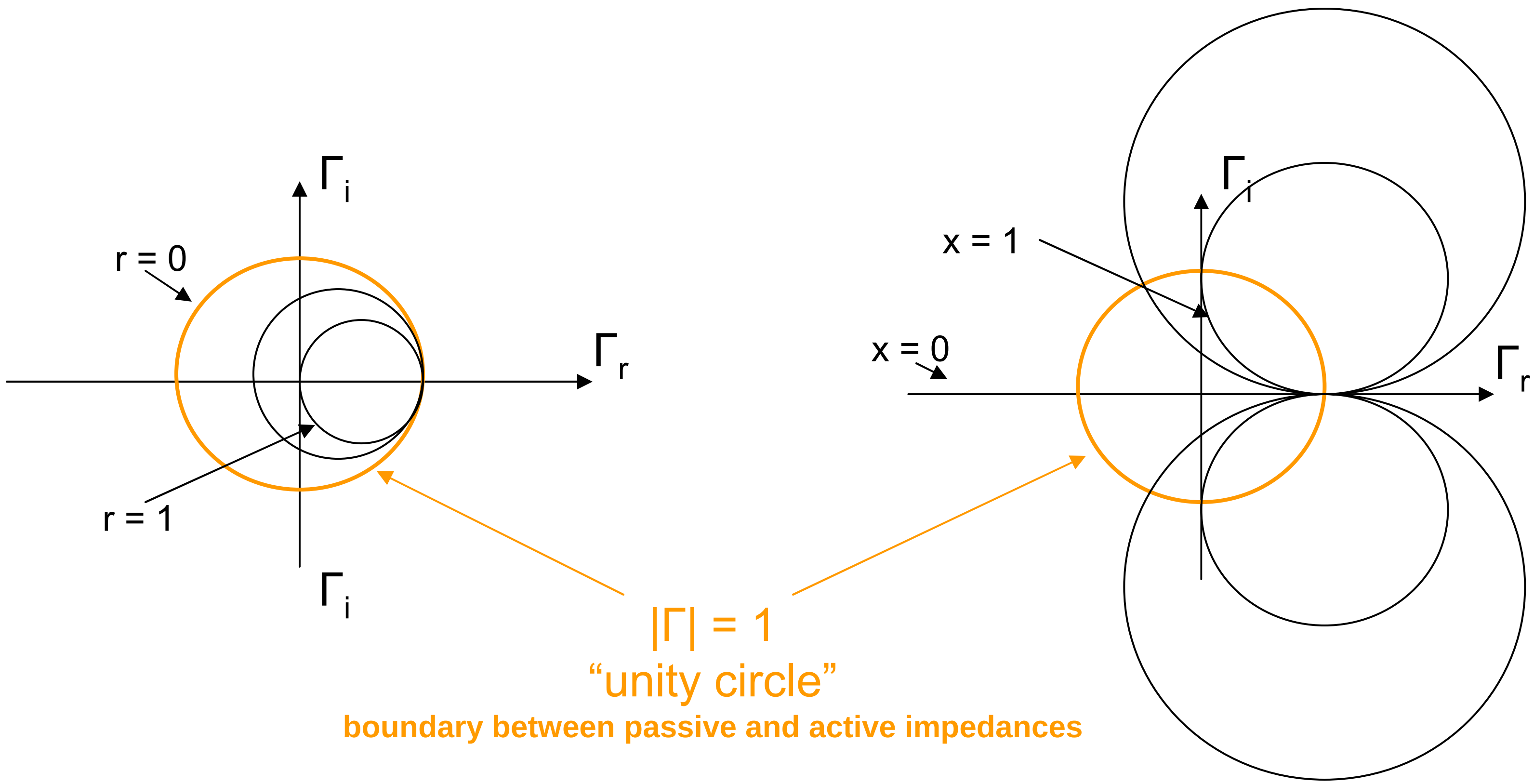
$r = (1-\Gamma_r^2-\Gamma_i^2)/[(1-\Gamma_r)^2+\Gamma_i^2)]$

$x= 2\Gamma_i/[(1- \Gamma_r)^2+\Gamma_i^2)]$

Smith chart "r" and "x" circles … cont.

Then, circles in the Γ-plane parameterized by "r" and "x" are:

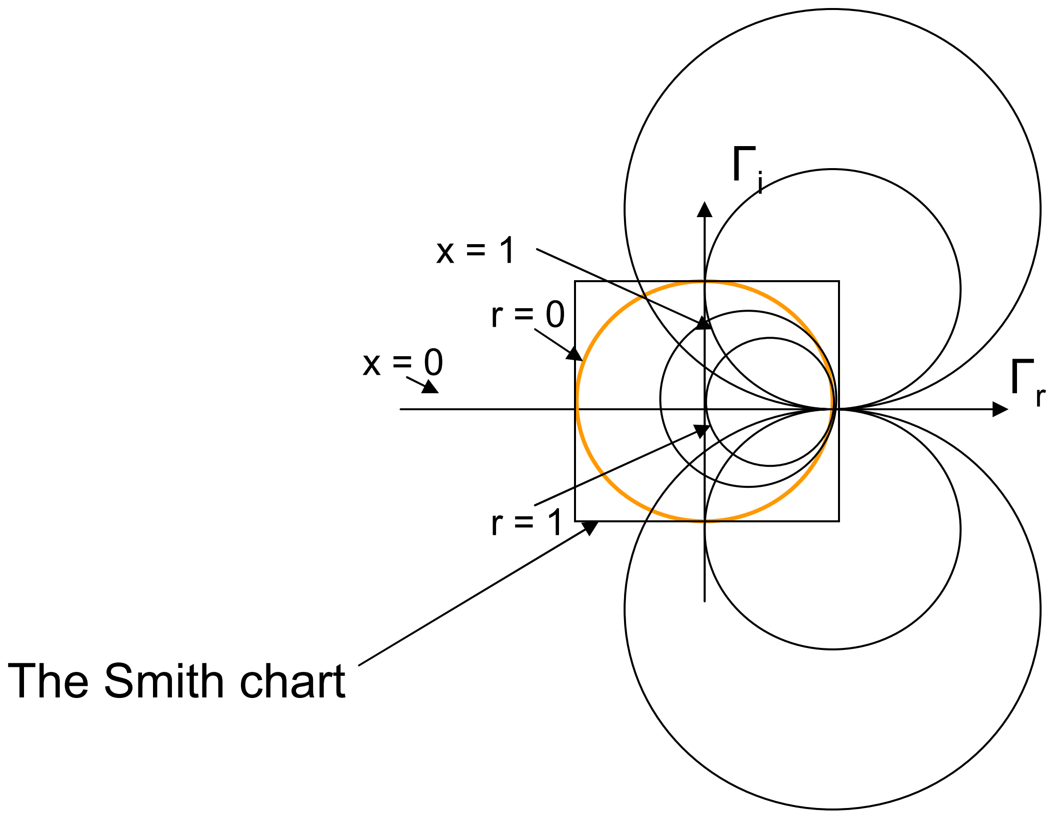
$[\Gamma_r - r/(r+1)]^2 + \Gamma_i^2 = [1/(1+r)]^2$ and $(\Gamma_r - 1)^2 + (\Gamma_i - 1/x)^2 = (1/x)^2$



Smith chart "r" and "x" circles … cont.

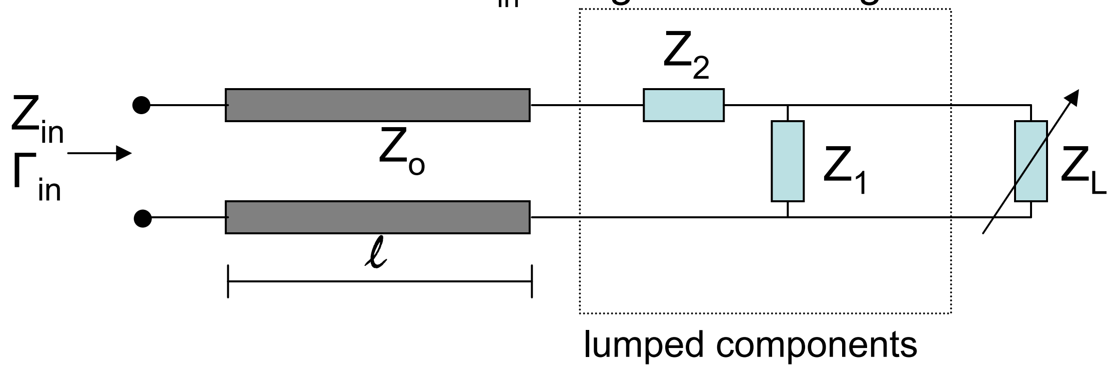
The Smith chart for $|\Gamma|$ less or equal to 1 (unity circle) represents all the passive impedances. Values of Γ higher than 1require power gain and represent impedances that can only be produced by active devices.

Overlaying Γ circles parameterized by "r" and "x" leads to the Smith chart below.



Example of Smith chart methods: solving a problem

An engineer makes measurements for Z_{in} using the following network



The lumped-components box has fixed impedances Z_1 and Z_2.
All these impedances are made of passive association of resistors, inductors and capacitors.
Z_L is a variable impedance created by moving a short along a lossless T.line stub.
A piece of lossless T.line of length, ℓ, transforms the impedance from the lumped component box to the Z_{in} impedance to be measured.

4 measurements were made for Z_{in} for 4 different values of Z_L, and they resulted:
Zin(1) = 0.290+j0.420
Zin(2) = 0.026+j0.174
Zin(3) = 0.130+j0.690
Zin(4) = 0.362-j0.045

When the engineer showed the 4 results to his manager, his manager replied one of the measurements was wrong. How can he tell and which is the wrong one?