EECS 217

Lecture 5: The Smith Chart

Prof. Niknejad

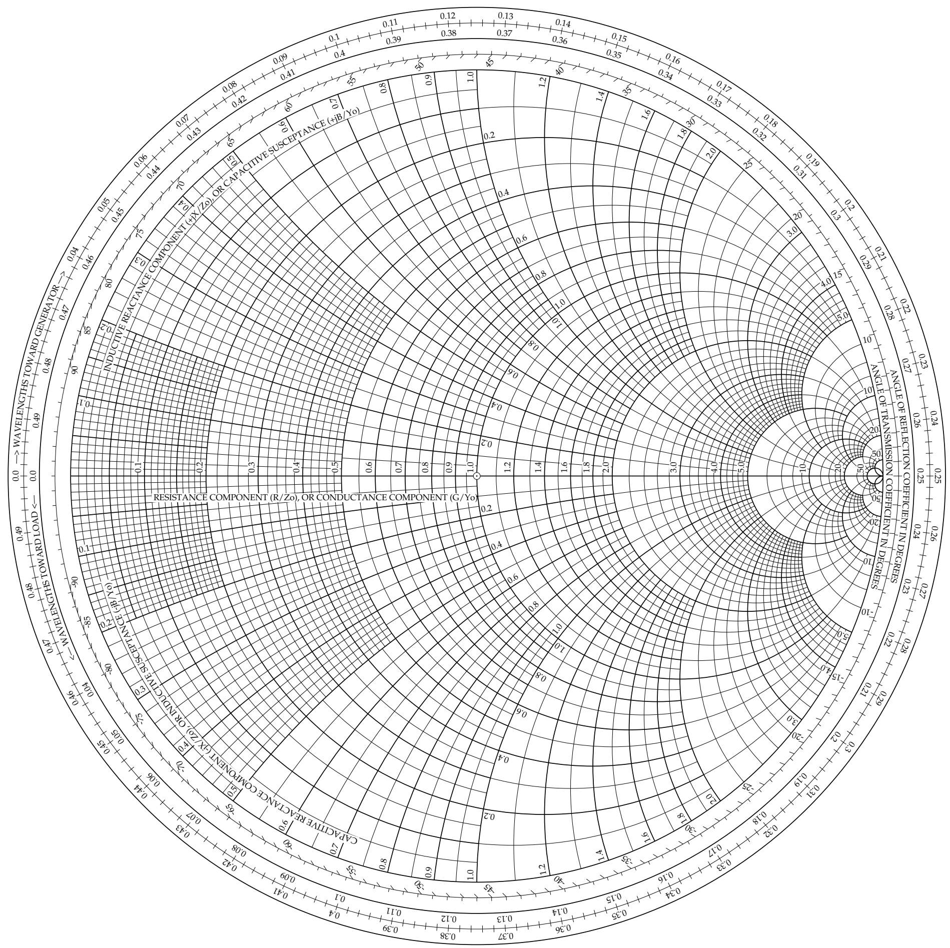
University of California, Berkeley

The Smith Chart

- The Smith Chart is simply a graphical calculator for computing impedance as a function of reflection coefficient $z = f(\rho)$
- More importantly, many problems can be easily visualized with the Smith Chart
- This visualization leads to a insight about the behavior of transmission lines
- All the knowledge is coherently and compactly represented by the Smith Chart
- Why else study the Smith Chart? It's beautiful!
- There are deep mathematical connections in the Smith Chart. It's the tip of the iceberg! Study complex analysis to learn more.

An Impedance Smith Chart

- Without further ado, here it is!



Generalized Reflection Coefficient

- In sinusoidal steady-state, the voltage on the line is a T-line

$$v(z) = v^+(z) + v^-(z) = V^+(e^{-\gamma z} + \rho_L e^{\gamma z})$$

- Recall that we can define the reflection coefficient anywhere by taking the ratio of the reflected wave to the forward wave

$$\rho(z) = \frac{v^-(z)}{v^+(z)} = \frac{\rho_L e^{\gamma z}}{e^{-\gamma z}} = \rho_L e^{2\gamma z}$$

- Therefore the impedance on the line ...

$$Z(z) = \frac{v^+ e^{-\gamma z}(1 + \rho_L e^{2\gamma z})}{\frac{v^+}{Z_0} e^{-\gamma z}(1 - \rho_L e^{2\gamma z})}$$

Normalized Impedance

- ...can be expressed in terms of $\rho(z)$

$$Z(z) = Z_0 \frac{1+\rho(z)}{1-\rho(z)}$$

- It is extremely fruitful to work with normalized impedance values $z = Z/Z_0$

$$z(z) = \frac{Z(z)}{Z_0} = \frac{1+\rho(z)}{1-\rho(z)}$$

- Let the normalized impedance be written as $z = r + jx$ (note small case)
- The reflection coefficient is "normalized" by default since for passive loads $|\rho| \le 1$. Let $\rho = u + jv$

Dissection of the Transformation

- Now simply equate the $\Re$ and $\Im$ components in the above equaiton

$$r + jx = \frac{(1+u) + jv}{(1-u) - jv} = \frac{((1+u+jv)(1-u+jv)}{(1-u)^2 + v^2}$$

- To obtain the relationship between the (r,x) plane and the (u,v) plane

$$r = \frac{1 - u^2 - v^2}{(1-u)^2 + v^2}$$

$$x = \frac{v(1-u) + v(1+u)}{(1-u)^2 + v^2}$$

- The above equations can be simplified and put into a nice form

Completing Your Squares...

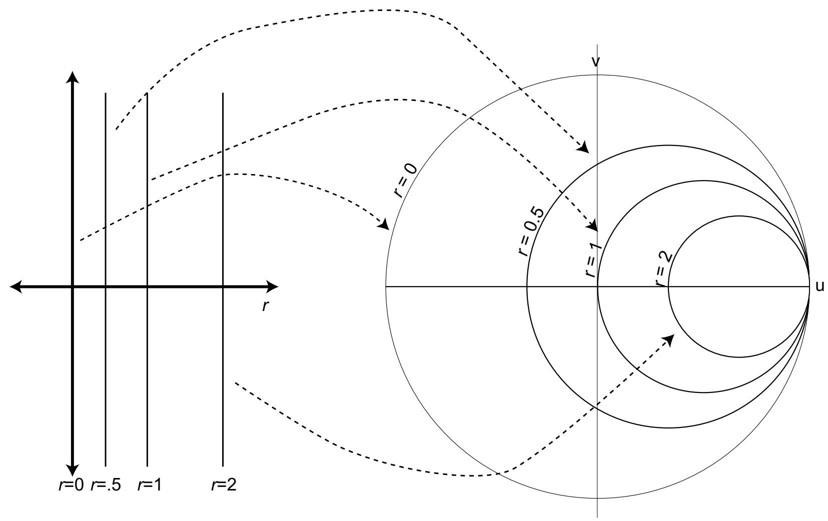
- If you remember your high school algebra, you can derive the following equivalent equations

$$\left(u - \frac{r}{1+r}\right)^2 + v^2 = \frac{1}{(1+r)^2}$$

$$(u-1)^2 + \left(v - \frac{1}{x}\right)^2 = \frac{1}{x^2}$$

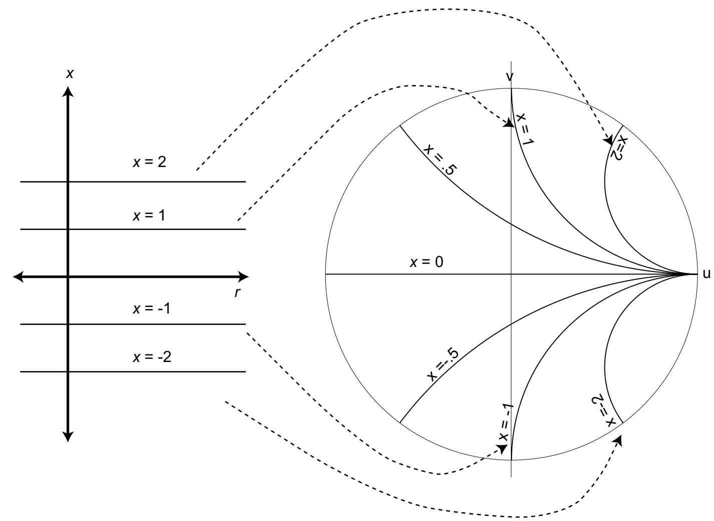
- These are circles in the (u,v) plane! Circles are good!
- We see that vertical and horizontal lines in the (r,x) plane (complex impedance plane) are transformed to circles in the (u,v) plane (complex reflection coefficient)

Resistance Transformations



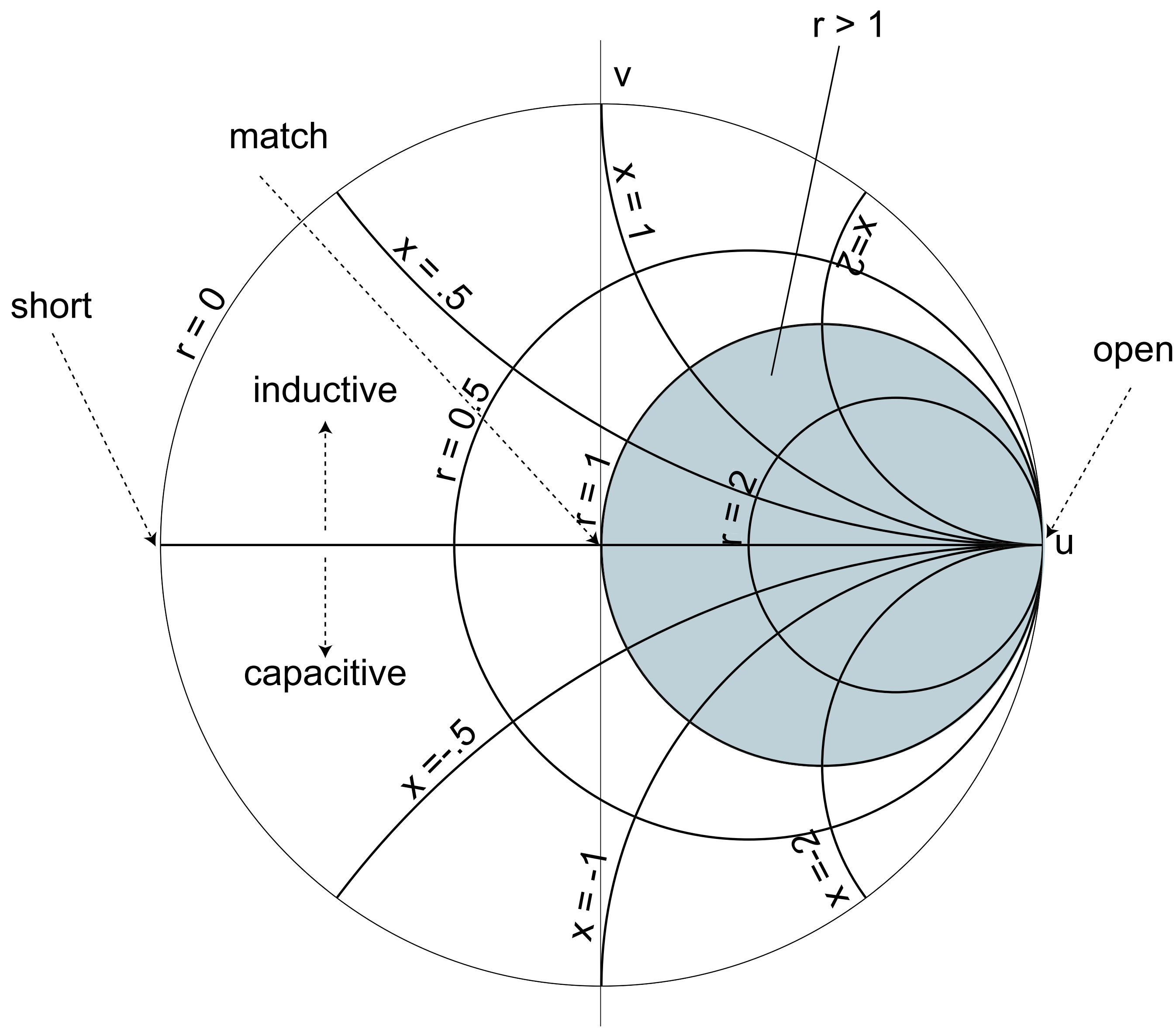
- $r = 0$ maps to $u^2 + v^2 = 1$ (unit circle)
- $r = 1$ maps to $(u - 1/2)^2 + v^2 = (1/2)^2$ (matched real part)
- $r = .5$ maps to $(u - 1/3)^2 + v^2 = (2/3)^2$ (load R less than Z_0)
- $r = 2$ maps to $(u - 2/3)^2 + v^2 = (1/3)^2$ (load R greater than Z_0)

Reactance Transformations

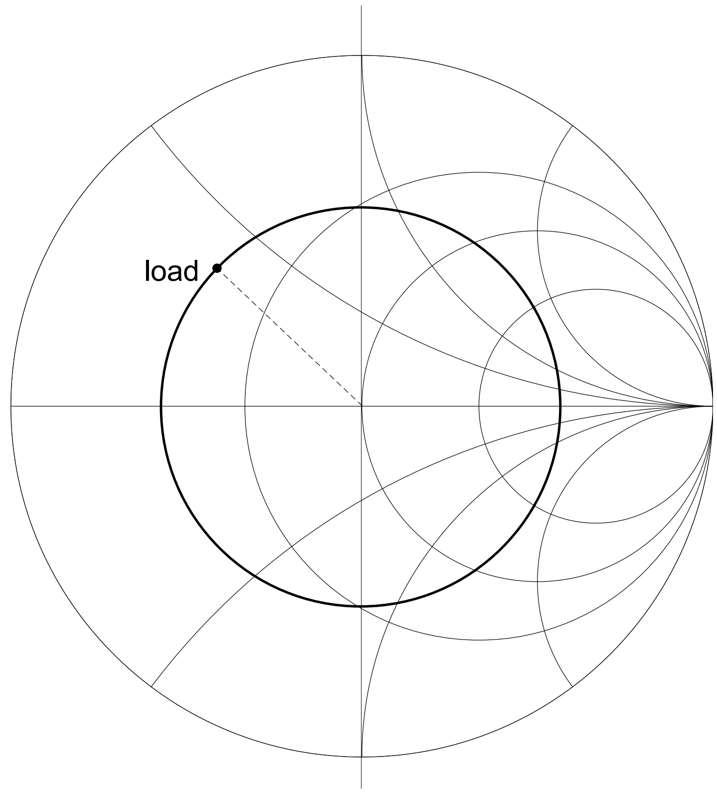


- $x = \pm 1$ maps to $(u-1)^2 + (v \mp 1)^2 = 1$
- $x = \pm 2$ maps to $(u-1)^2 + (v \mp 1/2)^2 = (1/2)^2$
- $x = \pm 1/2$ maps to $(u-1)^2 + (v \mp 2)^2 = 2^2$
- Inductive reactance maps to upper half of unit circle
- Capacitive reactance maps to lower half of unit circle

Complete Smith Chart



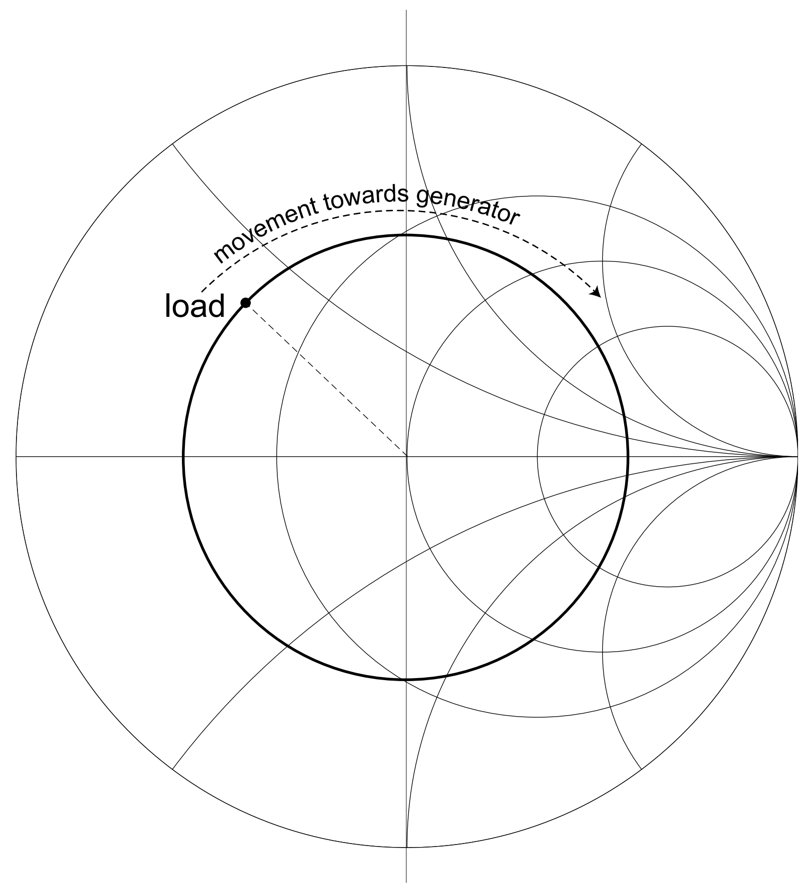
Load on Smith Chart



- First map z_L on the Smith Chart as ρ_L
- To read off the impedance on the T-line at any point on a lossless line, simply move on a circle of constant radius since $\rho(z) = \rho_L e^{2j\beta}$

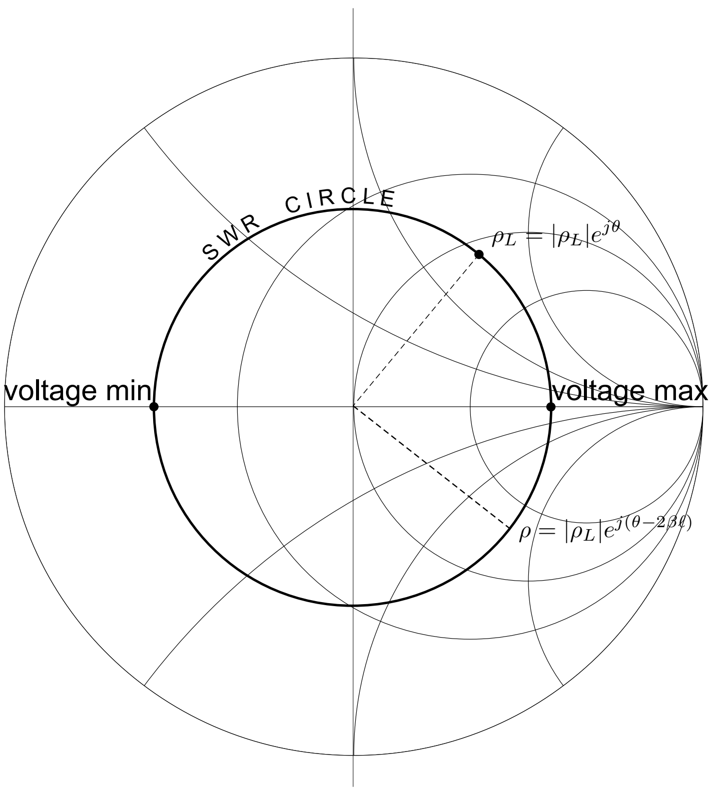
Motion Towards Generator

- Moving towards generator means $\rho(-\ell) = \rho_L e^{-2j\beta\ell}$, or clockwise motion
- For a lossy line, this corresponds to a spiral motion
- We're back to where we started when $2\beta\ell = 2\pi$, or $\ell = \lambda/2$
- Thus the impedance is periodic (as we know)



SWR Circle

Since SWR is a function of $|\rho|$, a circle at origin in (u,v) plane is called an SWR circle



- Recall the voltage max occurs when the reflected wave is in phase with the forward wave, so $\rho(z_{min}) = |\rho_L|$
- This corresponds to the intersection of the SWR circle with the positive real axis
- Likewise, the intersection with the negative real axis is the location of the voltge min

Example of Smith Chart Visualization

- Prove that if Z_L has an inductance reactance, then the position of the first voltage maximum occurs before the voltage minimum as we move towards the generator
- A visual proof is easy using Smith Chart
- On the Smith Chart start at any point in the upper half of the unit circle. Moving towards the generator corresponds to clockwise motion on a circle. Therefore we will always cross the positive real axis first and then the negative real axis.

Impedance Matching Example

- Single stub impedance matching is easy to do with the Smith Chart
- Simply find the intersection of the SWR circle with the $r = 1$ circle
- The match is at the center of the circle. Grab a reactance in series or shunt to move you there!

Admittance Chart

- Since $y = 1/z = \frac{1-\rho}{1+\rho}$, you can imagine that an Admittance Smith Chart looks very similar
- In fact everything is switched around a bit and you can buy or construct a combined admittance/impedance smith chart. You can also use an impedance chart for admittance if you simply map $x \rightarrow b$ and $r \rightarrow g$
- Be careful ... the caps are now on the top of the chart and the inductors on the bottom
- The short and open likewise swap positions

Admittance on Smith Chart

- Sometimes you may need to work with both impedances and admittances.
- This is easy on the Smith Chart due to the impedance inversion property of a $\lambda/4$ line

$$Z' = \frac{Z_0^2}{Z}$$

- If we normalize Z' we get y

$$\frac{Z'}{Z_0} = \frac{Z_0}{Z} = \frac{1}{z} = y$$

Admittance Conversion

- Thus if we simply rotate π degrees on the Smith Chart and read off the impedance, we're actually reading off the admittance!
- Rotating π degrees is easy. Simply draw a line through origin and z_L and read off the second point of intersection on the SWR circle