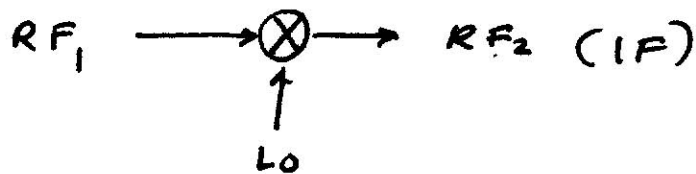


#24

MIXERS (STEPHEN MAAS)

A MIXER IS A FREQUENCY TRANSLATION DEVICE :



$$RF_1 = A_1 \cos(\omega_1 t)$$

$$LO = A_3 \cos(\omega_{LO} t)$$

$$RF_2 = RF_1 \otimes LO$$

$$= A_2 \cos(\omega_2 t)$$

$$\omega_2 = \omega_1 \pm \omega_{LO}$$

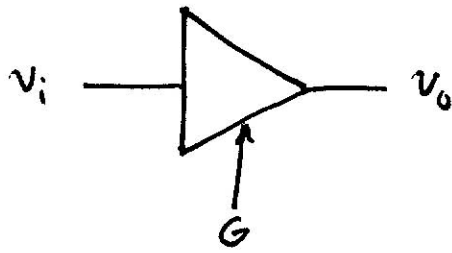
THIS IS NOT A LINEAR TIME-INVARIANT CIRCUIT!

A MULTIPLIER ACTS LIKE AN IDEAL MIXER:

$$V_M = A_1 A_2 \cos \omega_1 t \cdot \cos \omega_2 t \times G$$

$$= \frac{A_1 A_2 \times G}{2} \{ \cos(\omega_1 + \omega_2)t + \cos(\omega_1 - \omega_2)t \}$$

A TIME-VARYING CIRCUIT ALSO ACTS LIKE A MIXER



$$v_o = G \cdot v_i$$

FOR FIXED "G", CIRCUIT IS LINEAR

$$\text{LET } G = A_2 \cos \omega_2 t$$

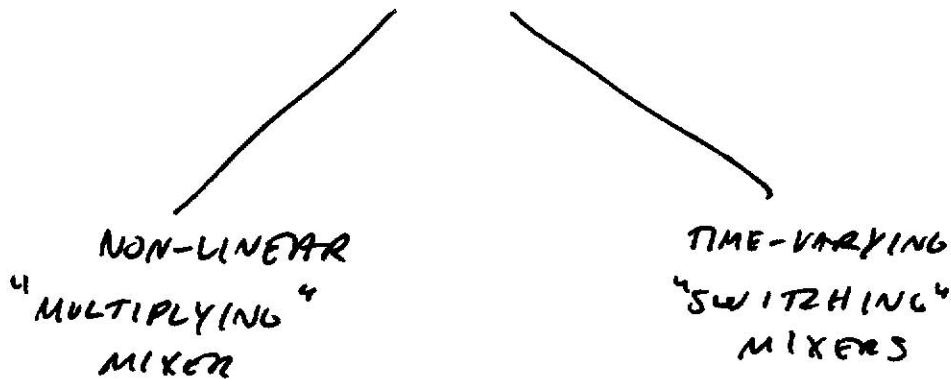
$$v_i = A_1 \cos \omega_1 t$$

$$v_o = G \cdot v_i = A_1 A_2 \cos \omega_1 t \cos \omega_2 t$$

→ ALSO A MIXER

A LINEAR TIME-VARYING CIRCUIT CAN BEET LIKE A MIXER.

TWO FAMILIES OF MIXERS:



NON-LINEAR MIXER : $y = f(x) = a_1 x + a_2 x^2 + \dots$

$$y_3 = f(x_1 + x_2) = a_1 (x_1 + x_2) + a_2 \underbrace{(x_1 + x_2)^2}_{\text{MIXING}} + \dots$$

$$= a_1 (x_1 + x_2) + a_2 (x_1^2 + x_2^2 + \underbrace{2x_1 x_2}_{\text{MIXING}}) + \dots$$

ANY DEVICE WITH SQ LAW CAN MIX

PERIODICALLY TIME-VARYING MIXER

$$v_o(t) = p(t) v_i(t)$$

$$p(t+T) = p(t)$$

$$= \sum_{n=-\infty}^{\infty} c_n e^{j\omega_0 n t} v_i(t)$$

$$c_n = \frac{1}{T} \int_0^T p(t) e^{-j\omega_0 n t} dt$$

$$v_i(t) = A(t) \cos \omega_1 t = A(t) \left(\frac{e^{j\omega_1 t} + e^{-j\omega_1 t}}{2} \right)$$

$$v_o(t) = A(t) \sum_{n=-\infty}^{\infty} c_n \frac{e^{j(\omega_0 n t + \omega_1 t)} + e^{j(\omega_0 n t - \omega_1 t)}}{2}$$

$$\swarrow \quad \frac{c_1}{2} e^{j\omega_0 t + \omega_1 t} + \frac{c_{-1}}{2} e^{-j(\omega_0 t - \omega_1 t)}$$

$$c_1 = c_{-1}$$

$$= c_1 \cos(\omega_0 t - \omega_1 t)$$

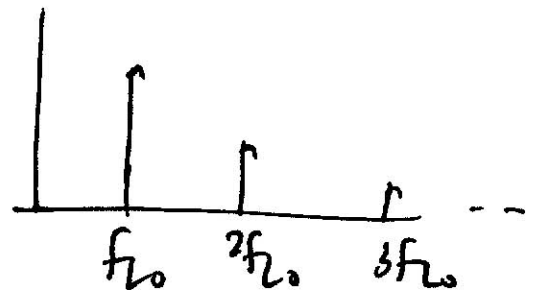
DESIRED SIGNAL ... FILTER
OUT UNWANTED SIGNALS

MULTIPLICATION IN TIME IS CONVOLUTION IN FREQ...

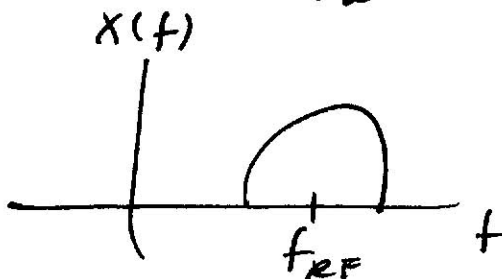
$$y(t) = p(t) x(t)$$

$$Y(f) = X(f) * P(f)$$

$$P(f) = \sum_{-\infty}^{\infty} C_n \delta(f - n f_{L0})$$

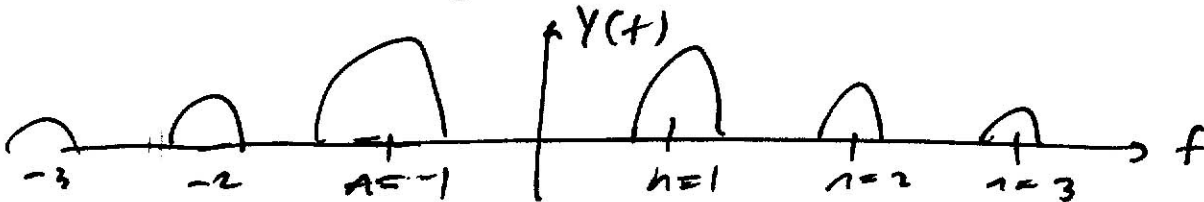


$$\begin{aligned} Y(f) &= \int_{-\infty}^{\infty} \sum_{-\infty}^{\infty} C_n \delta(\sigma - n f_{L0}) X(f - \sigma) d\sigma \\ &= \sum_{-\infty}^{\infty} C_n \left(\int_{-\infty}^{\infty} \delta(\sigma - n f_{L0}) X(f - \sigma) d\sigma \right) \\ &= \sum_{-\infty}^{\infty} C_n X(f - n f_{L0}) \end{aligned}$$



TRANSLATE SPECTRUM

$$f = f_{RF} + n f_{L0}$$



MIXER CHARACTERISTICS:

- CONVERSION GAIN (OR LOSS)
- NOISE FIGURE
- BANDWIDTH
- DISTORTION
- SPURIOUS RESPONSE

$$f_{IF} = m f_{RF} + n f_{LO} \quad (m, n)$$

- FEED THROUGH
- DC POWER
- LO / RF / IF ISOLATION (REJECTION)

SPURIOUS SIGNALS:

CONSIDER A DIODE MIXER

$$g(t) = \left. \frac{\partial I}{\partial V} \right|_{V=V_{LO}} = \frac{q I_0}{kT} e^{\frac{q V_{LO}}{kT}}$$

$$\approx \frac{q}{kT} I(V_{LO}(t))$$

$$I(V) = I_0 (e^{qV/kT} - 1)$$

$$g(t) = \sum_{k=-\infty}^{\infty} G_k e^{jk\omega_p t}$$

ω_p : LO FREQ

$$i(t) = \sum_{k=-\infty}^{\infty} I_k e^{j(k\omega_p + \omega_s)t} = g(t) V_s e^{j\omega_s t}$$

MIXING PRODUCT

BUT THE VOLTAGE WAVEFORM WILL ALSO HAVE FREQ COMPONENTS AT ALL MIXING PRODUCTS

$$\text{LET } \omega_o \equiv |\omega_s - \omega_p|$$

$$i(t) = \sum_{k=-\infty}^{\infty} I_k e^{j(k\omega_p + \omega_0)t}$$

$$v(t) = \sum_{k=-\infty}^{\infty} V_k e^{j(k\omega_p + \omega_0)t}$$

$$g(t) = \sum_{k=-\infty}^{\infty} G_k e^{jk\omega_p t}$$

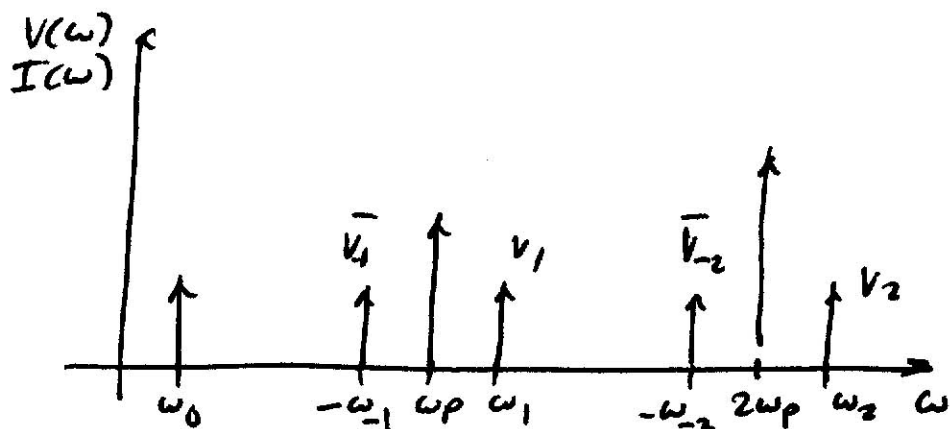
$$i(t) = v(t)g(t)$$

$$\sum_{l=-\infty}^{\infty} I_l e^{j(\omega_0 + l\omega_p)t} = \sum_m \sum_n G_m V_n e^{j(\omega_0 + (n+m)\omega_p)t}$$

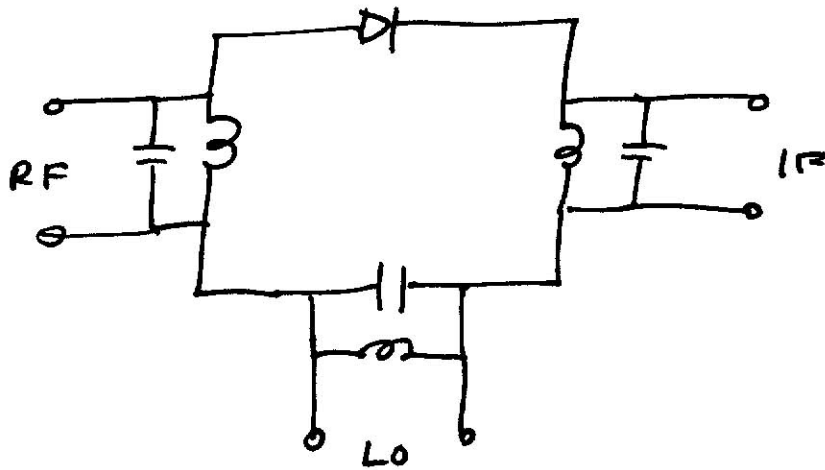
$$\begin{pmatrix} \bar{I}_{-N} \\ \bar{I}_{-N+1} \\ \vdots \\ \bar{I}_{-1} \\ I_0 \\ I_1 \\ \vdots \\ I_N \end{pmatrix} = \begin{pmatrix} G_0 & G_{-1} & G_{-2} & \dots & G_{-2N} \\ G_1 & G_0 & & & G_{-2N+1} \\ & & \ddots & & \vdots \\ & & & \ddots & \vdots \\ G_{N-1} & & & & \vdots \\ G_N & & & & \vdots \\ \vdots & & & & G_{-1} \\ & & & & G_0 \\ G_{2N} & & & & \vdots \end{pmatrix} \begin{pmatrix} \bar{V}_{-N} \\ \vdots \\ \vdots \\ V_0 \\ V_1 \\ \vdots \\ V_N \end{pmatrix}$$

$$\omega_n = n\omega_p + \omega_0$$

$$\omega_{-n} = -\omega_n$$



SINGLE DIODE MIXER



IDEAL "LC" IS A SHORT AT FREQ ~~FOR REMOVED~~ ^{AWAY} FROM RESONANCE. B/C OF THIS ALL PORTS

ISOLATED :

NO LO FEEDTHRU TO IF
NO IF FEEDTHRU TO IF
NO LO FEEDTHRU TO RF

;

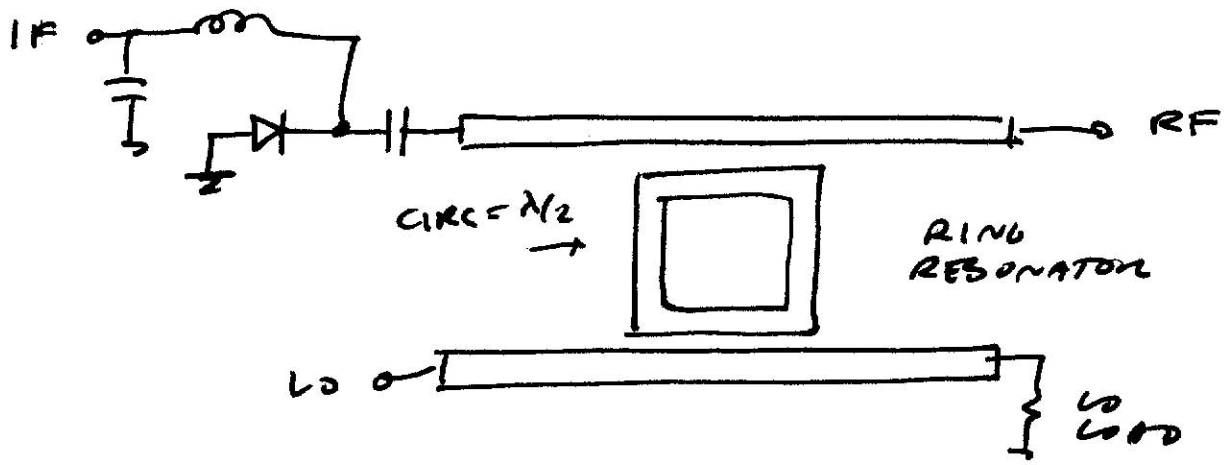
SINCE DIODE IS SHORT CIRCUITED AT ALL FREQ EXCEPT RF, LO, IF

$$\begin{pmatrix} I_{IF} \\ I_{RF} \end{pmatrix} = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} \begin{pmatrix} V_{IF} \\ V_{RF} \end{pmatrix}$$

↖
ALL OTHER FREQ PRODUCTS SHORTED

DIODE CAN BE CHOSEN TO PROVIDE MATCH AT ALL PORTS

PRACTICAL IMPLEMENTATION:



RING COUPLES POWER FROM L0 TO RF LINE. SINCE DIODE IS MATCHED, RF POINT DOES NOT SEE A REFLECTION. RF IS ISOLATED FROM L0 DUE TO NARROW RESONANT FREQ

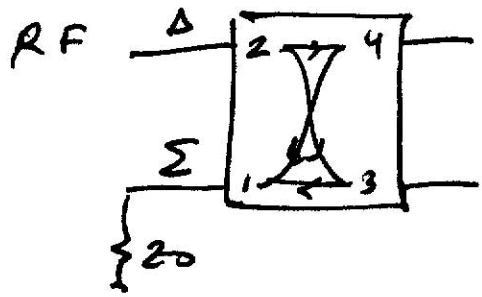
BALANCED DIODE MIXERS

USE BALUN OR HYBRIDS

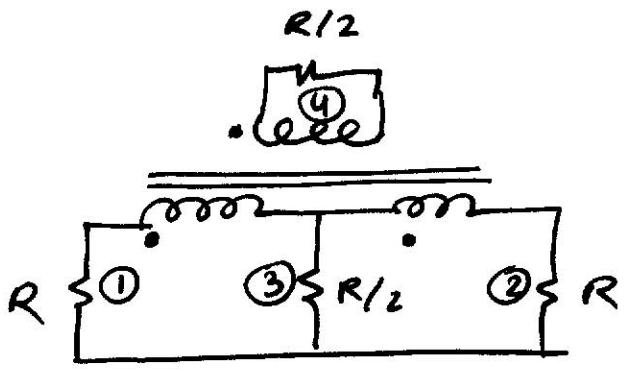
RECALL:

$$S_{90} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & -j & 1 \\ 0 & 0 & 1 & -j \\ -j & 1 & 0 & 0 \\ 1 & -j & 0 & 0 \end{pmatrix}$$

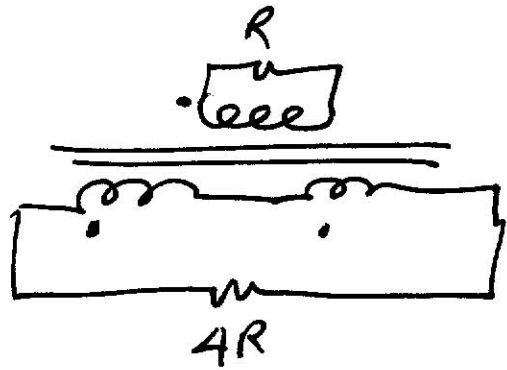
$$S_{180} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \\ 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{pmatrix}$$



TRANSFORMERS AS HYBRIDS / BALUNS

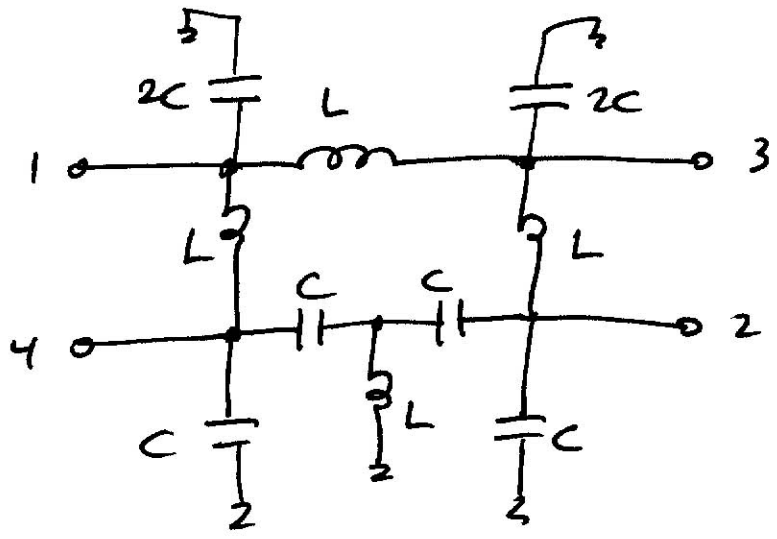


HYBRID



BALUN

LUMPED ELEMENT HYBRID

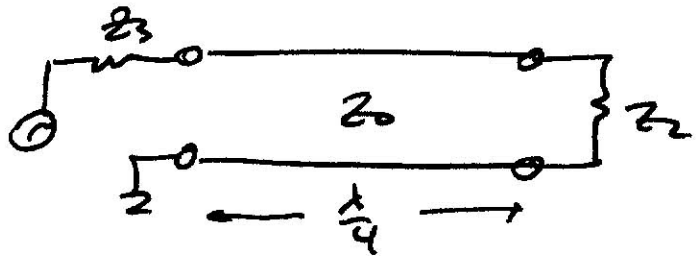


$2C \sim 20\%$

180 HYBRID

$$\omega L = \frac{1}{\omega C} = \sqrt{2} Z_0$$

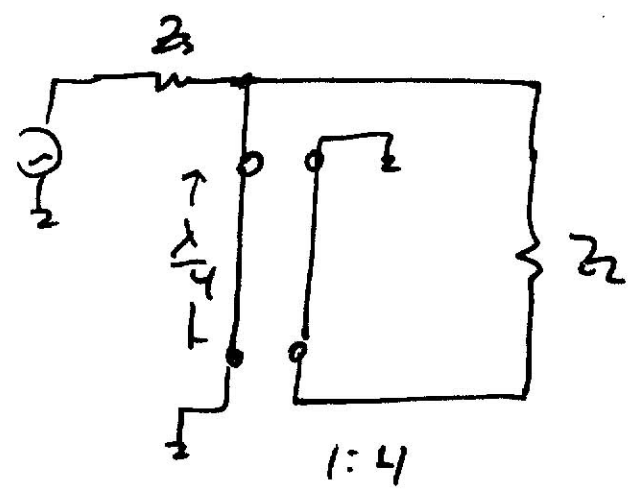
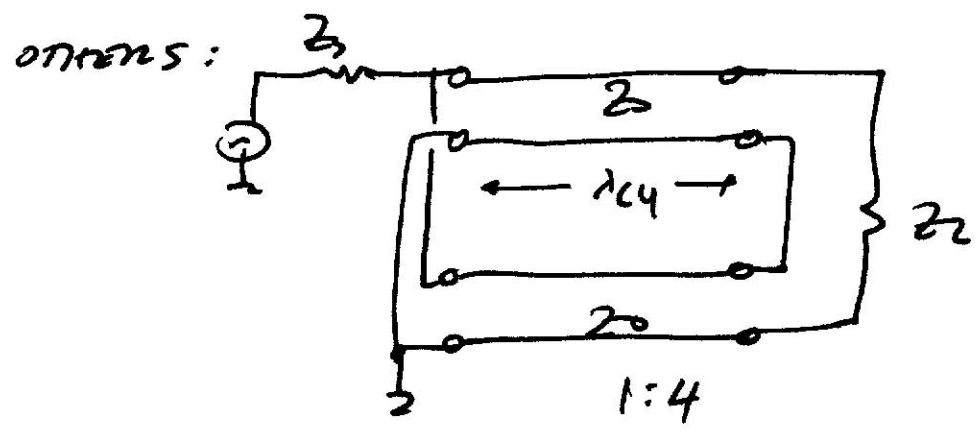
T-LINE BALUNS



$$Z_0 = \sqrt{Z_1 Z_2}$$

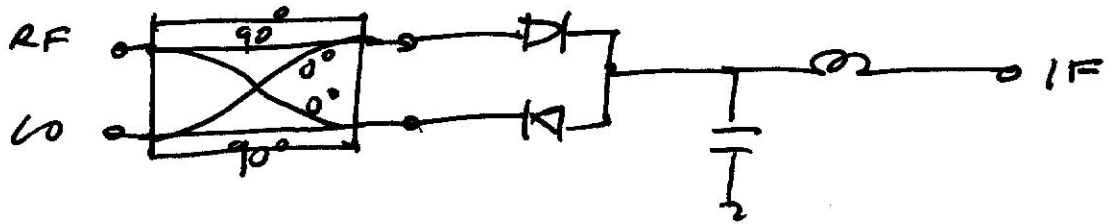
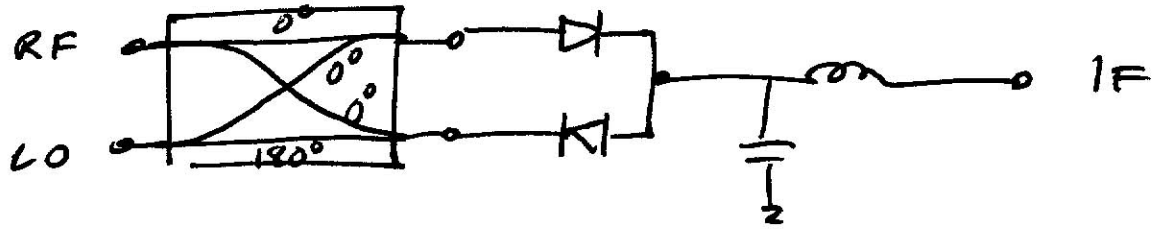
$$Z_{00} = \frac{1}{2} Z_0$$

$Z_{00} \approx 10 Z_{00}$
FOR GOOD PERF



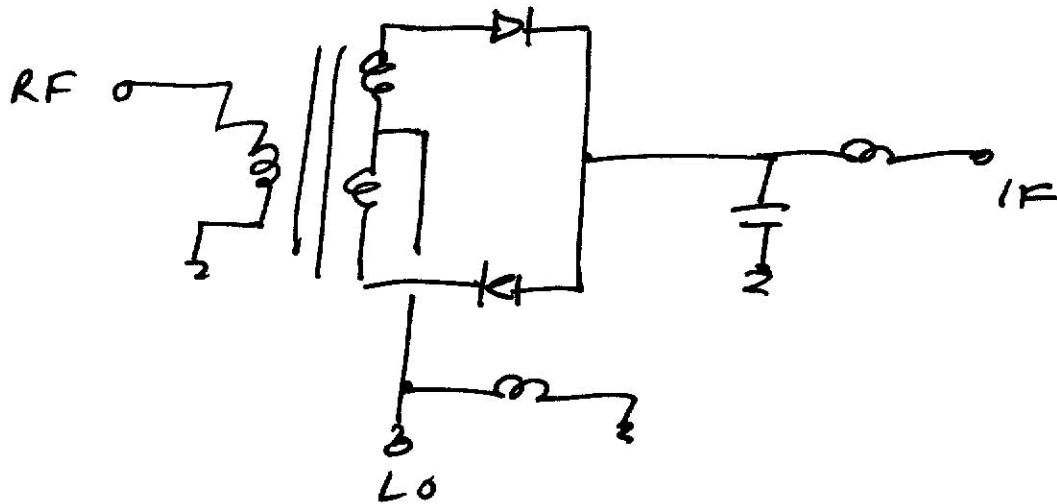
CONVENIENT GROUND
RETURN

SINGLY BALANCED DIODE MIXERS



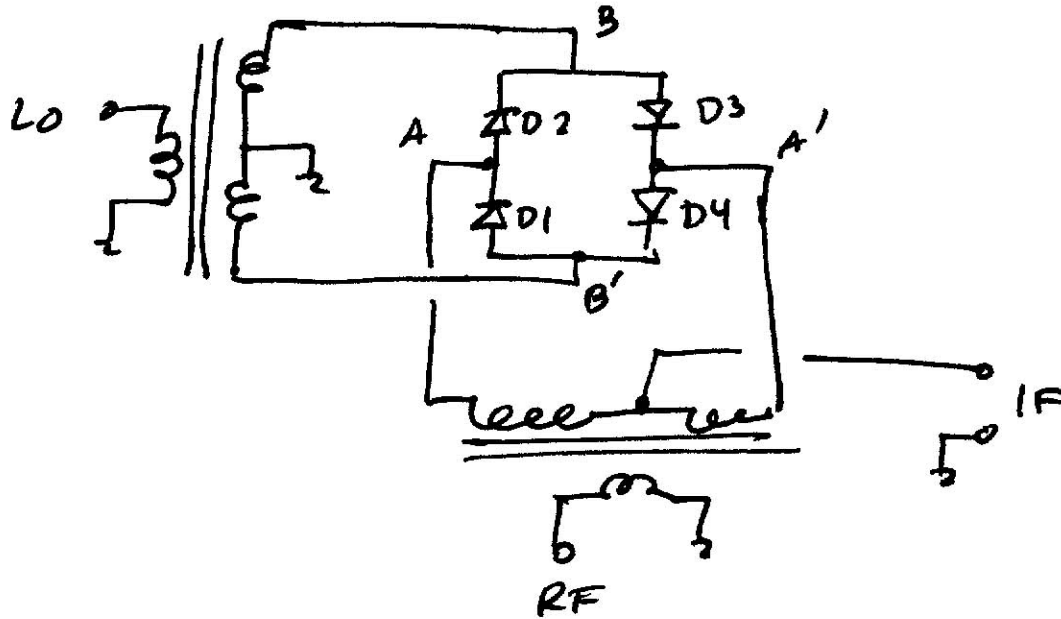
- BETTER RF & LO ISOLATION
- REJECTION OF SPURIOUS SIGNALS
- REJECTION OF AM NOISE FROM LO

TRANSFORMER REALIZATION WITH 180° HYBRID



DOUBLY BALANCED MIXERS

- REJECT ALL EVEN ORDER SPURIOUS RESPONSES
- IF, RF, LO ALL ISOLATED



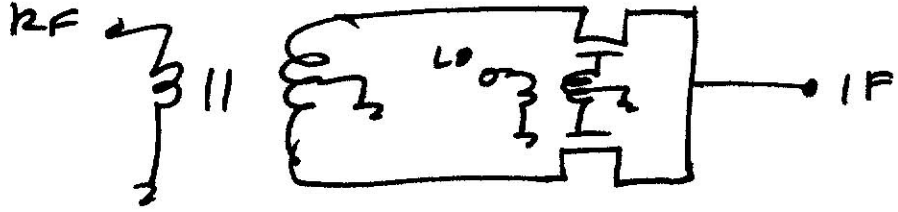
AA' VIRTUAL GROUNDS FOR LO
 BB' VIRTUAL GROUNDS FOR RF

LO+	B+, B'-	D1 & D2	OFF	OFF
		D3 & D4	ON	ON
LO-	B-, B'+	D3 & D4	OFF	ON
		D1 & D2	ON	OFF

RF PART IS COUPLED TO IF PORT WITH CHANGING POLARITY

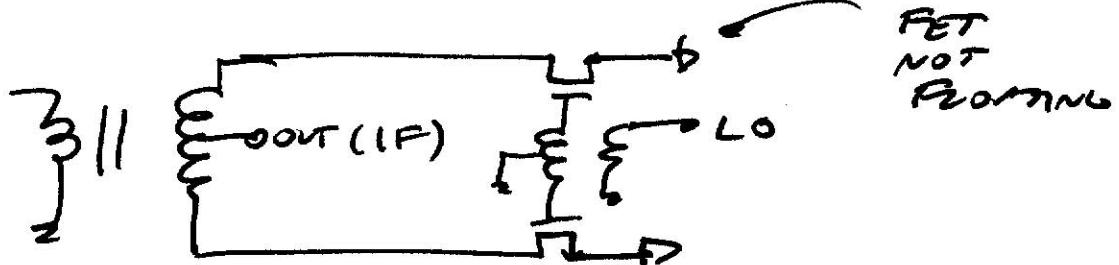
$$V_{IF} = S(t) V_{RF}(t)$$

MOS REALIZATION

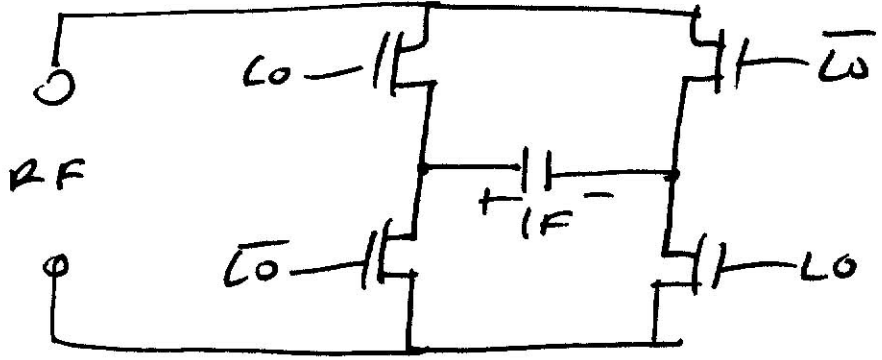


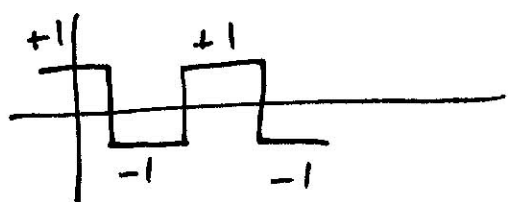
MULTIPLY RF BY ± 1

BETTER ...



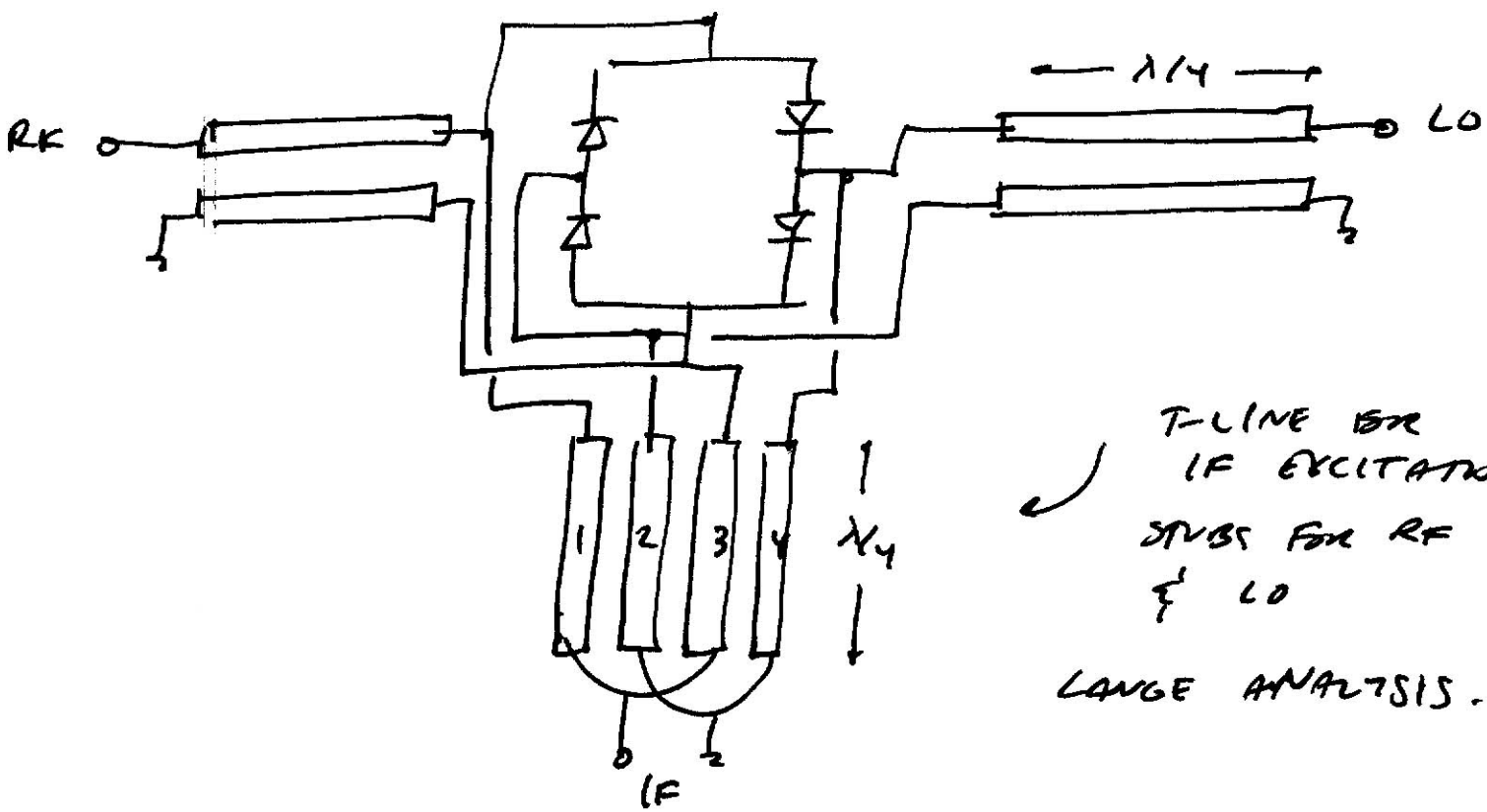
FULLY BALANCED : VERY SIMPLE MIXER





- $S(f)$ HAS NO EVEN HARMONICS
- $S(f)$ HAS NO DC COMPONENT: IF CONTAINS NO RF
- MIXING WITH EVEN COMPONENTS OF RF CANNOT OCCUR
- IF CURRENT EXCITED TRANSFORMERS (IN EVEN MODE \rightarrow NO IF VOLTAGE AT SECONDARIES)

MICROWAVE REALIZATION:



IF BAND CAN OVERLAP W/ RF/LO BANDS