EECS 217

Lecture 20: Two-Port Noise

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Noise Figure Review

- Recall that the noise figure of a two-port is defined by

$$F = \frac{P_{N_s} + P_{N_a}}{P_{N_s}} = 1 + \frac{P_{N_a}}{P_{N_s}}$$

- In the above equation P_{N_s} is the noise due to the source resistance whereas P_{N_a} is the noise added to the signal by the amplifier. Since $P_{N_a} \geq 0$ for any two-port, the noise figure $F \geq 1$.

- A noiseless system has $F = 1$. For instance, a two-port consisting of ideal L's and C's (lossless) has an $F = 1$.

- If a *matched* two-port has loss, then the noise figure is equal to the loss. To see this, we use the following equivalent formulation

$$F = \frac{P_{sig}(P_{N_s} + P_{N_a})}{P_{sig}P_{N_s}} = \frac{P_{sig}}{P_{N_s}} \frac{P_{N_s} + P_{N_a}}{P_{sig}} = \frac{SNR_i}{SNR_o}$$

- Thus we recall that $F(\,\mathrm{dB})$ can also be interpreted as the *decrease* in SNR as a result of the two-port.

- Thus, for a constant noise floor (or equal system resistance) any reduction in the signal adds to the F dB for dB.

Noise Figure and Input/Output Reference

$$P_{sig} + P_{N,s} \longrightarrow \boxed{+} \longrightarrow \triangleright G$$

$$P_{N_a}$$
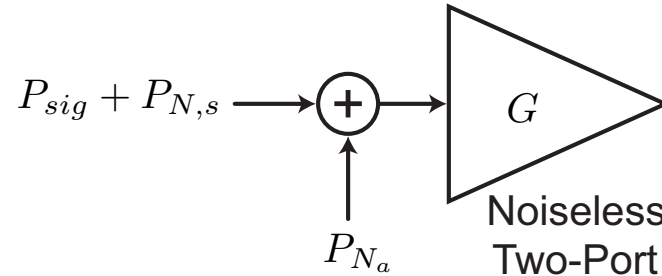
Noiseless
Two-Port

- While the noise of a two-port originates from *inside* the two-port, it's convenient to pretend that the two-port is noiseless and to imagine that the noise added to either the input or or output of the amplifier.

- The input referred noise is more commonly used. Also, the noise power is used in the above equations, not the noise voltage or current. From circuit theory, we can take any two-port and simplify it to a noiseless two-port with an input referred voltage and current. In general, the voltage and current are correlated.

- The total input noise can be derived from the definition of F

$$F - 1 = \frac{P_{N_a}}{P_{N_s}}$$

$$P_{N_a} = P_{N_s}(F - 1)$$

Cascade Noise Figure
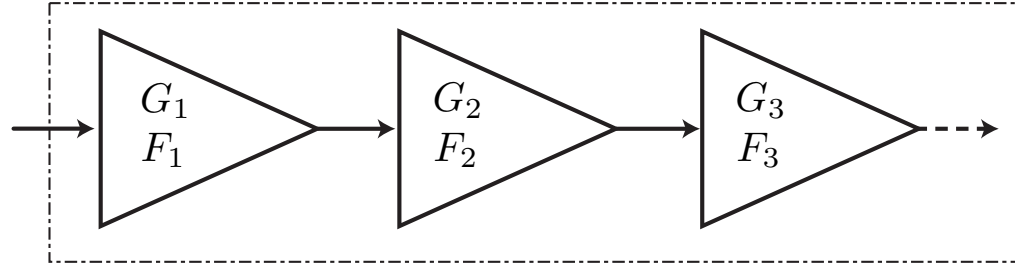
- If several amplifier stages or two-ports are connected in cascade, we wish to know the overall noise figure. We can use the concept of a noiseless two-port cascade and simply input refer all the noise sources.

- The noise from the second stage can be input refereed if the power gain is known and the stages are matched

$$P_{N_{ia,2}} = \frac{P_{N_s}(F_2 - 1)}{G_1}$$

- The noise from the k'th stage can be similarly input referred

$$P_{N_{ia,k}} = \frac{P_{N_s}(F_k - 1)}{G_1 \cdot G_2 \cdots G_{k-1}}$$

Cascade Noise Figure (cont)

- Therefore the total noise figure is given by

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \cdots$$

- The above formula is very important as it highlights the importance in the first stage of the cascade. The overall noise figure of the cascade is bounded by the noise of the first stage whereas the noise figure of other stages is divided by the gain preceding the stage.

- The first stage amplifier, or the *low noise amplifier* (LNA), is thus one of the most important blocks in a microwave receiver. The noise of the first this stage must be as low as possible and the gain as high as possible in order to reject the subsequent noise in the system, especially the mixer (which tends to have a high noise figure).

- Any attenuation before the LNA must be minimized (cables, mismatch, loss in filters, switches, and diplexers).

Noise Voltage/Current
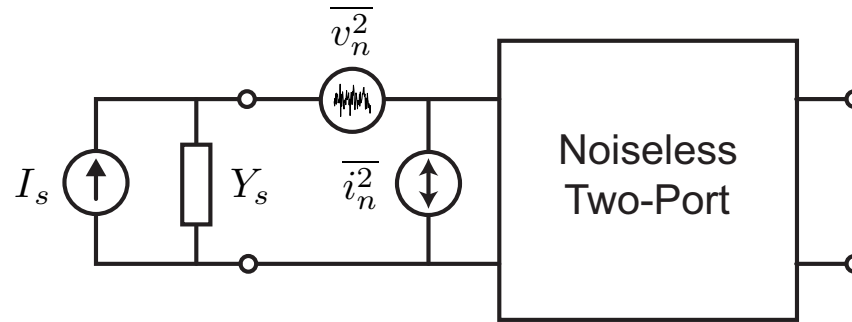
- Let's calculate the noise figure of a general two-port shown above. The total input current into the device is easily calculated by superposition

$$i_i = \frac{Y_{in}}{Y_{in} + Y_s} i_s + \frac{Y_{in}}{Y_{In} + Y_s} i_n + \frac{1}{Z_{in} + Z_s} v_n$$

- Here we have added noise currents and voltages as if they were deterministic signals. We need to exercise caution when computing the power entering the two by carefully considering the correlation among the sources.

- The above expression can be simplified by noting that

$$\frac{1}{Z_{in} + Z_s} v_n = \frac{Y_{in}}{Y_s + Y_{in}} Y_s v_n$$

Input Noise Power

- The total noise current is thus given by

$$i_i = \frac{Y_{in}}{Y_{in} + Y_s}(i_s + i_n + Y_s v_n)$$

- Now the total input RMS power is proportional to $\overline{i_i^2}\Re(Z_{in})$, and the ratio of powers can be written simply

$$F = \frac{\overline{i_i^2}}{\overline{i_i^2}\Big|_{i_n=0,v_n=0}} = 1 + \frac{\overline{(i_n + Y_s v_n)^2}}{\overline{i_s^2}}$$

- The last equality follows from the fact that the source noise is independent of the two-port noise.

- It's fruitful to now express the input current i_n as a sum of two part, a part uncorrelated from v_n called i_u and a part that is correlated i_c

$$i_n = i_u + Y_C v_n$$

- The admittance Y_C is called the correlation admittance.

Two-Port Noise Figure (cont)

- With the above substitution, we have

$$= 1 + \frac{\overline{(i_u + (Y_C + Y_s)v_n)^2}}{\overline{i_s^2}}$$

- But now by the fact that the numerator terms are independent by design

$$= 1 + \frac{\overline{i_u^2} + |Y_C + Y_s|^2\overline{v_n^2}}{\overline{i_s^2}}$$

- Let $v_n = 4kTR_n$ and $i_u = 4kTG_n$ so that we can express the above in the following form

$$F = 1 + \frac{G_n}{G_s} + \frac{R_n}{R_s}|Y_C + Y_s|^2$$

$$= 1 + \frac{G_n}{G_s} + \frac{R_n}{R_s}((G_C + G_s)^2 + (B_C + B_s)^2)$$

Noise Optimum Source Admittance

- For a power match we know that the optimum source impedance is given by Y_{in}^*. But as we shall see, this is not necessarily the optimum source impedance if we wish to minimize the noise figure.

- The optimum is easily found by taking partials of F. In particular note that the minimum F as a function of B_s is given by $-B_C$. In terms of G_s

$$\frac{\partial F}{\partial G_s} = -\frac{G_u}{G_s^2} + \frac{R_n}{G_s} 2(G_C + G_s) - \frac{(G_C + G_s)^2 R_n}{G_s^2} = 0$$

- The solution exists can be shown to correspond to a minima

$$G_{s,opt} = \sqrt{G_C^2 + \frac{G_u}{R_n}}$$

$$B_{s,opt} = -B_C$$

- Note that in general $Y_{S,opt} \neq Y_{in}^*$ and thus the noise match does not correspond to the gain match, and thus a compromise is necessary to find the best performance.

F_{min}

- The minimum achievable noise figure is given by

$$F_{min} = 1 + 2G_C R_n + 2\sqrt{R_n G_u + (R_n G_C)^2}$$

- Note that F_{min} can be attained with a unique choice $Y_{s,opt}$. With some algebraic manipulations, it can be shown that for any other source admittance

$$F = F_{min} + \frac{R_n}{G_s}\left|Y_s - Y_{s,opt}\right|^2$$

- The noise figure increases proportional to the distance squared between the optimum source impedance and a given source impedance. The proportionality factor is a property of the transistor R_n/G_s. A transistor with a small R_n is thus desirable.

- Also note that at a single frequency, the two-port noise property of a device is completely characterized by four numbers, $Y_{s,opt}$, R_n and F_{min}.