EECS 217

Lecture 1: Introduction to Microwave Circuits

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What Are Microwave Circuits?

- First we must understand where circuit theory comes from. Crudely speaking, circuit theory is an approximation to Maxwell's Eq. valid when structure dimensions are small relative to the wavelength (at the highest frequency of interest).
- Alternatively, circuit theory is valid when the speed of light is infinite $c \to \infty$.
- Ex: At $f = 60\,\mathrm{Hz}$, we have $\lambda = \frac{c}{f} = \frac{3\times10^8}{60} = 0.5 \times 10^7$

  If we arbitrarily require that the dimension be a factor of a thousand smaller than the wavelength, we have

$$\frac{\ell}{\lambda} = 10^{-3} \to \ell = 5\,\mathrm{km}$$

Circuits at GHz Frequencies

- Now let's consider $f = 1\,\mathrm{GHz}$. This corresponds to the popular cellular bands. Now $\lambda = c/f = 30\,\mathrm{cm}$, so using the same requirement we have

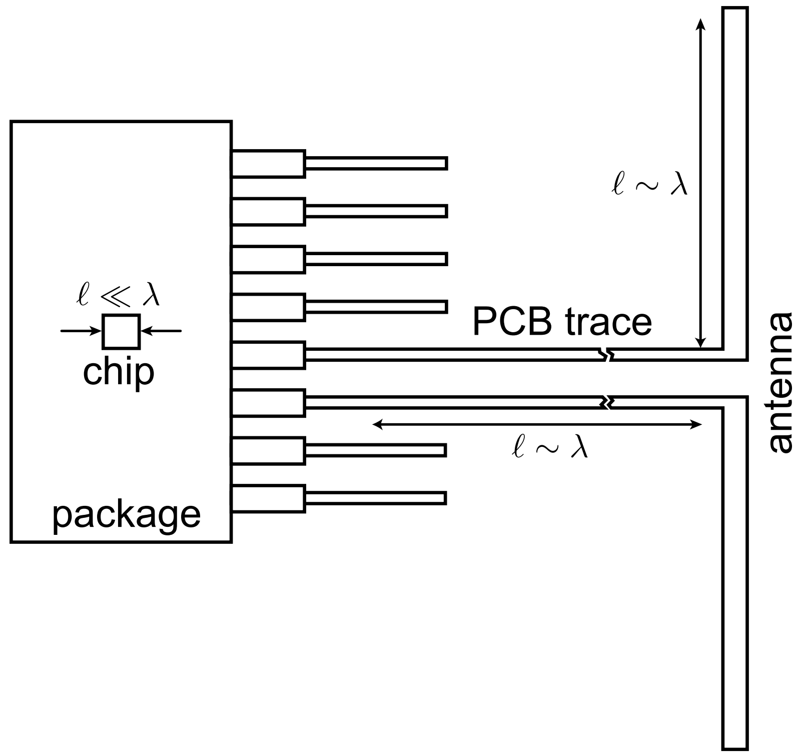
$$\frac{\ell}{\lambda} = 10^{-3} \rightarrow \ell = 0.3\,\mathrm{mm}$$

- This is a lot more restrictive! We see that this is strictly only valid for relatively small structures on the Si chip. So inside a small transistor with a dimension of tens of microns, certainly circuit theory is valid at this frequency.
- But recall that $\lambda = v/f = c/\sqrt{\epsilon\mu}f$, so inside the Si substrate the wavelength for TEM waves drops by roughly $\sqrt{12}$.

Microwave Circuit Theory

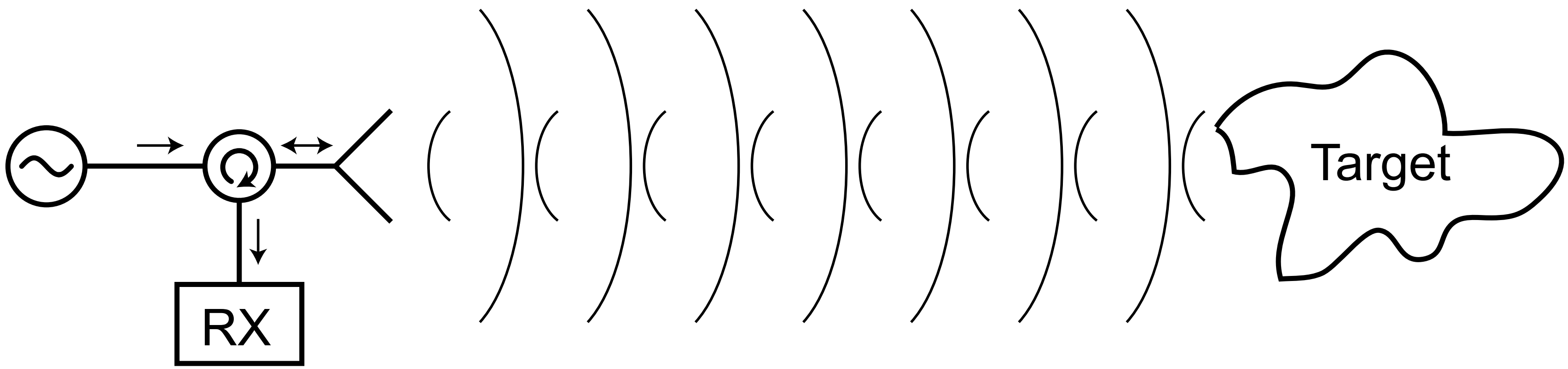
- Microwave circuit theory is an extension of circuit theory to higher frequencies where the circuit dimensions approach the wavelength, $\ell \sim \lambda$.
- We need this theory in order to avoid solving Maxwell's Equations!
- We can also use our intuition and experience from circuit theory (e.g. lumped filter design) and apply it to higher frequencies. We have to be careful in applying our inuition.
- For instance, a transmission line of lenght $\lambda/4$ converts an open circuit termination into a short circuit! This behavior is very counterintuitive from a lumped circuit theory perspective.

Chip/Package/Board



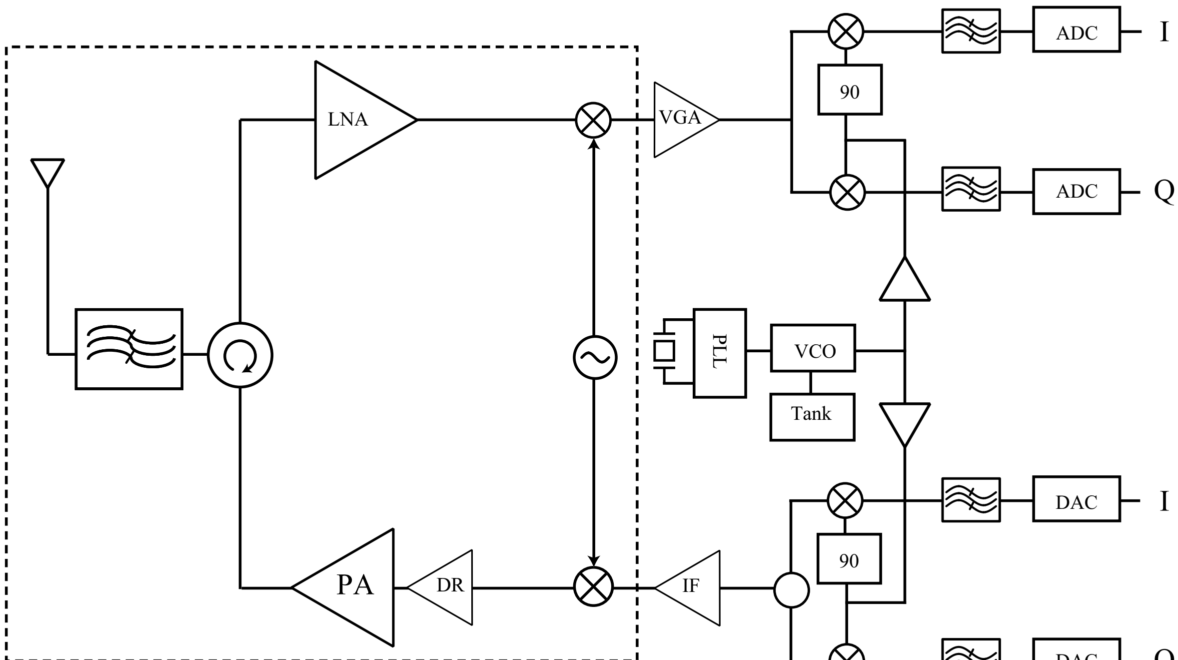
- In the above example, the structures “on-chip” may behave like lumped elements (transistors, inductors, capacitors, etc.). The leads, board traces, and radiation structures, though, are “large” relative to the wavelength and require Microwave Theory.

Microwave Radar



- Radar, invented by Sir Robert Watson-Watt in 1935, and developed at MRL during WWII ('40 - '45), allows us to detect distant objects by observing the microwave scattering from a target.
- In this course we'll learn to build the basic active and passive building blocks, such as the oscillators, amplifiers, mixers, and circulator.

Radio Block Diagram



Microwave/RF Circuits

- The block diagram above is a typical super-heterodyne transceiver architecture. The LNA (low-noise amplifier), the PA (power amplifier), the LO (local oscillator), and mixers all operate at the "carrier" frequency. We will learn how to design such building blocks operating close to the limits of technology ($10\,\mathrm{GHz} - 100's\,\mathrm{GHz}$).

Maxwell's Equations

- We begin with Maxwell's famous equations:

$$\oint_S \mathbf{D} \cdot d\mathbf{S} = \int_V \rho dV \qquad \nabla \cdot \mathbf{D} = \rho$$

$$\oint_S \mathbf{B} \cdot d\mathbf{S} = 0 \qquad \nabla \cdot \mathbf{B} = 0$$

$$\oint_C \mathbf{E} \cdot d\ell = -\frac{\partial}{\partial t} \int_S \mathbf{B} \cdot d\mathbf{S} \qquad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\oint_C \mathbf{H} \cdot d\ell = \frac{\partial}{\partial t} \int_S \mathbf{D} \cdot d\mathbf{S} + \int_S \mathbf{J} \cdot d\mathbf{S} \qquad \nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}$$

Maxwell's Eq. (cont)

$\mathbf{E}$	Electric Field	$[E] = \frac{\mathrm{V}}{\mathrm{m}}$
$\mathbf{H}$	Magnetic Field	$[H] = \frac{\mathrm{A}}{\mathrm{m}}$
$\mathbf{D}$	Electric Flux Density	$[D] = \frac{\mathrm{C}}{\mathrm{m}^2}$
$\mathbf{B}$	Magnetic Flux Density	$[B] = \frac{\mathrm{Weber}}{\mathrm{m}^2} = T = \frac{\mathrm{V}\cdot\mathrm{s}}{\mathrm{m}^2}$

- It's important to note that these equations follow from experimental observations:
- Gauss' Law (equivalent to Coulomb Force equation) from the inverse square law. Also, no magnetic monopoles have ever been observed.
- Faraday's law of electromagnetic induction. Ampere's law in addition to displacement current for consistency (equivalently charge conservation).

Constitutive Relations

- *Force Law:* For a charge q moving at velocity $\mathbf{v}$ through an electromagnetic field, the forice experienced by the charge is given by

$$\mathbf{f} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

- *Ohm's Law:* For a conductor with conductivity σ ($\mathrm{S/m}$), the current density $\mathbf{J}$ is given by

$$\mathbf{J} = \sigma \mathbf{E}$$

- In this class σ is a scalar so the current is in the direction of the field $\mathbf{E}$.
- *Convection Current:* For a charge density ρ moving with velocity $\mathbf{v_p}$, the current density is

$$\mathbf{J} = \rho \mathbf{v_p}$$

Permittivity (Dielectric Constant)

- The electric flux density $\mathbf{D}$ is related to the electric field intensity $\mathbf{E}$ by

$$\mathbf{D} = \epsilon \mathbf{E} = \epsilon_r \epsilon_0 \mathbf{E}$$

$$\epsilon_0 \approx 8.854 \times 10^{-12}\,\mathrm{F/m}$$

- The relative permittivity ϵ_r characterizes the effect of the atomic and molecular dipoles in the material. For most microwave materials we assume ϵ is a scalar constant. Most problems are characterized by homogeneous, isotropic, linear, time-invariant materials.
- The frequency response of ϵ is very important as the imaginary component of ϵ gives rise to loss at microwave frequencies.

Permeability

- The magnetic flux density $\mathbf{B}$ is related to the magnetic field $\mathbf{H}$ by

$$\mathbf{B} = \mu\mathbf{H} = \mu_r\mu_0\mathbf{H}$$

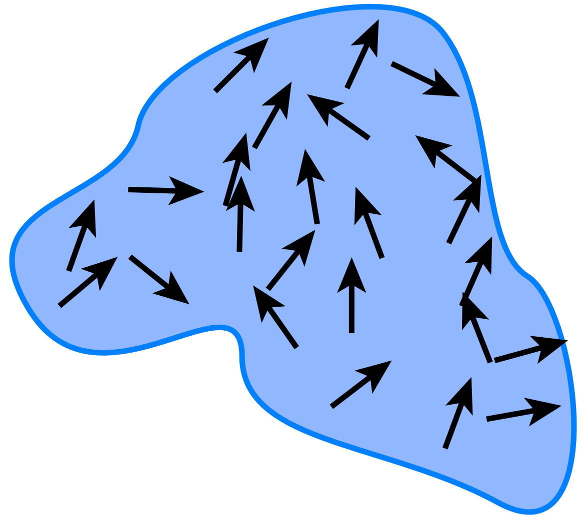
$$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{H/m}$$

- The relative permeability μ_r measures the effect of constituent atomic and/or molecular magnetic dipole moments.
- Most materials in nature are diamagnetic. The induced magnetic fields *oppose* the applied field. But the response is usually very weak and so $\mu \sim 1$. This is due primarily to the response of the electron "orbit" in an atom.

Paramegnetic Materials

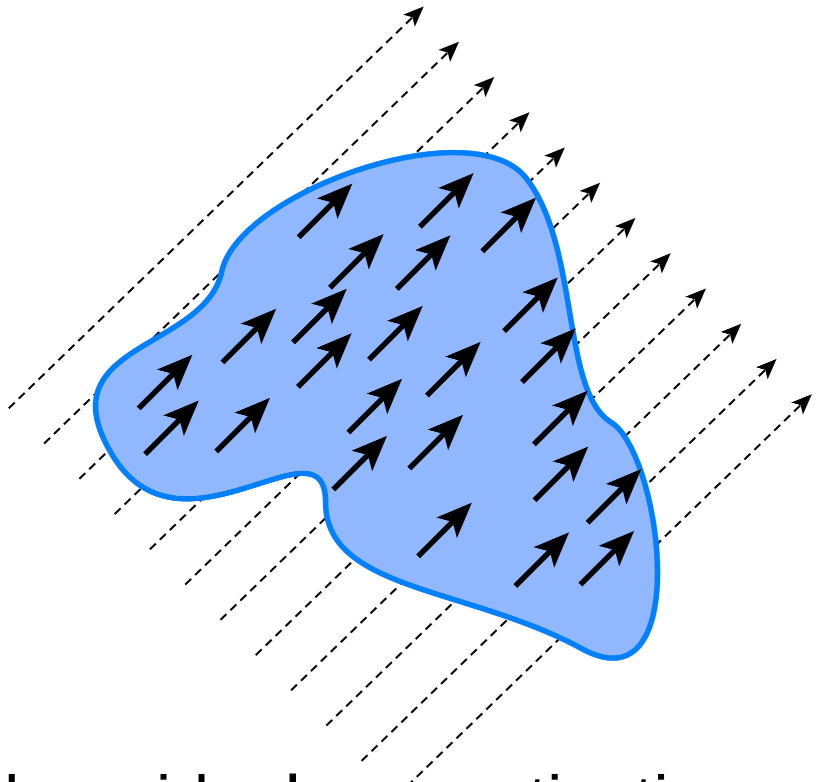
- Some materials have a natural net magnetic dipole moment. Such materials give a paramagnetic response which can arise from dipole moments in an atom, molecules, crystal defects, and conduction electrons. The dipole moments tend to align with the magnetic field but are deflected from complete alignment by their thermal activity.
- The fields resulting from the partial alignment *adds* to the applied field so $\mu \geq 1$. It can be shown that this effect is also relatively weak so that at room temperature $\mu \approx 1 + 10^{-5}$

Ferromagnetics and Ferrimagnetics



- Materials with residual magnetization exist where it is energetically favorable for internal magnetic dipoles to align spontaneously below a certain (Curie) temperature. These *ferromagnetic* and *ferrimagnetic* materials exhibit non-linear and large μ factors $\sim 10^3 - 10^6$.

Ferromagnetics and Ferrimagnetics



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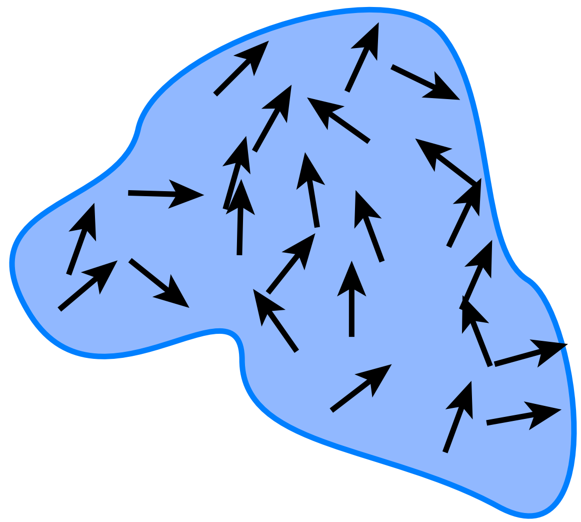

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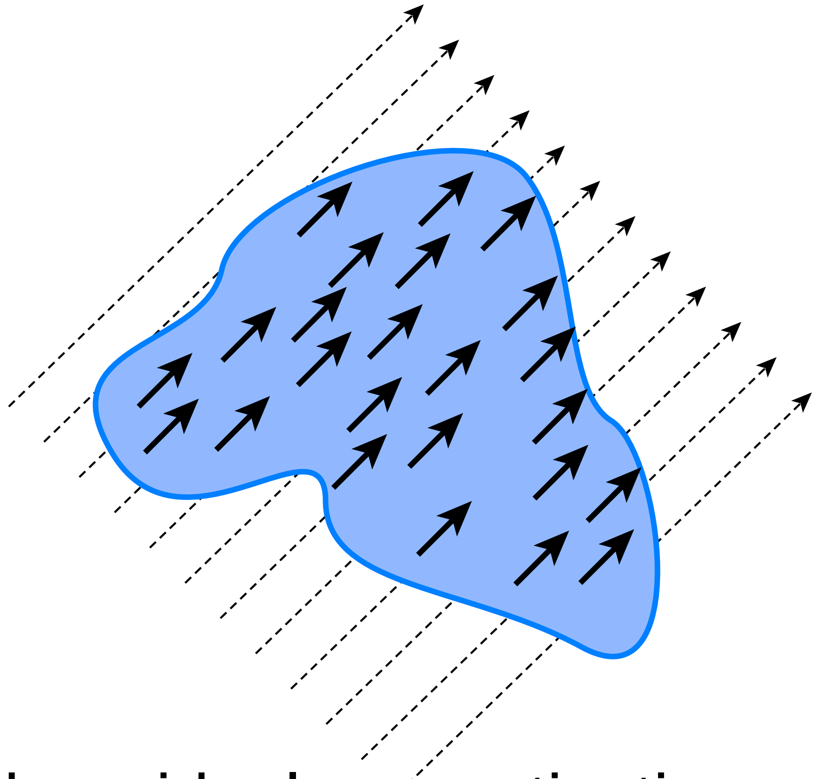

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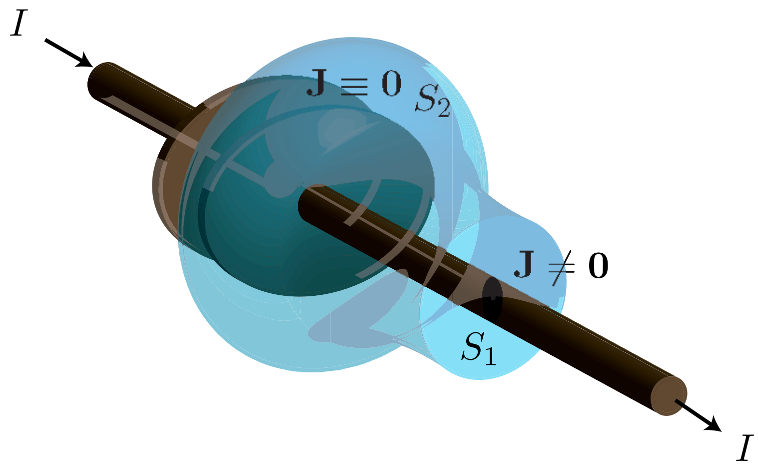
Displacement Current

- Ampere discovered that the magnetic field in static situtations can be calcuated by $\nabla \times \mathbf{H} = \mathbf{J}$. Equivalently,

$$\int_S \nabla \times \mathbf{H} \cdot d\mathbf{S} = \oint_C \mathbf{H} \cdot d\ell = \int_S \mathbf{J} \cdot d\mathbf{S} = I$$

- The above equation is a mathematical identity for *any* surface bounded by the contour C. Now Maxwell realized a flaw in this equation when you consider an AC current and a capacitor.
- Since $\mathbf{J} \equiv 0$ on surface S_2, but $\mathbf{J} \neq 0$ for surface S_1, this leads to a contradiction. (see next page)

Displacement Current (cont)



- To resolve this, Maxwell introduced a displacement current to Ampere's eq.

$$\nabla \times \mathbf{H} = \mathbf{J} + \mathbf{J_d}$$

- Furthermore, since $\nabla \cdot \nabla \times \mathbf{A} \equiv 0$, this implies that $\nabla \cdot \mathbf{J} = 0$, violating charge conservation, unless we introduce J_d. Maxwell defined

$$\mathbf{J_d} = \frac{\partial \mathbf{D}}{\partial t}$$

- to account for charge conservation since $\nabla \cdot \mathbf{D} = \rho$. This means that magnetic fields can be generated by currents or by changing electric fields.

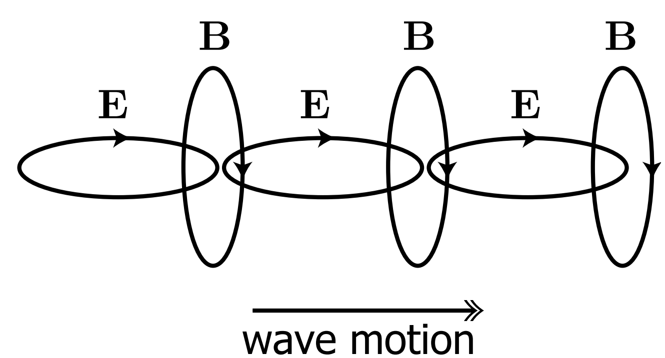
Radiation and EM Propagation

- The introduction of displacement current allows EM waves. To see this intuitively, note that in a charge-free region of space

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu\epsilon\frac{\partial \mathbf{E}}{\partial t}$$

$$\frac{\partial \mathbf{B}}{\partial t} \longrightarrow \frac{\partial \mathbf{E}}{\partial t} \longrightarrow \frac{\partial \mathbf{B}}{\partial t} \longrightarrow \dots$$



- This exchange happens at the speed of light.

Wave Equation in 3D

- We can derive the wave equation directly in a coordinate free manner using vector analysis

$$\nabla \times \nabla \times \mathbf{E} = \nabla \times -\mu \frac{\partial \mathbf{H}}{\partial \mathbf{t}} = \mu \frac{\partial \left(\nabla \times \mathbf{H} \right)}{\partial t}$$

- Substitution from Maxwell's eq.

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} = \epsilon \frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \times \nabla \times \mathbf{E} = -\mu\epsilon \frac{\partial^2 E}{\partial t^2}$$

Wave Eq. in 3D (cont)

- Using the identity $\nabla \times \nabla \times \mathbf{E} = -\nabla^{\mathbf{2}}\mathbf{E} + \nabla\left(\nabla \cdot \mathbf{E}\right)$
- Since $\nabla \cdot \mathbf{E} = 0$ in charge free regions

$$\nabla^{\mathbf{2}}\mathbf{E} = \mu\epsilon \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

- In Phasor form we have $k^2 = \omega^2 \mu\epsilon$

$$\nabla^{\mathbf{2}}\mathbf{E} = -k^2\mathbf{E}$$

- Now it's trivial to get a 1-D version of this equation

$$\nabla^2 E_x = \mu\epsilon \frac{\partial^2 E_x}{\partial t^2} \qquad \frac{\partial^2 E_x}{\partial x^2} = \mu\epsilon \frac{\partial^2 E_x}{\partial t^2}$$