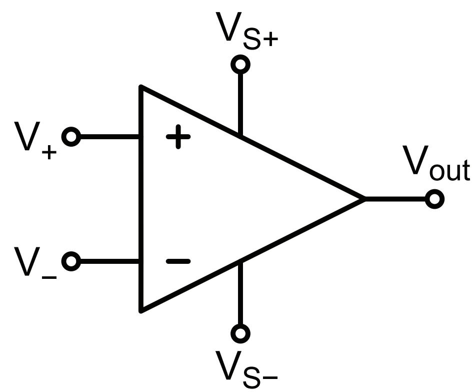
EE 42/100
Lecture 8: Op-Amps

Rev C 2/8/2012 (9:54 AM)

Prof. Ali M. Niknejad

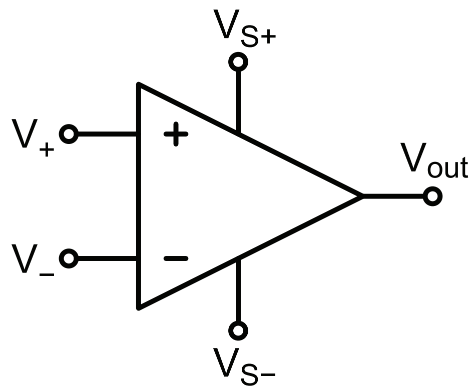
University of California, Berkeley

Operational Amplifiers



- Invented in 1941by Bell Labs engineer Karl D. Swartzel Jr. using vacuum tubes. It found wide application in WW-II.
- First monolithic IC op-amp was designed by Bob Widlar at Fairchild Semiconductor.
- The 741 op-amp is perhaps the best known op-amp in the world. Many other op-amps use the same pin configuration as the 741.
- The output voltage is usually millions of times larger than the voltage presented at the inputs.
- Op-amps are ubiquitous low cost components used in countless applications for analog signal processing (gain, filtering, signal conditioning).

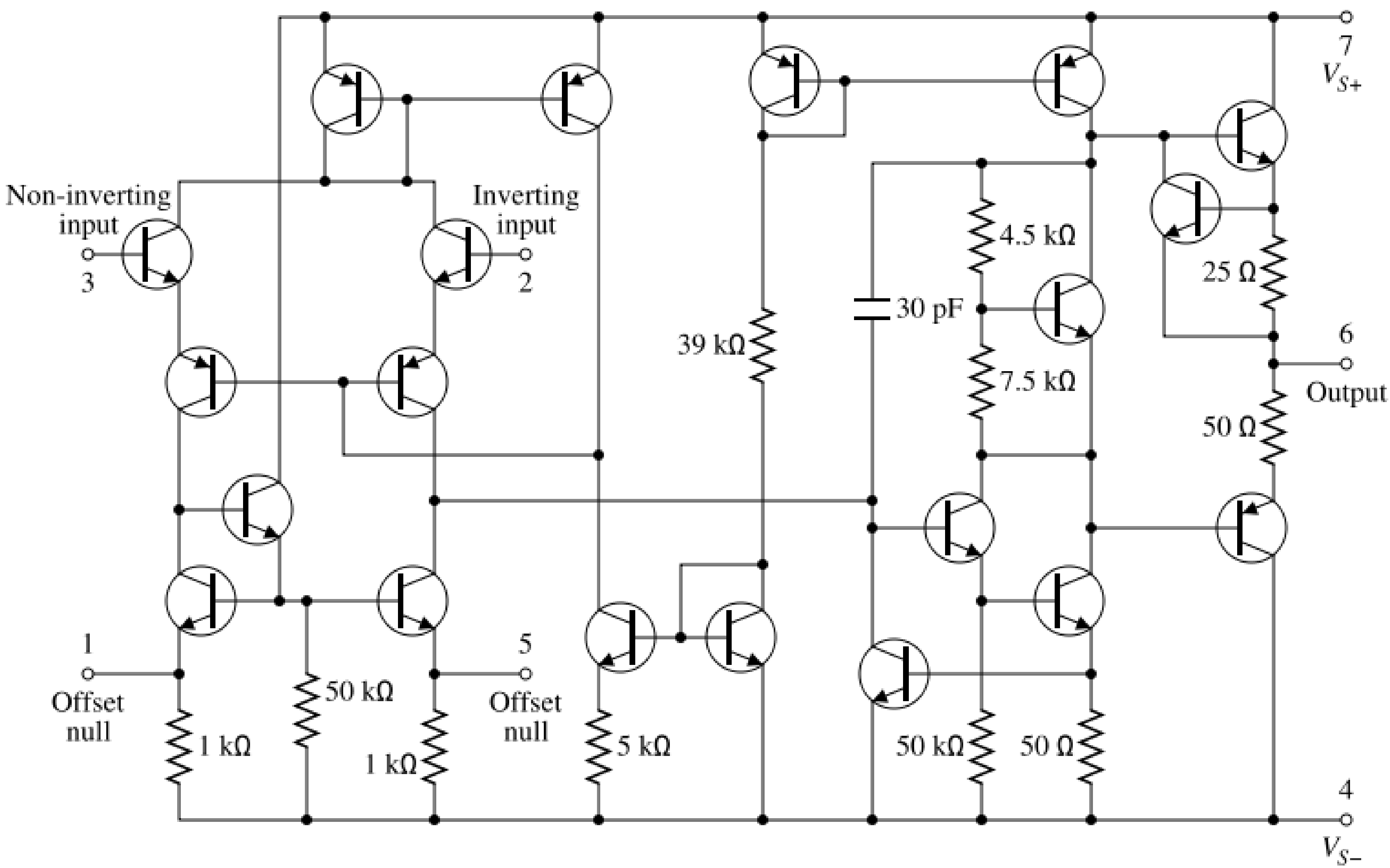
Operational Amplifier Pins



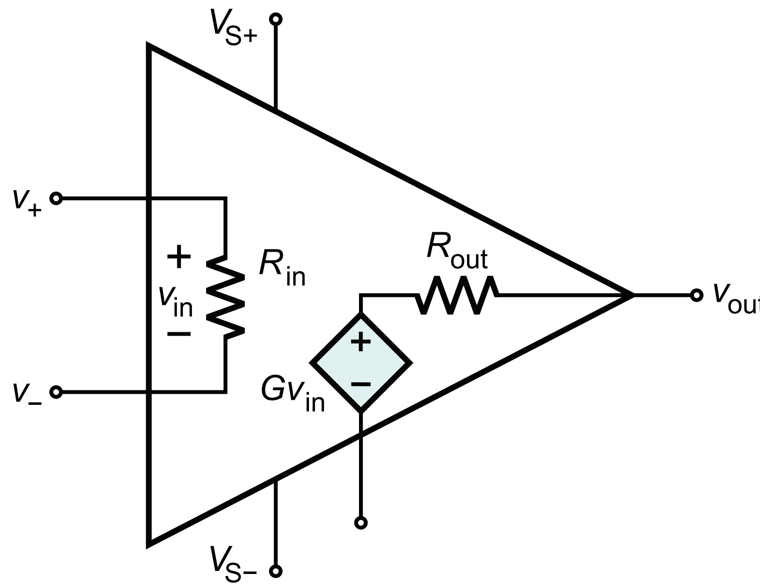
- The op-amp has 6 pins. There are the supply pins, where we connect a positive and negate voltage, and three signal pins, the two inputs, and an output. There is also a ground pin (not shown).
- The signal pins are usually AC voltages whereas the supply voltages are DC voltages.
- Some op-amps work with a single supply, in which case the negative rail is ground.
- Commonly known as the op-amp, is a high gain amplifier with a *differential input*.

$$v_o = A \cdot (v^+ - v^-)$$

Classic 741 Schematic

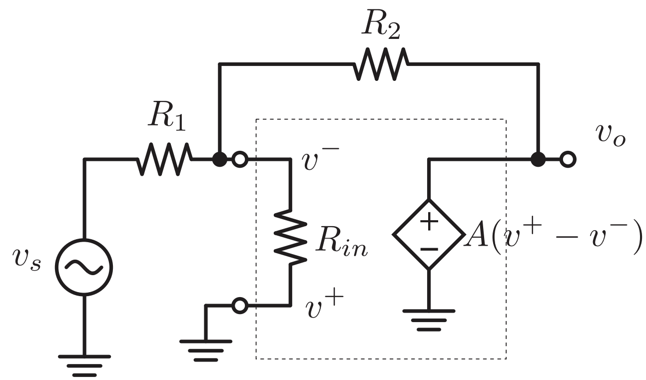


Equivalent Circuit Model (hard)



- We model the complex op-amp by using the simple equivalent circuit shown above. The most salient features are the high gain A (tyipcally a million or more), very high input resistance R_{in}, and low output resistance R_{out}.
- Because of the large gain, only a few microvolts of input signal is required to saturate the op-amp output. Thus the amplifier is very impractical if used without *feedback*. In fact, the gain of the op-amp is a very poorly controlled parameter, often varying wildly with temperature or from part-to-part. How do you design with such an imperfect component? *Feedback.*

Example Calculation



- The above example shows a typical op-amp configuration where the output signal is fed-back to the negative input terminals. This is called negative feedback.
- This seems strange at first because we are subtracting the output from the input, but as we shall see, this is a self-regulation mechanism that results in a very precise amplifier.
- Write KCL at the input node of the amplifier

$$(v^- - v_o)G_2 + v^- G_{in} + (v^- - v_s)G_1 = 0$$

Voltage Gain of Circuit

- But the output voltage in this case is simply given by $v_o = -Av^-$, where A is very large, which means that $v^- = -v_o/A$ is a very small voltage

$$\left(\frac{-v_o}{A} - v_o\right)G_2 + \frac{-v_o}{A}G_{in} + \left(\frac{-v_o}{A} - v_s\right)G_1 = 0$$

- which allows us to write the complete expression for gain

$$\frac{v_o}{v_s} = \frac{-AG_1}{G_2(A+1) + G_{in} + G_1}$$

- Assuming that the op-amp has a very large gain, the above equation simplifies

$$\frac{v_o}{v_s} \approx \frac{-AG_1}{G_2(A+1)} \approx \frac{-G_1}{G_2} = \frac{-R_2}{R_1}$$

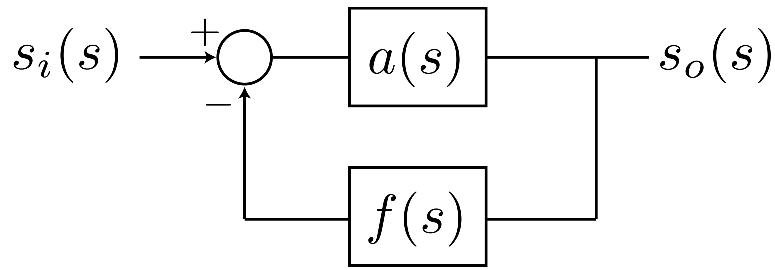
Differential rather than Difference Amplifier

- Why do we call an op-amp a differential amplifier rather than a difference amplifier?
- In the inverting amplifier configuration, we can calculate the effective input voltage by

$$(v^+ - v^-) = \frac{v_o}{A} = \frac{\frac{-R_2}{R_1} v_s}{A} \approx 0$$

- A *difference* amplifier is perhaps a better name, but somewhat misleading since as we see the input voltage must be small for the op-amp to operate correctly (hence a "differential" voltage).
- We will build a true *difference* amplifier with op-amps later.

Op-Amp Feedback System



- In the derivation we have a nice result that the voltage gain of the overall circuit is just set by the ratio of two resistors, which can be made very precise and can track temperature.
- The internal gain of the amplifier A does not appear in the final expression, which means if it varies due to temperature or from part to part, it plays a negligible role in setting the gain.
- So we sacrificed gain to arrive at a solution that is much more robust. This is the concept of negative feedback and it is used widely in electronic systems (biological, chemical, and mechanical systems use it too).
- The idea is to sample a fraction of the output and compare it to the input. By forcing equality between the sample and the fraction of the output, the gain is determined by the fraction rather than by the raw gain of the amplifier.
- Note that positive feedback is not used, since it has a saturating (rather than regulating) characteristic.

Dynamic Range of Amplifier

- So what did we gain when we designed an op-amp with such a high value of gain? For one, it's an extremely versatile device that can be reconfigured to have any gain range by simply selecting the feedback components (R_1 and R_2).
- *Subtle Point*: Unlike a non-feedback ("open loop") amplifier, the input linear range can be made larger since regardless of the input voltage source magnitude, the differential input of the op-amp is always small $(v^+ - v^-) = v_o/A \approx 0$.

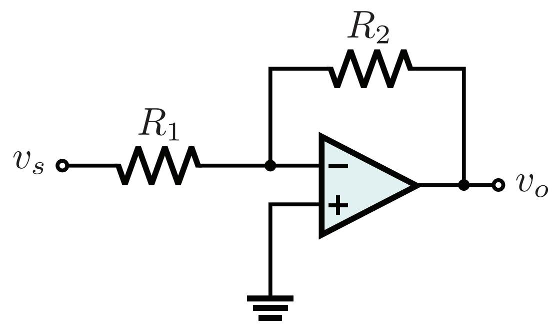
Ideal Op-Amp: Golden Rules

- The ideal op-amp model introduced thus far is useful for circuit simulation, but too complicated for hand analysis. In fact, since the gain of the op-amp is so large, we can make several simplifying assumptions:

Both inputs are at the same voltage.
No current flows in or out of either input.

- As a consequence of the first rule, the input impedance of the amplifier is nearly infinite. Even if the amplifier has a relatively modest input impedance, when feedback is applied, or in "closed loop" configuration, the input impedance is driven to very high values.

Example Calculation: Inverting Amplifier



- Let's redo the calculations for the so-called "inverting amplifier" using the Golden Rules
- By the first golden rule, the inverting input of the op-amp must be at ground potential ($v^+ = v^-$), which is often called a "virtual ground". That's because this voltage moves very little as an input signal is applied.
- Write KCL at the input of the op-amp (which is at the virtual ground potential):

$$\frac{v_s}{R_1} = -\frac{v_o}{R_2}$$

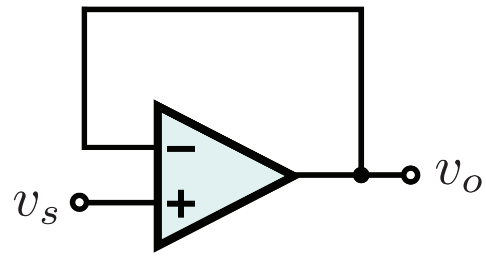
- Note that the term for the input current of the op-amp is missing, due to the golden rule. Then we have

$$\frac{v_o}{v_s} = \frac{-R_2}{R_1}$$

Inverting Amp Input/Output R

- Since the input of the amplifier is at a virtual ground, the voltage source v_s only “sees" the resistance R_1, which is approximately the input resistance of the circuit.
- At the output, the action of the feedback lowers the output resistance, and so the output looks like a nearly ideal voltage source, which means that the op-amp has transformed the voltage source into a nearly perfect voltage source! In other words, the op-amp buffers the source and presents it as a nearly perfect source (with very small source resistance).

Voltage Follower



- This is the idea behind the voltage follower, which has an input-output relation that at first seems trivial:

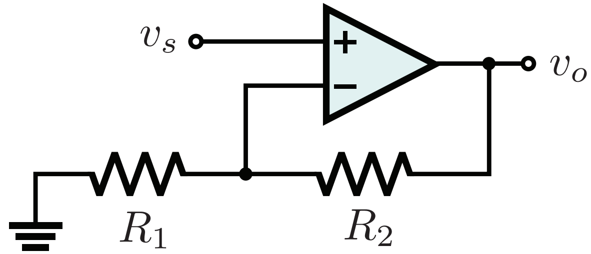
$$v^s = v^+ = v^- = v_o$$

- Which means the output voltage is just the same as the input voltage, or it *follows* the input. But notice that the input impedance seen by the source is nearly infinite, since no current flows into the op-amp. This means that the op-amp does not load the source in this configuration. Likewise, the output impedance is very low, which means that an imperfect source can be buffered and made to appear as an ideal voltage source.

Microphone Example

- A crystal microphone has a source resistance of 10 $\mathrm{M}\Omega$. We wish to drive a speaker with has an impedance of only 8Ω. Last lecture we realized that using a simple linear amplifier is not practical, because of the loading issues.
- But a voltage buffer followed by an amplifier solves these problems.

Non-Inverting Amplifier



- For the non-inverting amplifier, we still apply feedback to the negative terminal. The input is supplied to the positive terminal.
- Applying the first golden rule as before, we have

$$v^- = v^+ = v_s$$

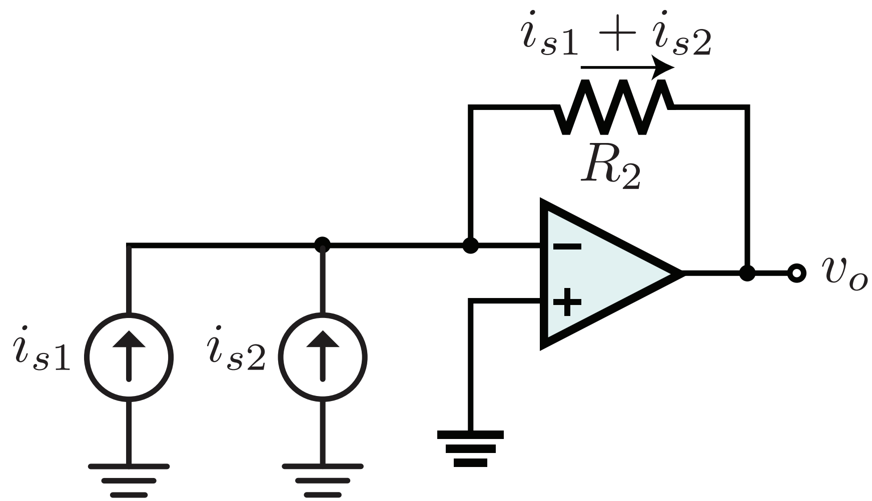
- Now applying the second golden rule, since the input current of the op-amp is zero, there is a perfect voltage divider from the output of the op-amp to the negative terminal

$$v^+ = v_o \frac{R_1}{R_1 + R_2}$$

- Which means that

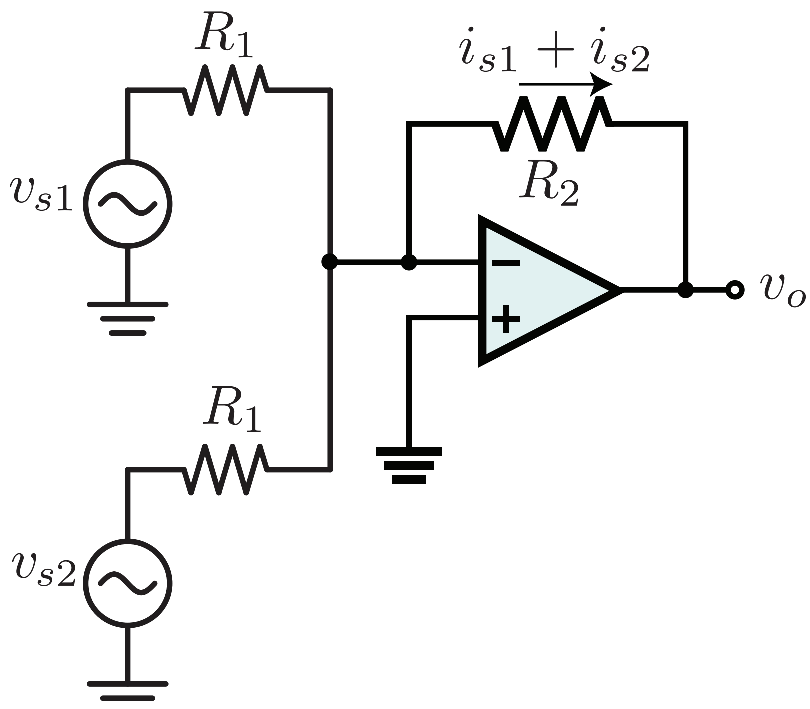
$$\frac{v_o}{v_s} = 1 + \frac{R_2}{R_1}$$

Current Summing Amplifier



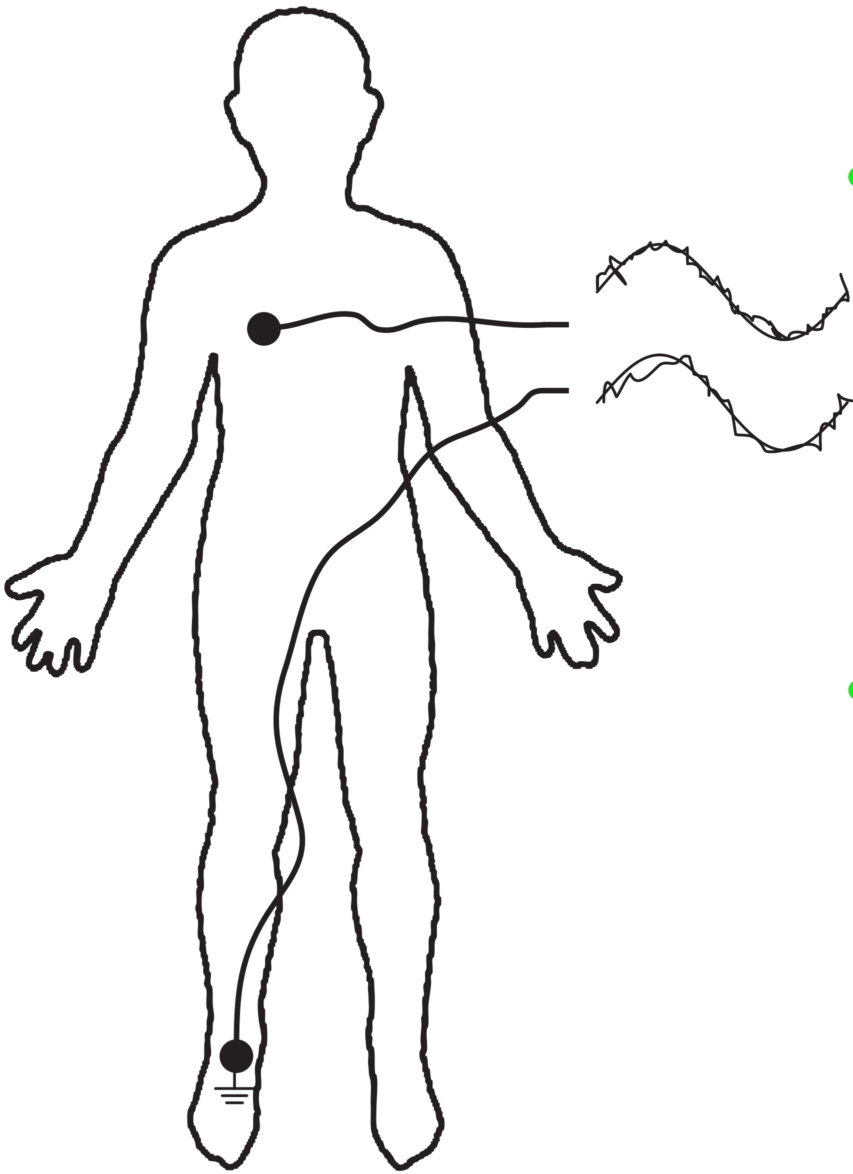
- An important observation is that in the inverting amplifier, the current injected into the negative terminal of the op-amp is routed to the output and converted into a voltage through R_2. If multiple currents are injected, then the *sum* of the currents is converted to a voltage.

Voltage Summing Amplifier



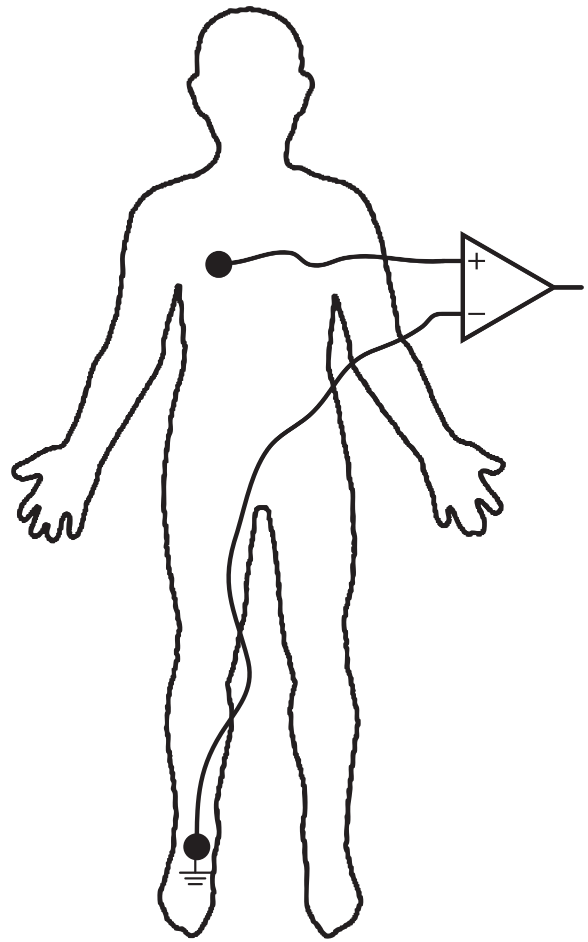
- Each source is converted into a current and then summed in a similar fashion as the currents. Note that the virtual ground means that no current is “lost” when the currents are put in parallel (due to the output resistance).

Differential Amplification and Noise



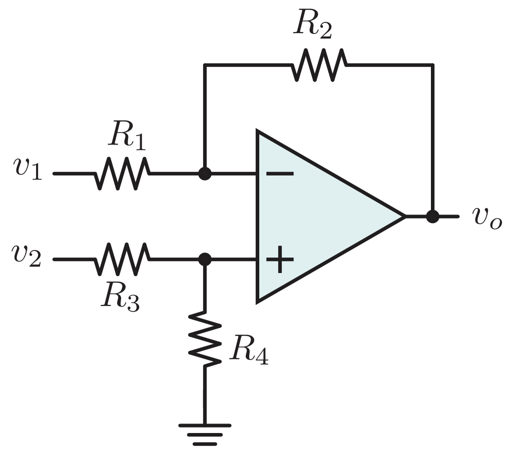
- In many systems, the desired signal is weak but it's accompanied by a much larger undesired signal. A good example is the ECG measurement on the human body. The human body picks up a lot of 60 Hz noise (due to capacitive pickup) and so a very weak ECG signal (mV) is accompanied by a large signal ($\sim 1V$) that we wish to reject.
- Fortunately the noise pickup is in *common* with both leads of the ECG because the body is essentially an equipotential surface for the noise pickup. If we take the difference between two points, it disappears.

Common-Mode and Differential-Mode Gain



- We thus need an amplifier that can detect a small *differential mode* voltage in the presence of a potentially very strong *common mode* voltage.

Difference Amplifier



- Using superposition, we can quickly calculate the transfer function. For port 1, the amplifier is simply an inverting stage

$$v_o^1 = v_1 \frac{-R_2}{R_1}$$

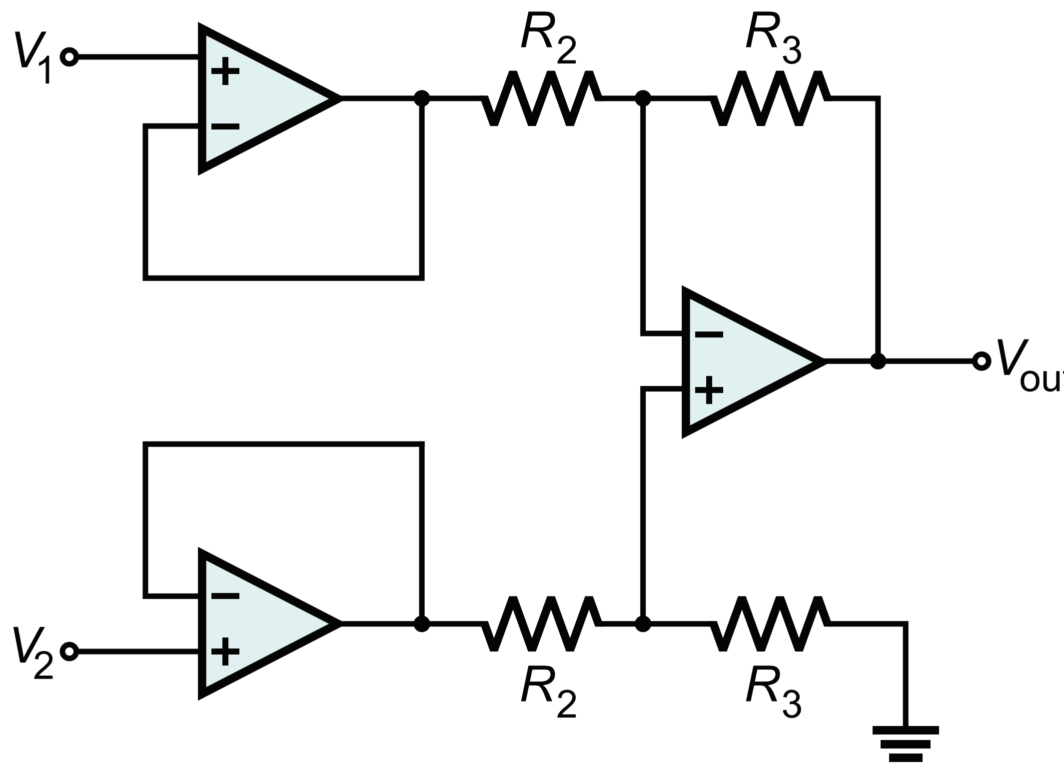
- For port 2, it's a non-inverting stage, except we only tap off a fraction of v_2

$$v_o^2 = v_2 \frac{R_4}{R_3 + R_4}(1 + \frac{R_2}{R_1})$$

- Take the sum and set $R_1 = R_3$ and $R_2 = R_4$, we have

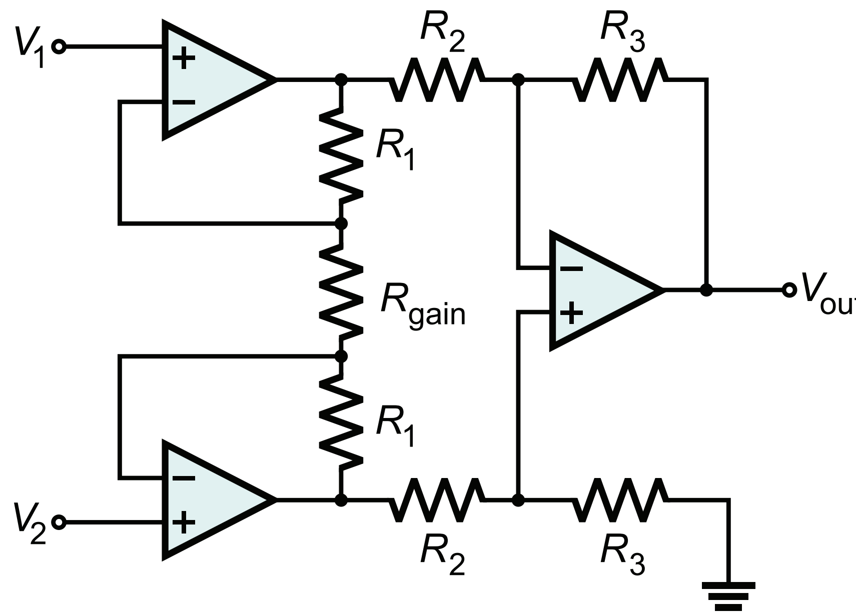
$$v_o = \frac{R_2}{R_1}(v_2 - v_1)$$

Instrumentation Amplifier



- Two unity-gain buffers are used to buffer the input signal. A difference amplifier is used to provide a gain of R_3/R_2.

Instrumentation Amplifier Version 2



$$v_o = \left(1 + \frac{2R_1}{R_{gain}}\right) \frac{R_3}{R_2} (v_2 - v_1)$$

- The input buffers now have gain due to the fact that the circuit is configured as a non-inverting amplifier. Why is the resistor R_{gain} appear between the amplifiers?
- *Subtle:* Note that we could simply put in two resistors to ground, but a more elegant way to realize it is to put in between the top and bottom. For balanced inputs, the resistor is split in two and grounded in the middle (another virtual ground). For unbalanced inputs, no current flows and the resistor is not there, so there's no common-mode gain!