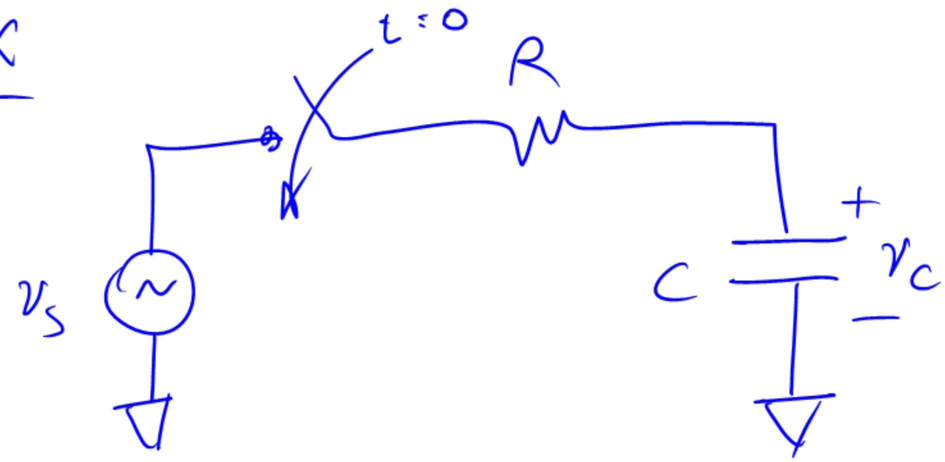
EX



$$V_s = V_0 \cos \omega t$$

$$v_c(0) = 0V$$

STEP 1 SETUP NODAL EQ.

BY INSPECTION

$$v_s = iR + v_c$$

KVL

$$i = i_c = C \frac{dV_c}{dt}$$

$$\boxed{V_s = RC \frac{dV_c}{dt} + V_c}$$

GOVERNING EQ.

$$v_c + RC\frac{dv_c}{dt} = V_s$$

STEP 2 : FIND THE HOMOGENEOUS (ZERO INPUT) SOLUTION

$$v_h + RC\frac{dv_h}{dt} = 0$$

"NATURAL" RESP

$$v_h = Ae^{Bt} \qquad v_h' = A \cdot B e^{Bt}$$

$$\cancel{A}\cancel{e^{Bt}} + RC\,\cancel{A} \cdot B \cdot \cancel{e^{Bt}} = 0$$

$$1 + BRC = 0 \qquad B = -1/RC$$

$$v_h = A e^{-t/\tau}$$

$\tau = RC$

TIME CONSTANT OF CIRCUIT

STEP 3 FORCED RESPONSE

$$v_f + RC \frac{dv_f}{dt} = v_s = V_0 \cos \omega t$$

TRY

$$v_f = A \cos \omega t \qquad v_f' = -A\omega \sin \omega t$$

$$A \cos \omega t + RC \cdot (-A\omega \sin \omega t) \neq V_0 \cos \omega t$$

TRY

$$v_f = A \cos \omega t + B \sin \omega t$$

CONST TERM — d/dt TERM

$$v_f' = -A\omega \sin \omega t + B\omega \cos \omega t$$

$$A \cos \omega t + B \sin \omega t + RC(-A\omega \sin \omega t + B\omega \cos \omega t) = V_0 \cos \omega t$$

cos > $A + \tau B\omega = V_0$ sin > $B - A\omega\tau = 0$, $B = A\omega\tau$

$\sum$ COSINE TERMS $= V_0$ $\sum$ SINE TERMS $= 0$

$$B = A\omega\tau$$

$$A + \tau\omega(A\omega\tau) = V_0$$

$$A = \frac{V_0}{1+(\omega\tau)^2}$$

$$B = \frac{\omega\tau \quad V_0}{1+(\omega\tau)^2}$$

$$V_f(t) = \frac{V_0}{1+(\omega\tau)^2}\left(\cos\omega t + \omega\tau\sin\omega t\right)$$

$$\cos x\cos y - \sin x \sin y = \cos(x+y)$$

$$V_f(t) = \frac{V_0}{1+(\omega\tau)^2}\left(\cos\omega t + \omega\tau \sin\omega t\right)$$

$$= \frac{V_0}{1+(\omega\tau)^2} A \cos(\omega t + \phi)$$

$$A = \sqrt{1^2 + a^2} = \sqrt{1+(\omega\tau)^2}$$

$$\phi = -\tan^{-1}(\omega\tau)$$

$$V_f(t) = \frac{V_0}{\left(1+(\omega\tau)^2\right)^{1/2}} \cos(\omega t + \phi)$$

AMPLITUDE $< V_0$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\sin^2 x = 1 - \cos^2 x$$

$$= \cos^2 x - 1 + \cos^2 x$$

$$\frac{\cos 2x + 1}{2} = \cos^2 x$$

$$I \cos x + Q \sin x = A\cos(x + \theta)$$

$$= \cos x \cos\theta - \sin x \sin\theta$$

$$I = \cos\theta \qquad \tan\theta = -\frac{Q}{I}$$

$$Q = -\sin\theta$$

$$I^2 + Q^2 = \cos^2\theta + \sin^2\theta = 1 \qquad \theta = \tan^{-1}(-Q/I)$$

STEP 4 COMPLETE SOLUTION

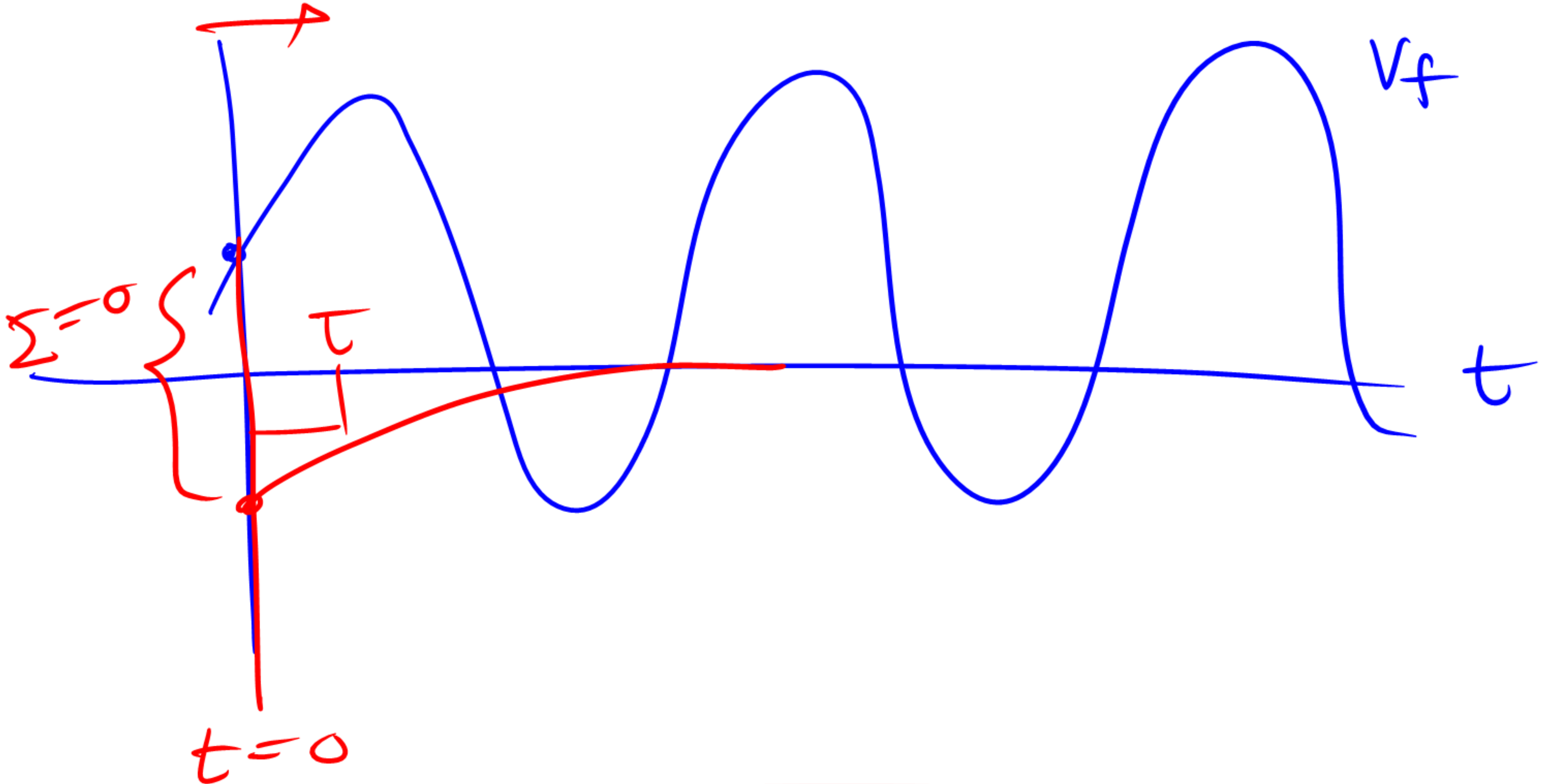
$$V_c(t) = V_n(t) + V_f(t)$$

$$= K e^{-t/\tau} + A \cos(\omega t + \phi)$$

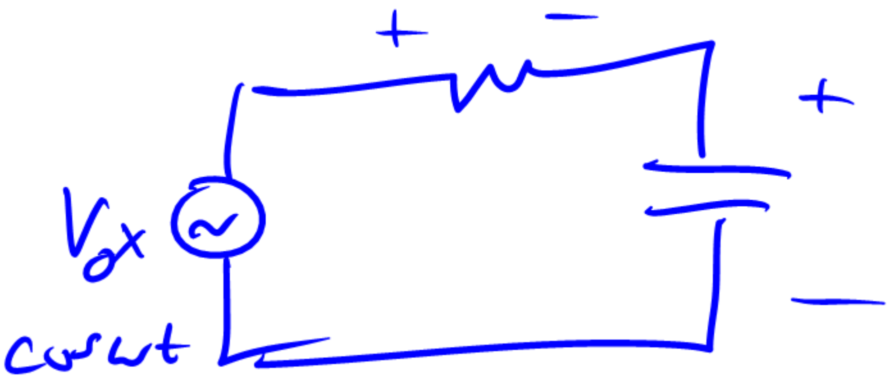
STEP 5 USE I.C. TO RESOLVE UNKNOWNS

$$v_c(0) = 0V = K + A \cos\phi$$

$$K = -A \cos\phi$$



$$A = \frac{V_0}{\sqrt{1+(\omega\tau)^2}}$$



$$v_c = A \cdot \cos(\omega t + \phi)$$

$$A = \frac{V_0}{\sqrt{1 + (\omega\tau)^2}}$$

ω FREQ OF SOURCE

τ TIME CONSTANT OF CIRCUIT

"LOW" FREQ INPUT

$\omega\tau \ll 1$ $\qquad$ $\omega \ll 1/\tau$

SAY $R = 1k\Omega \cdot 2\pi$

$C = 1\mu F$

$RC = 1ms$

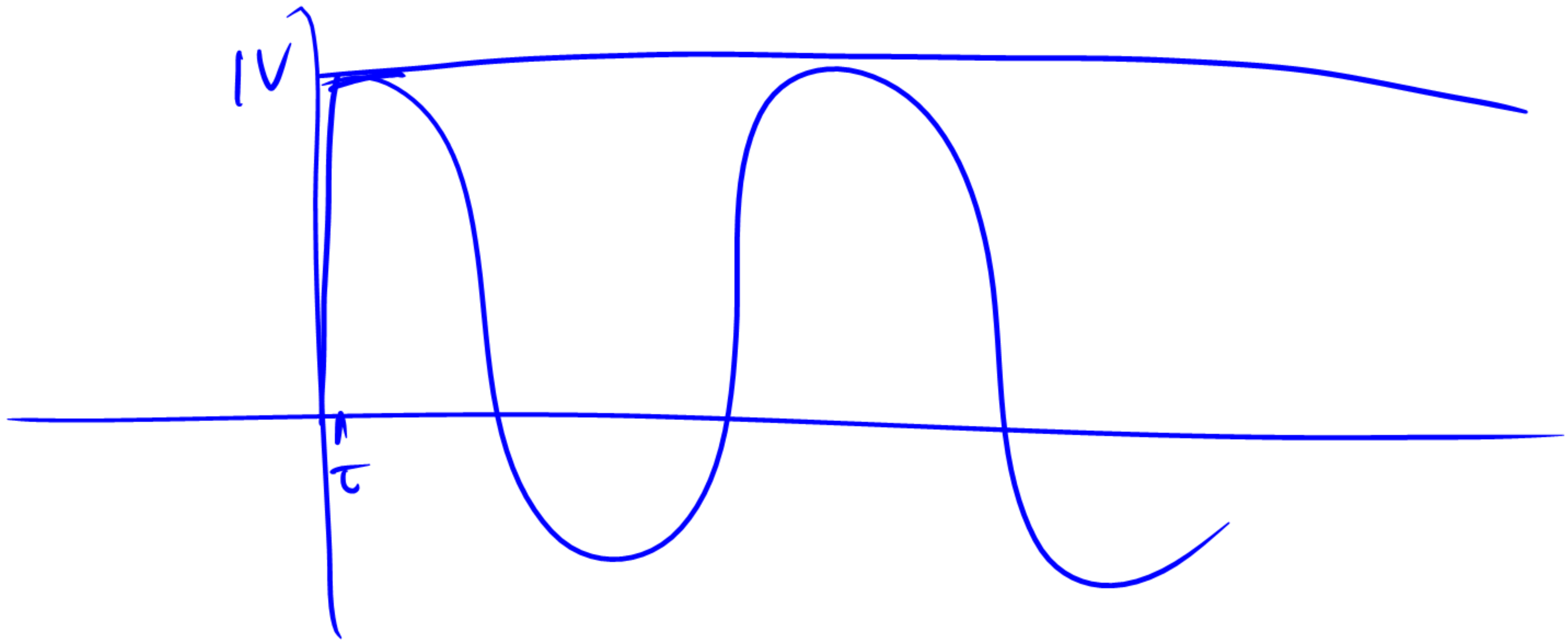
$\frac{1}{RC} = 2\pi \; k \; rad/s$

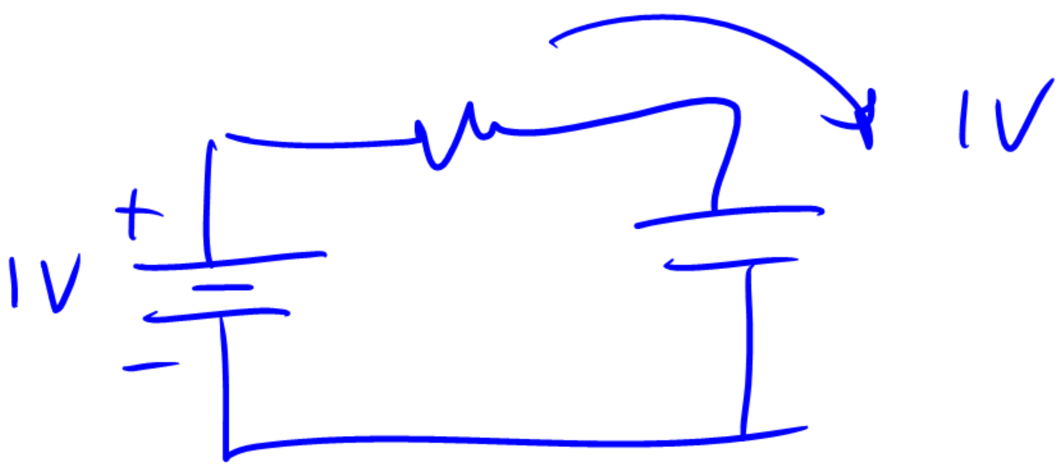
$\omega_c = \cancel{2\pi} f_c = \cancel{2\pi} \; k \; rad/s$

$f_c = 1 \; KHz$

$\Rightarrow$

$A = V_o$


1V
τ


1V
+
-
1V

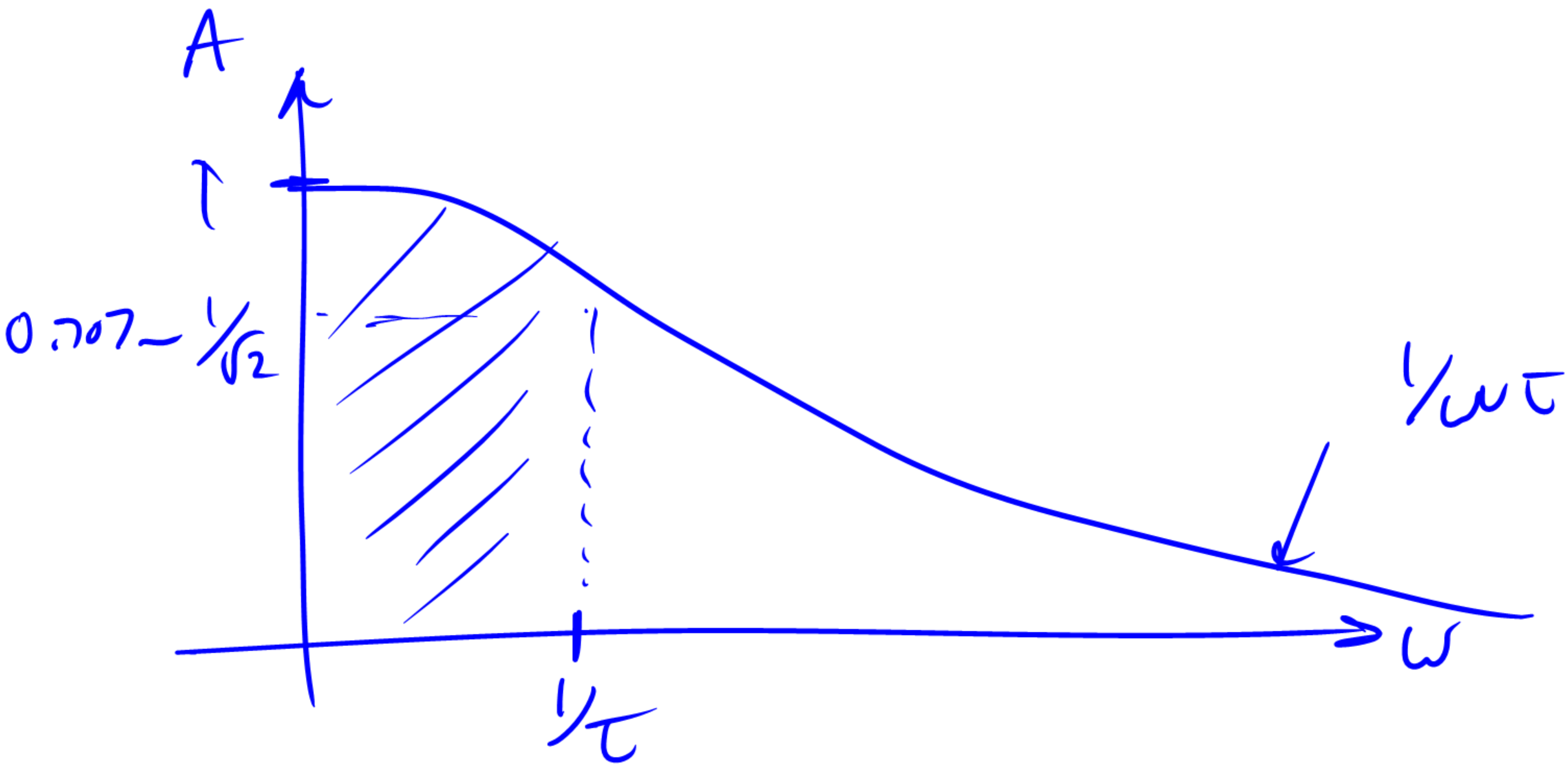
$\omega\tau \gg 1$ HIGH FREQ

$$f > f_c = 1\,\text{kHz} = \frac{1}{2\pi R \cdot C}$$

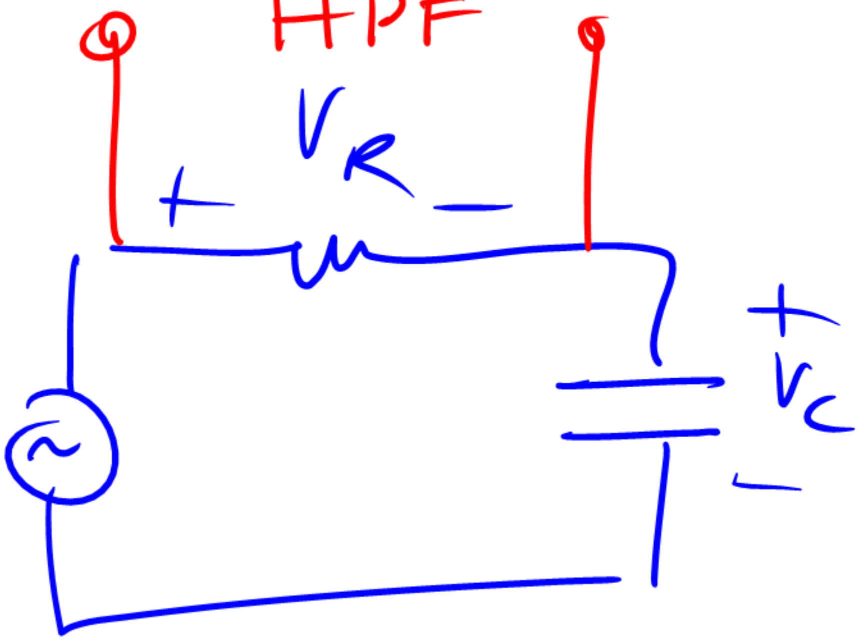
$$A = \frac{V_o}{\sqrt{\cancel{1} + \underbrace{(\omega\tau)^2}_{\gg 1}}} \simeq \frac{V_o}{\omega\tau}$$

$\omega\tau = 1$ CRITICAL FREQ

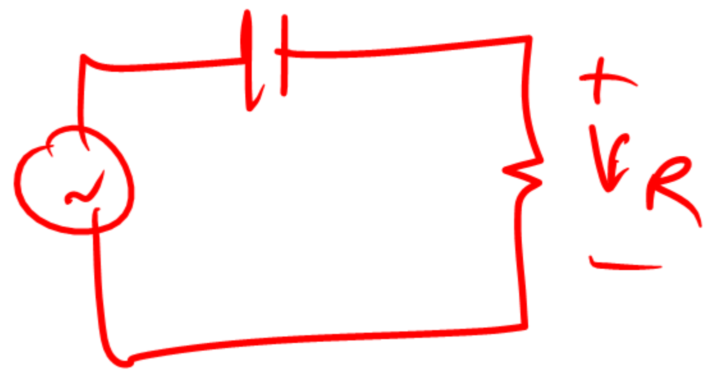
$$A = \frac{V_o}{\sqrt{1+1}} = \frac{V_o}{\sqrt{2}}$$


A
1
0.707 ~ 1/√2
1/ωτ
ω
1/τ

HPF



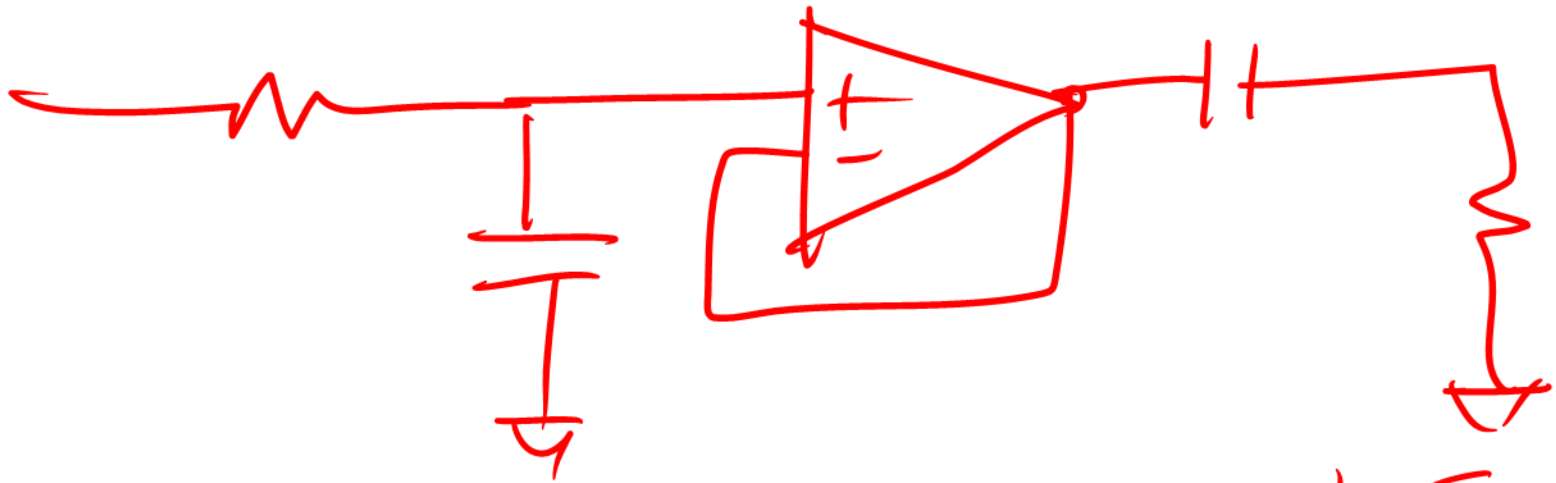
HIGH-PASS FILTER



$$V_R + V_C = V_S$$

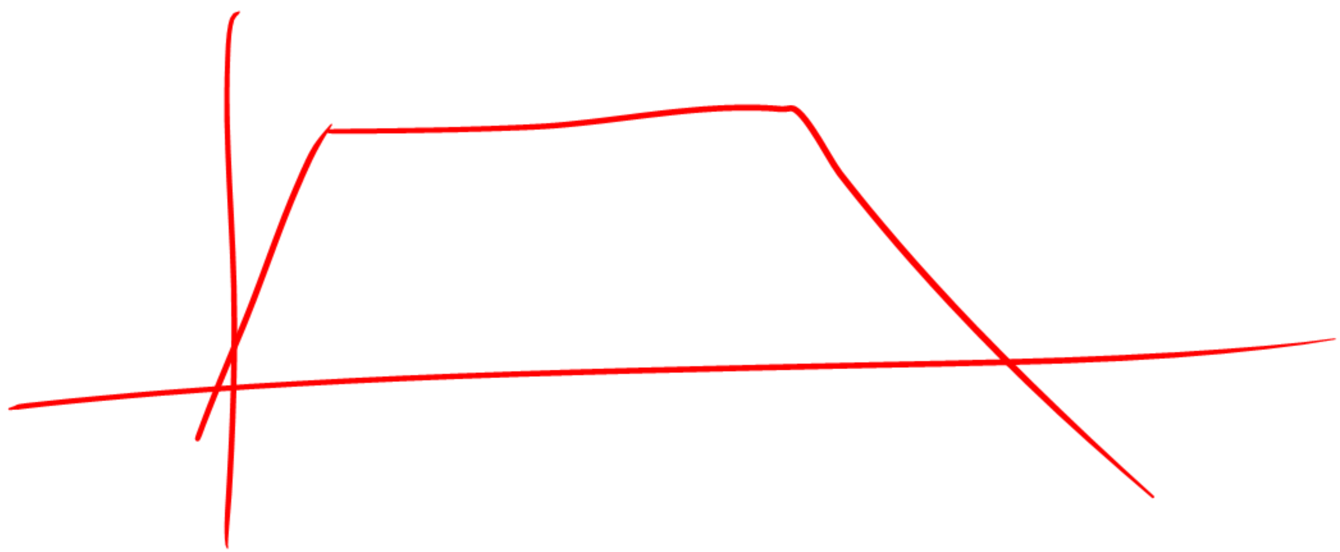
LOW FREQ: $V_R \approx 0$, $V_C \approx V_S$

HI FREQ: $V_R \approx V_S$, $V_C \approx 0$

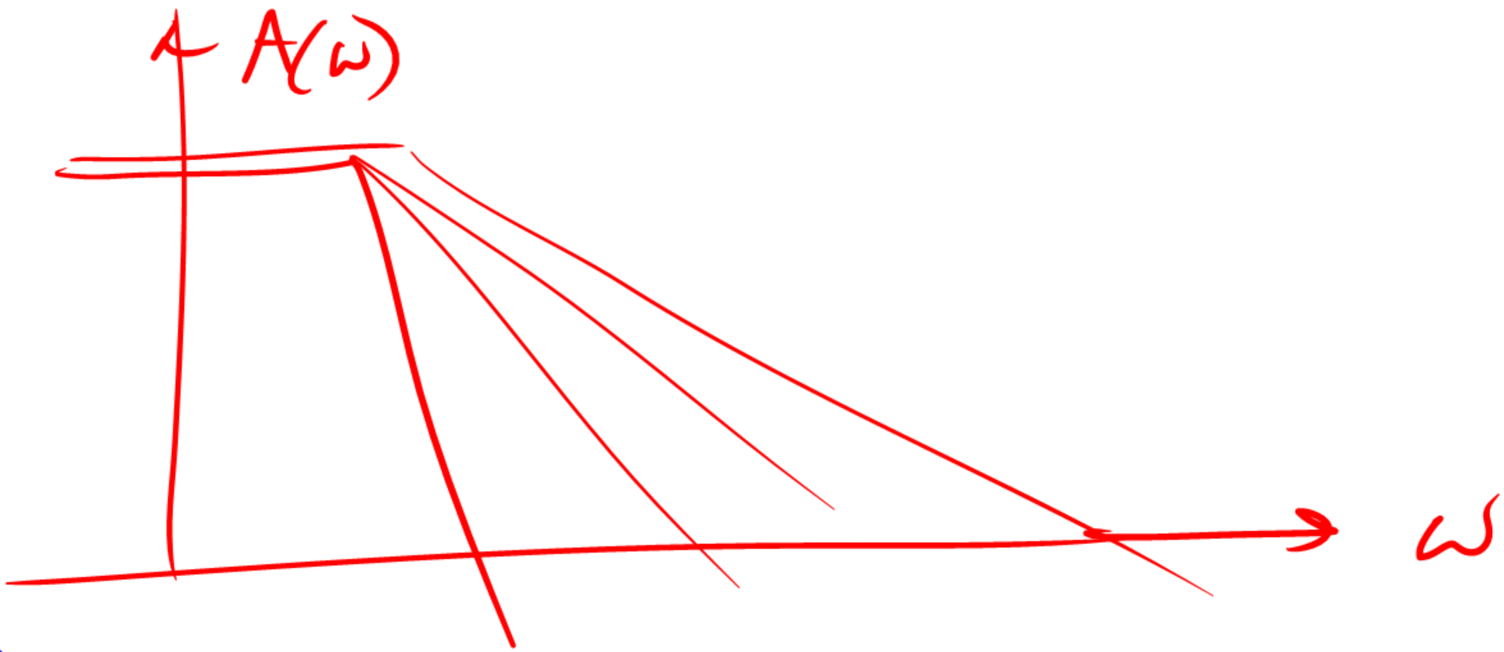


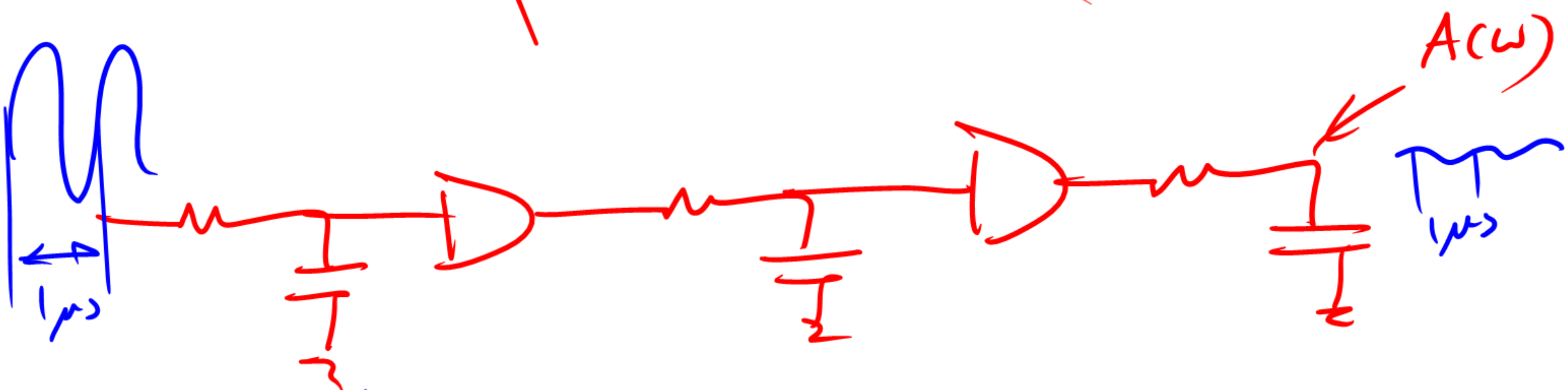
LPF ~20kHz

HPF ~100Hz



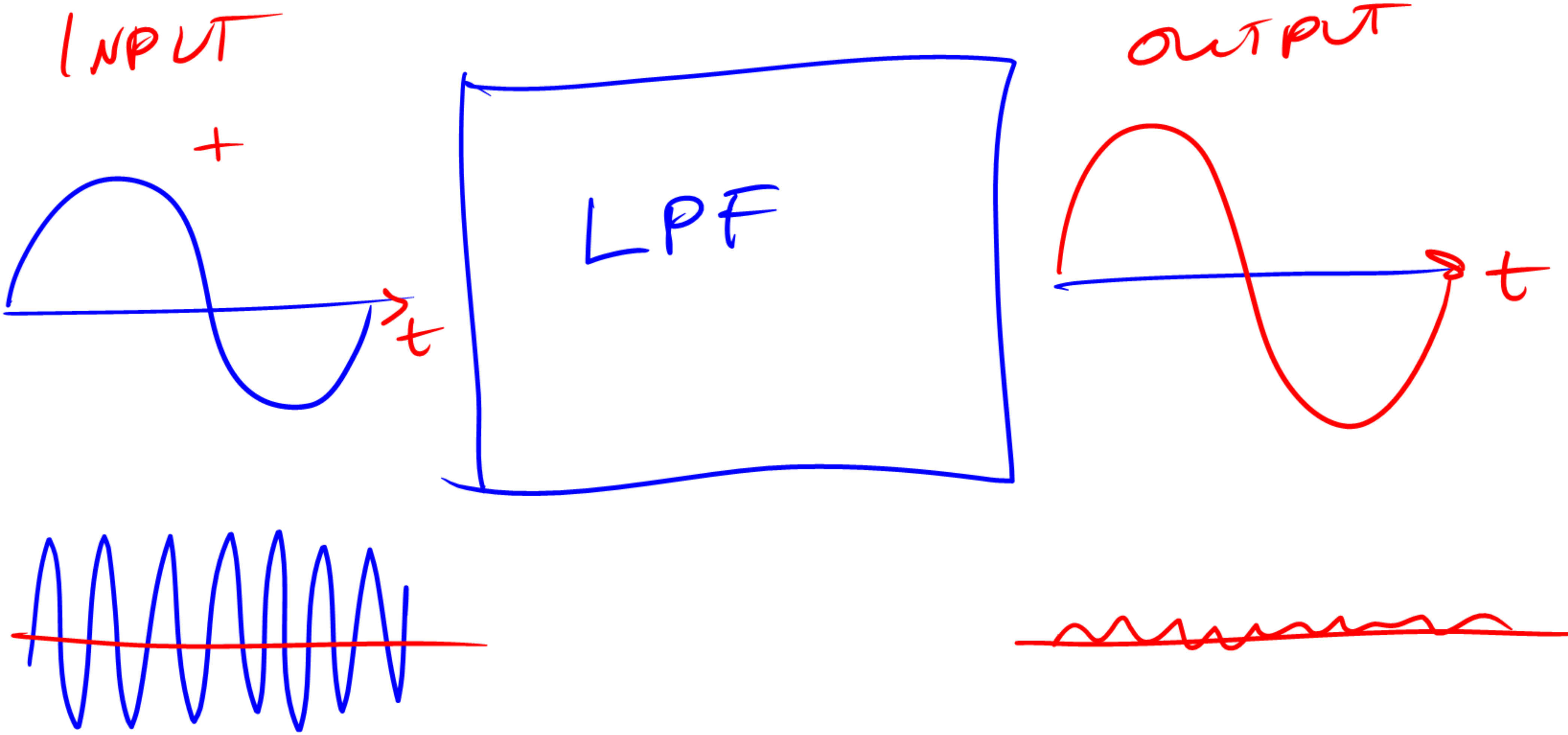
CASCADE FILTER





$\tau = 1\,ms$

$\omega\tau = 2\pi \cdot \frac{1}{\mu s} \cdot 1\,ms = \frac{2\pi}{1000}$


INPUT
+
t
LPF
OUTPUT
t