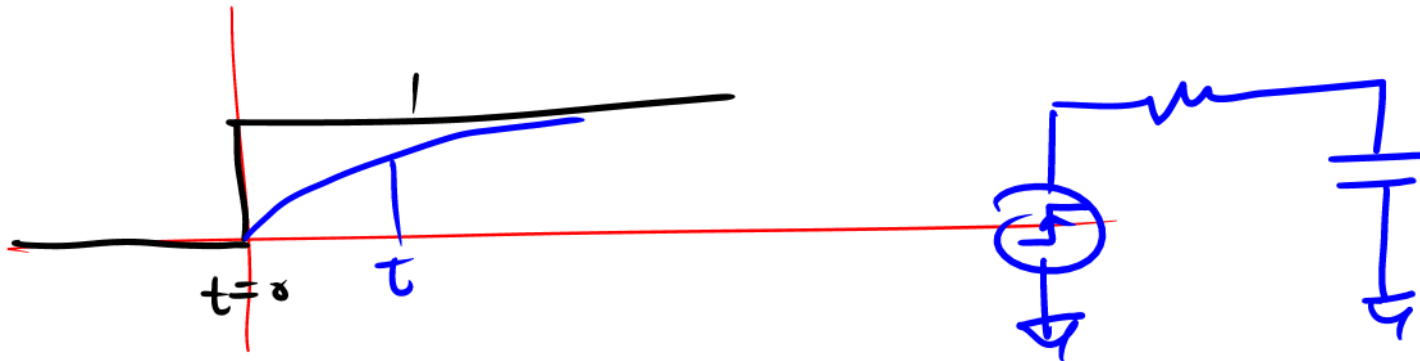


Step Response

- A unit step function is described by

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$

- While the waveform has an artificial “jump” (difficult to reproduce in real life), it’s a very useful model for real stimuli. It’s like a source that turns “on” at time $t = 0$.
- The solution procedure is therefore very similar to the example with the switches.



Step Response of Circuit

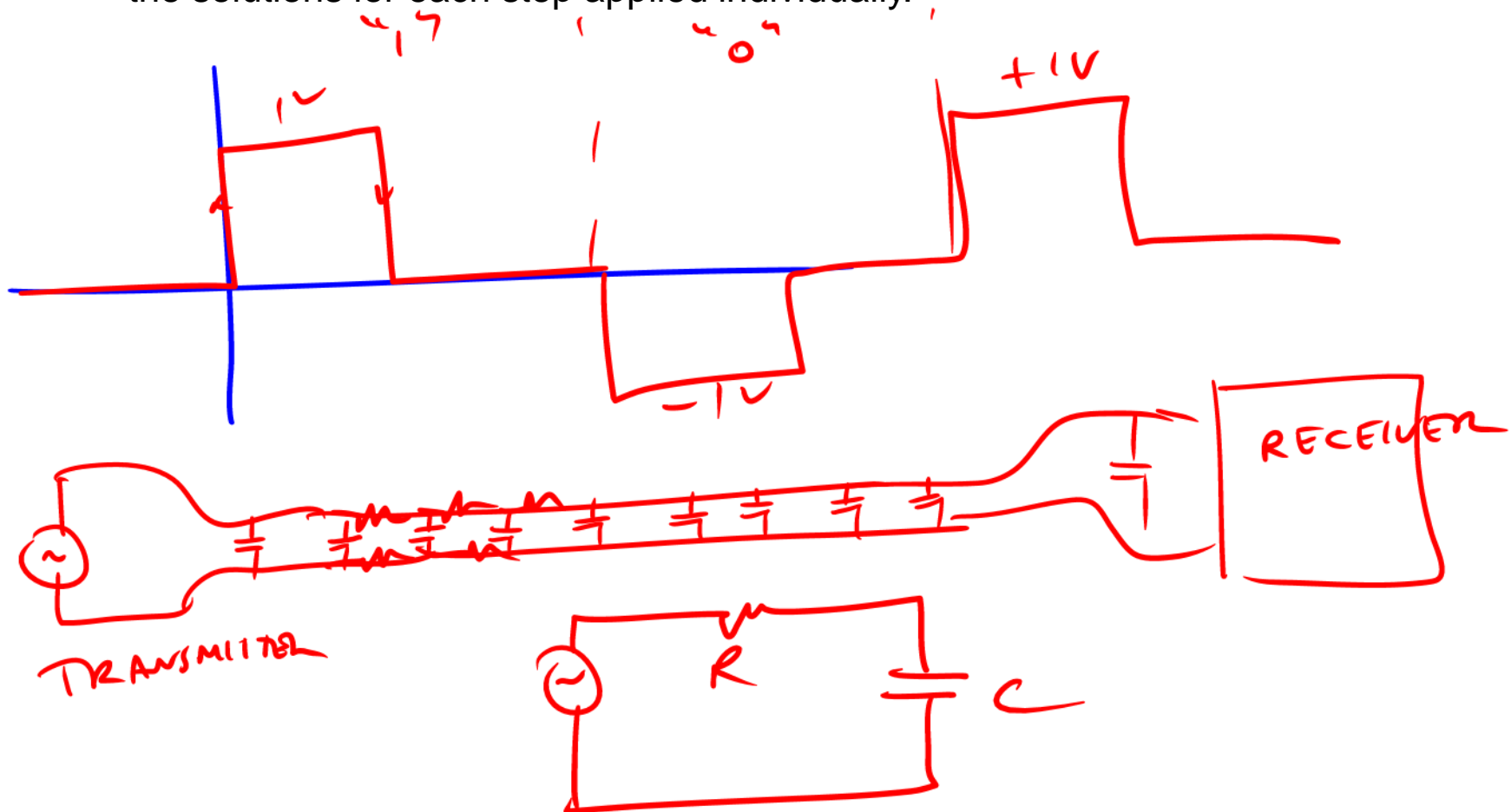
- The step response of a circuit is extremely useful. Not only does it have engineering significance of its own, but in a miraculous way it contains everything you need to know about a system! In other words, if you know the step response, you know the response of the circuit to any input!
- You can imagine that any input can be approximated by a series of small steps and then use superposition. But that's for another class...

Pulse Response

- Now suppose that we apply a pulse to our circuit. We can mathematically construct a pulse as

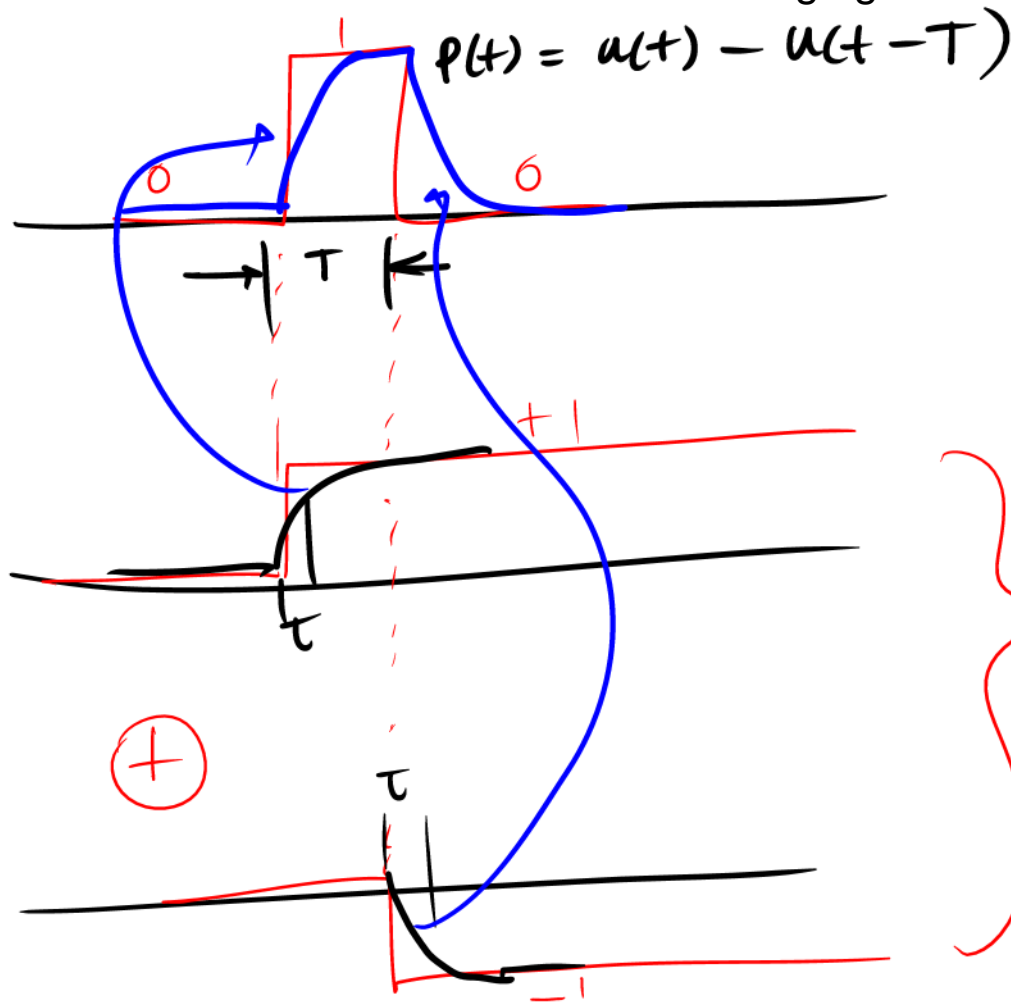
$$p(t) = u(t) - u(t - T)$$

- which is the sum of a unit step and a negative unit step applied at a later time T . We can now invoke superposition and say the solution is the same as the sum of the solutions for each step applied individually.



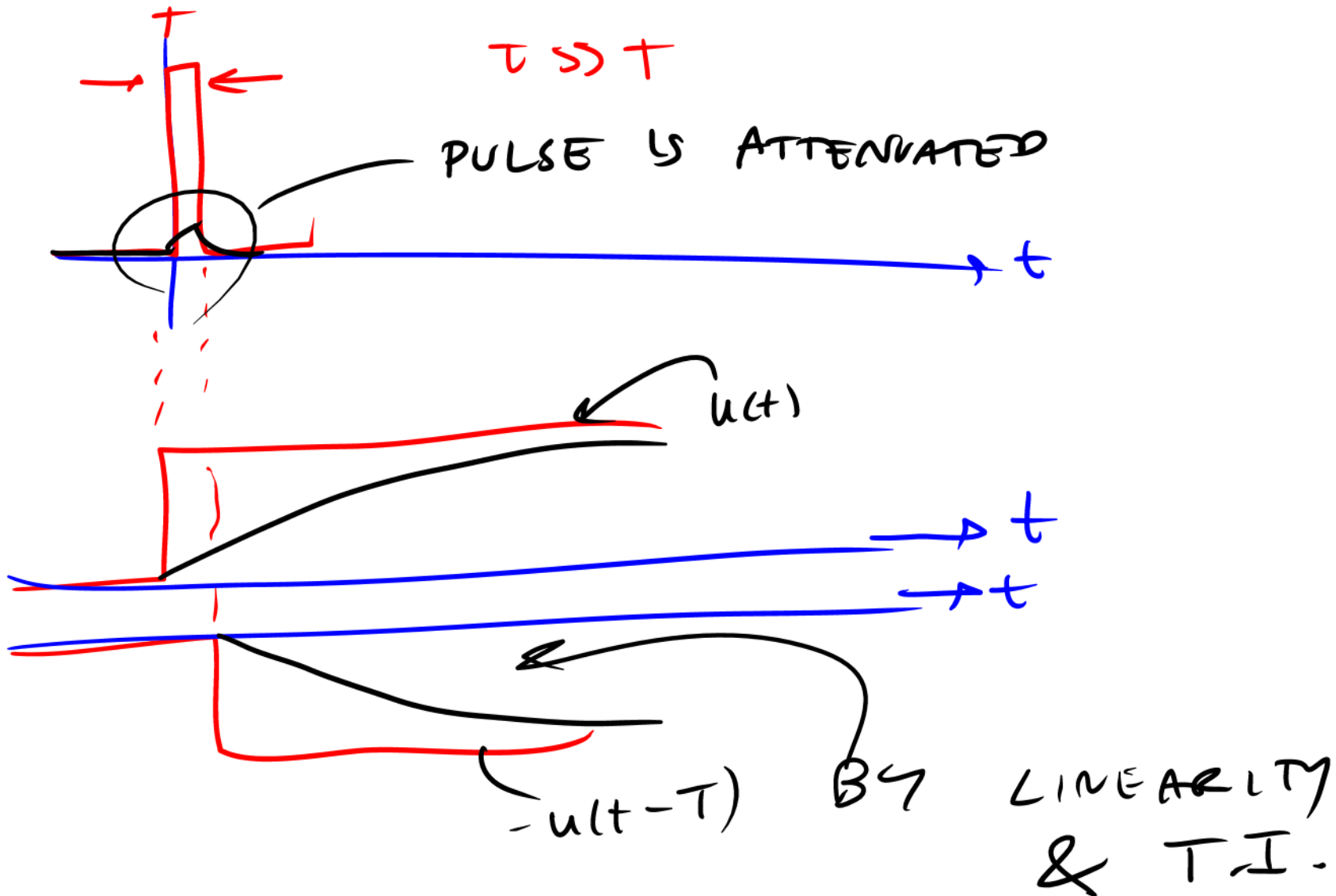
Pulse Reponse (cont)

- The solution for the second step is easy to do since it's just a sign inversion (absorbed into constants) and a delay. By time invariance, we actually know the solution already. The circuit does not care "when" we apply the step function, because the circuit itself is not changing.



Complete Pulse Response

- Simply sum the step responses and be sure to delay the output.



Filtering: Short Pulse Response

- Imagine a very narrow pulse is applied to the circuit, such that $T \ll RC$. In this case, the output is almost nil. In other words, the RC circuit will not respond to a very short “fast” stimulus. It acts like a filter.

DC Blocking Capacitor

- It's very common to connect two circuits using a “blocking” or “coupling” capacitor, as shown.
- The reason is that we often wish to pass signals from one block to another (say one amplifier to another), but we want to bias the stages differently (apply different DC levels). We desire a circuit that can pass AC signals but block DC signals.

DC Block (cont)

- Let's calculate the voltage across the load as shown. We know that the steady-state solution is zero because when the capacitor goes to an open circuit, the load is shunted to ground. But if the source changes, then the output will move.
- Roughly speaking, if the circuit changes faster than the RC time scale, the output will always be moving to respond to the input. But if it changes more slowly, the capacitor will “block” the signal.
- That's because in steady state the capacitor charges up the voltage of the source and so the voltage of the resistor is

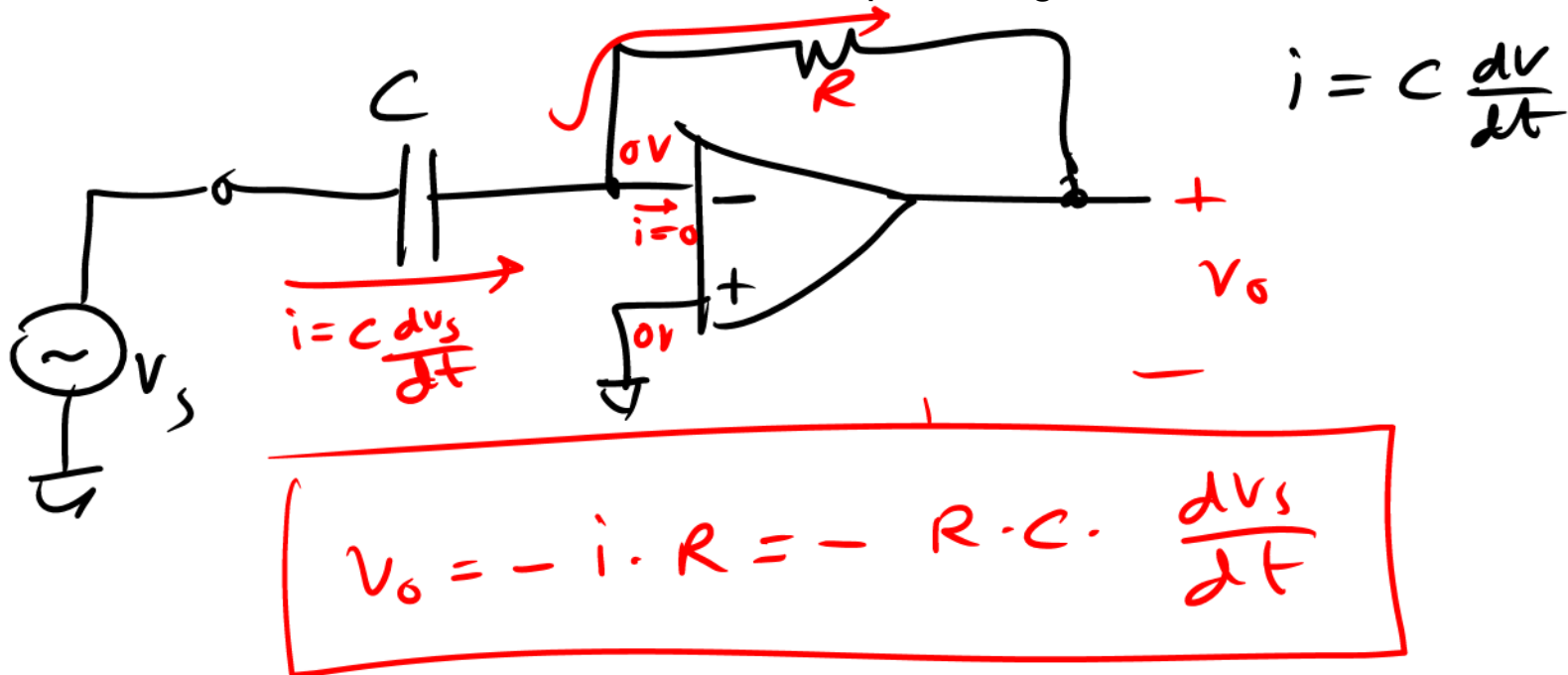
$$v_R(t) = v_s - v_c(t)$$

A Differentiator

- Suppose we use an op-amp to create a virtual ground as shown. Then the output voltage, which is simply given by the capacitor current multiplied by R , is given by

$$v_o(t) = -iR = -RC \frac{dv_s}{dt}$$

- The circuit calculates the derivative of the input voltage



An Integrator

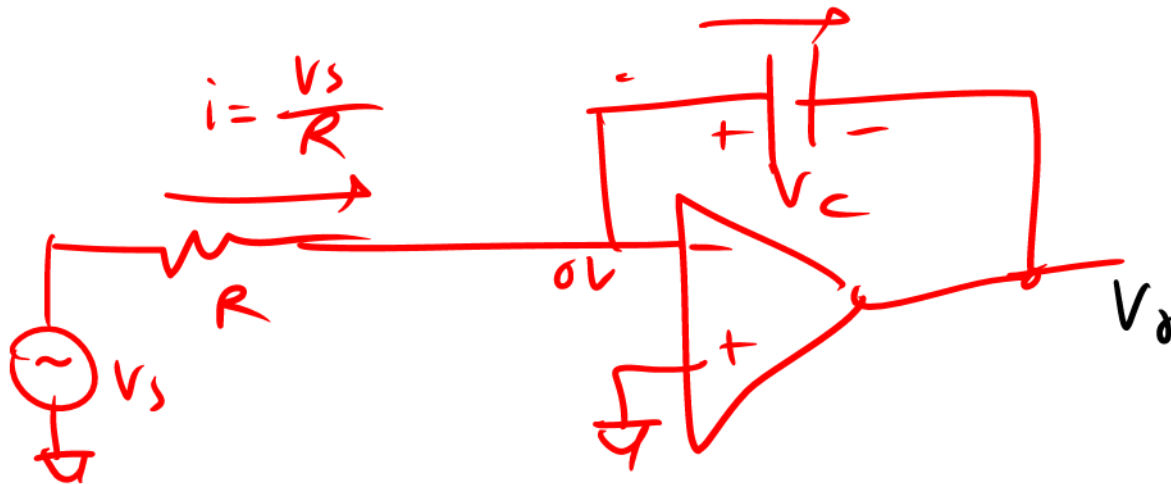
- If we swap the location of the capacitor and the resistor, it performs an integration operation.
- To see this, note that the output voltage is simply the voltage on the capacitor, which is given by the net charge flowing into it from the source

$$v(t) = -V_c$$

$$\frac{V_s}{R} = C \frac{dV_c}{dt} = -C \frac{dv_o}{dt}$$

$$v(t) = v(0) + \frac{-1}{RC} \int_{\tau=0}^{\tau=t} v_s(\tau) d\tau$$

- Note: τ is a “dummy” variable of integration.



More Complicated Example

- When we setup the equations, we see that the time constant is given by the product of C and the equivalent resistance seen by C .
- This can be understood if we find the Thevenin equivalent of the circuit for the homogenous solution. If we look out from the capacitors, we just see R_{th} .

More Capacitors

- If there are more capacitors in the circuit, we get a system of differential equations which can be solved using matrix techniques.
- For this class we will not cover the more complicated solution although we can comment on the general solution.
- The system of equations can be “diagonalized” by matrix factorization. The eigenvalues thus determine the time constants of the various states of the system. The independent states are determined by linear combinations of the capacitor charges, which are found from the eigenvectors.
- To find the general output, therefore, we have to take the sum over all the characteristic solutions, which all have different time constants. The solution will therefore be a sum of exponential solutions.