SOLUTIONS

University of California, Berkeley
EE 42/100

Spring 2010
Prof. A. Niknejad

Midterm Exam (closed book/notes)
Tuesday, February 23, 2010

Guidelines: Closed book. You may use a calculator. Do not unstaple the exam. In order to maximize your score, write clearly and indicate each step of your calculations. We cannot give you partial credit if we do not understand your reasoning. Feel free to use scratch paper.

Electron charge $q = 1.60217646 \times 10^{-19}$ C.

KCL: Kirchhoff's Current Law states that the net current flow into a node (or super-node or any closed surface) is zero.

$$\sum I_k = 0$$

KVL: Kirchhoff's Voltage Law states that the net voltage drop around any loop is zero.

$$\sum V_k = 0$$

Power flow into a component: (positive means power is absorbed or dissipated.

$$P = I \cdot V$$

Ohm's Law:

$$V = I \cdot R$$

or

$$I = G \cdot V$$

where $G = R^{-1}$.
Power dissipated in a resistor:

$$P = I^2 R = V^2 G$$

Resistors in series add:

$$R = R_1 + R_2 + \cdots$$

Conductances in parallel add:

$$G = G_1 + G_2 + \cdots$$

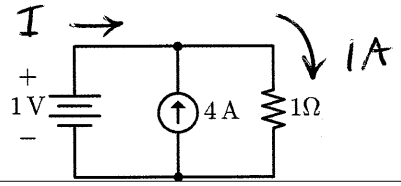
For two resistors in parallel, this implies

$$R_{||} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

When a chain of resistors are in series and connected to a voltage source, the voltage across the kth resistor is given by the voltage divider forma

$$V_k = \frac{R_k}{R_1 + R_2 + \cdots}$$

1. (16 points) Answer the following questions succinctly.



(a) (*4 points*) What is the power delivered or absorbed by the battery? Clearly state if the battery is delivering or absorbing power and find the numerical value.

$I + 4A - 1A = 0$

$I = -4A + 1A = -3A$

$P = 1V \cdot (-I) = +3W$

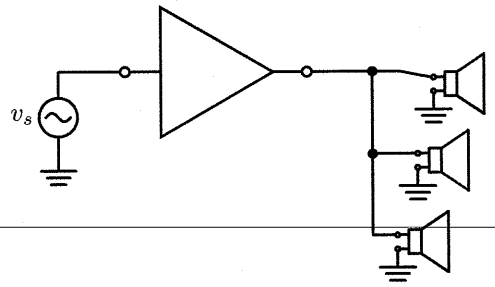
BATTERY IS ABSORBING 3W

(b) (*4 points*) A current of 15fA (f= 10^{-15}) flows through a resistor. On average, how many electrons flow into negative terminal of the resistor per second?

$I = 15fA = 15 \times 10^{-15} \frac{C}{s}$

$1e = 1.6 \times 10^{-19} C$

$N = \frac{I}{1e} \simeq 94,000$ electrons/sec

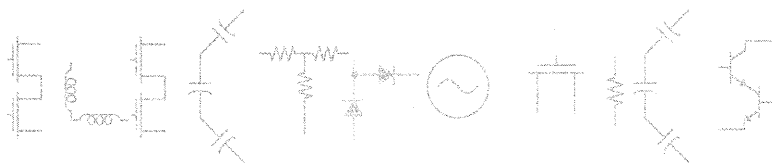


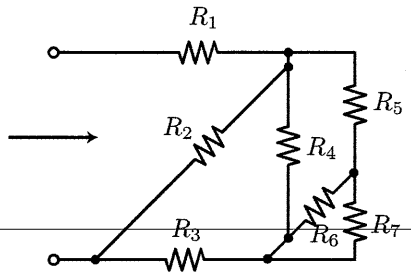
(c) (*4 points*) Calculate the voltage at the output driving the load speakers. Each speaker alone has an effective resistance of $R_L = 9\Omega$. The amplifier has an open-circuit voltage gain of 40dB ($A_v = 100$) and an input resistance of 10Ω and an output impedance of 3Ω. Assume the source has a voltage of $1V$.

$$R_L' = \frac{R_L}{3} = 3\Omega$$

$$V_o = \frac{R_L'}{R_L' + R_o} \times V_s \times A_V$$

$$= \frac{3\Omega}{6\Omega} \times 1V \times 100 = 50V$$



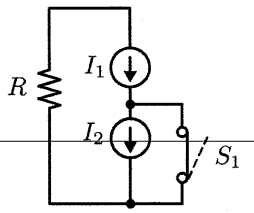


(d) (*4 points*) Find the equivalent resistance seen by looking into the terminals shown. Do not do any math but simply state the answer using the "||" and "+" operators. Use parenthesis to clarify your answer.

$$R = R_1 + R_2 \,||\, \left(R_4 \,||\, (R_5 + R_6 \,||\, R_7) + R_3 \right)$$

2. (*17 points*) For each schematic, describe any issues (if any) when combinations of switches are opened and closed. State which principals are violated in each case.

3

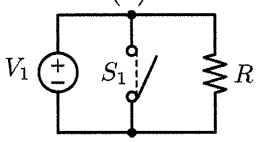


S_1 OPEN : $I_1 \neq I_2 \Rightarrow$ KCL VIOLATION

S_1 CLOSED : OK

3

(a)

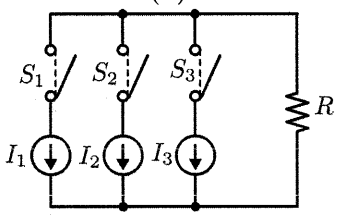


S_1 OPEN : OK

S_1 CLOSED : KVL VIOLATION
V_1 SHORTED

4

(b)

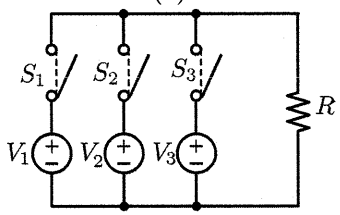


all S_k CLOSED : OK

S_k OPEN : KCL VIOLATION
I_k OPEN CIRCUITED

4

(c)



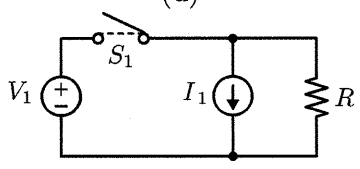
S_k OPEN : OK

ONLY ONE S_k CLOSED : OK

MORE THAN TWO CLOSED : KVL VIOLATION

3

(d)

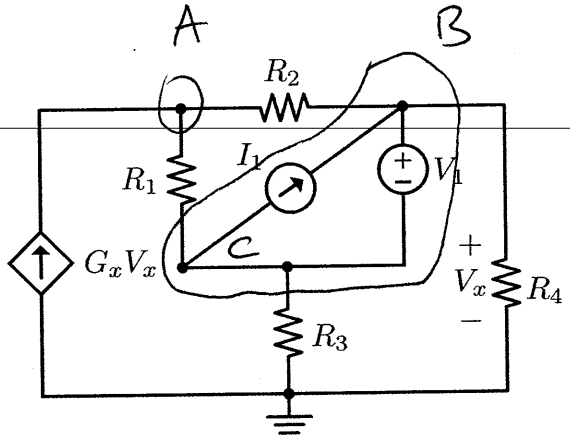


S_1 : OPEN : OK

S_1 CLOSED : OK

(e)

3. (*17 points*) For the following circuit, write nodal equations and put them into standard format, $Ax = b$. Assume the reference voltage is chosen as shown by the ground symbol.



$V_C = V_B - V_1$

$V_X = V_B$

KCL @ A

$$-G_X V_B + (V_A - V_B + V_1) G_1 + (V_A - V_B) G_2 = 0$$

$$V_A(G_1 + G_2) + V_B(-G_X - G_1 - G_2) = -G_1 V_1$$

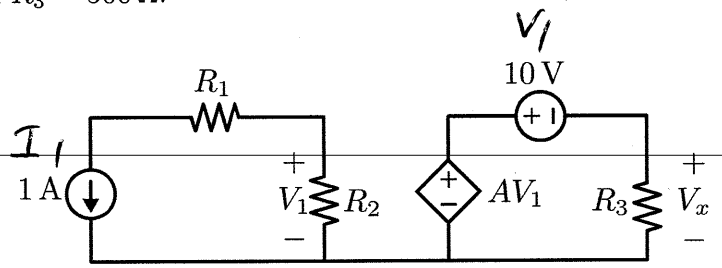
KCL @ SUPERNODE

$$V_B G_4 + (V_B - V_1) G_3 + (V_B - V_A) G_2 + (V_B - V_1 - V_A) G_1 = 0$$

$$V_A(-G_1 - G_2) + V_B(G_1 + G_2 + G_3 + G_4) = (G_3 + G_1) V_1$$

$$\begin{bmatrix} G_1+G_2 & -(G_1+G_2+G_X) \\ -(G_1+G_2) & G_1+G_2+G_3+G_4 \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} -G_1 V_1 \\ (G_1+G_3) V_1 \end{bmatrix}$$

4. (*16 points*) Use superposition to find V_x in the following circuit. $A = 10$, $R_1 = 1\,\text{k}\Omega$, $R_2 = 3\,\text{k}\Omega$, and $R_3 = 500\,\Omega$.



SET $I_1 = 0$ $\Rightarrow V_1 = 0$ $\Rightarrow$ $V_x^I = -10\,V$

SET $V_1 = 0$

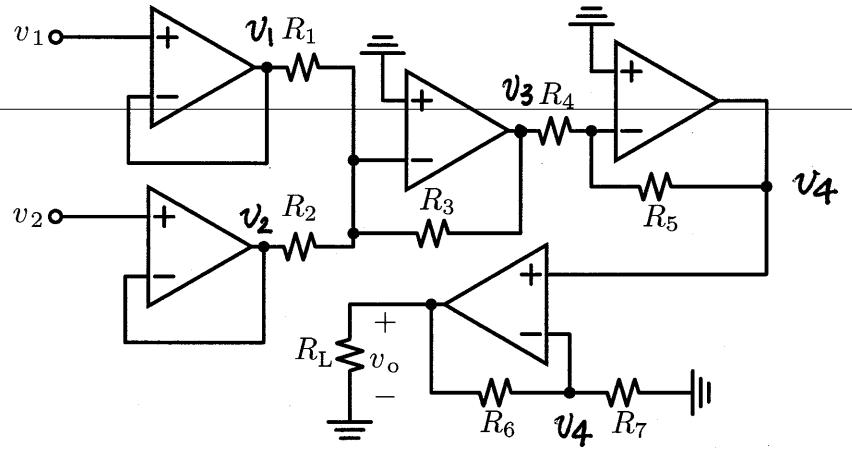
$V_1 = -1A \cdot R_2 = -3\,kV$

$V_x^V = A V_1 = 10 \cdot (-3\,kV) = -30\,kV$

$V_x = V_x^I + V_x^V = -30\,kV - 10V$

$\simeq -30\,kV$

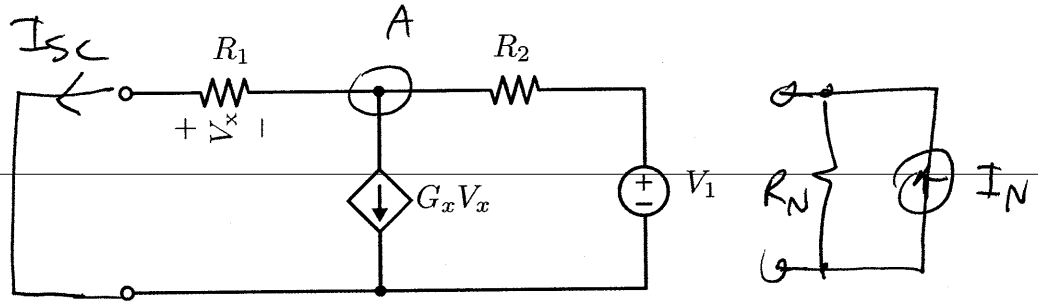
6. (*17 points*) Calculate the output voltage v_o as a function of v_1 and v_2. (*Hint:* Partition the circuit into stages.)



$$\begin{cases} \dfrac{v_1}{R_1} + \dfrac{v_2}{R_2} + \dfrac{v_3}{R_3} = 0 \Rightarrow v_3 = -\dfrac{R_3}{R_1} v_1 - \dfrac{R_3}{R_2} v_2 \\ \dfrac{v_3}{R_4} + \dfrac{v_4}{R_5} = 0 \Rightarrow v_4 = -\dfrac{R_5}{R_4} v_3 \\ \dfrac{v_o - v_4}{R_6} = \dfrac{v_4}{R_7} \Rightarrow v_o = \left(1 + \dfrac{R_6}{R_7}\right) v_4 \end{cases}$$

$$\Rightarrow \boxed{v_o = \left(1 + \frac{R_6}{R_7}\right)\left(-\frac{R_5}{R_4}\right)\left(-\frac{R_3}{R_1} v_1 - \frac{R_3}{R_2} v_2\right)}$$

5. (*17 points*) Find the Norton Equivalent for the following circuit.



FIND SHORT CIRCUIT CURRET:

KCL @ A: $V_A G_1 + G_x V_x + (V_A - V_1) G_2 = 0$

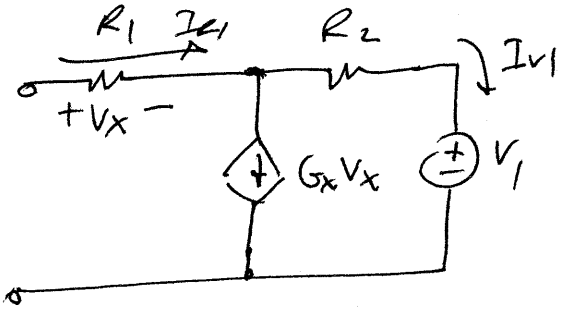
$V_x = -V_A$

$V_A (G_1 - G_x + G_2) = G_2 V_1$

$$V_A = \frac{G_2}{G_1 + G_2 - G_x} V_1$$

$$I_N = I_{SC} = G_1 V_A = \frac{G_1 G_2}{G_1 + G_2 - G_x} V_1$$

FIND OPEN CIRCUIT VOLTAGE



SINCE $I_{R_1} = 0$

$\Rightarrow V_x = 0$

$\Rightarrow G_x V_x = 0$

$\Rightarrow I_{V1} = 0$

$\Rightarrow V_{OC} = V_1$

$$R_N = \frac{V_{OC}}{I_N} = \frac{G_1 + G_2 - G_x}{G_1 G_2} = R_1 + R_2 - G_x R_1 R_2$$