EECS 242:
Receiver Architectures

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Outline

- Complex baseband equivalent of a bandpass signal
- Double-conversion single-quadrature (Superheterodyne)
- Direct-conversion (Single-conversion single-quad, homodyne, zero-IF)
- Weaver; Double-conversion double-quad
- Low-IF
- References

Complex Baseband

- Any passband waveform can be written in the following form:

$$s_p(t) = a(t)\cos\left[\omega_c t + \theta(t)\right]$$

$$s_p(t) = a(t)\cos\omega_c t\cos\theta(t) - a(t)\sin\omega_c t\sin\theta(t)$$

$$s_p(t) = \sqrt{2}s_c(t)\cos\omega_c t - \sqrt{2}s_s(t)\sin\omega_c t$$

$$s_c(t) \triangleq a(t)\cos\theta(t) = I(t) \qquad s_s(t) \triangleq a(t)\sin\theta(t) = Q(t)$$

$$a(t) = |s(t)| = \sqrt{s_c^2(t) + s_s^2(t)} \qquad \theta(t) = \tan^{-1}\frac{s_s(t)}{s_c(t)}$$

- We define the complex baseband signal and show that all operations at passband have a simple equivalent at complex baseband:

$$s(t) = s_c(t) + js_s(t) = I(t) + jQ(t)$$

$$s_p(t) = \mathrm{Re}\left\{\sqrt{2}s(t)e^{j\omega_c t}\right\}$$

$$||s||^{=}||s_p||^2$$

Orthogonality I/Q

- An important relationship is the orthogonality between the modulated I and Q signals. This can be proved as follows (Parseval's Relation):

$$x_c(t) = \sqrt{2} s_c(t) \cos \omega_c t \qquad x_s(t) = \sqrt{2} s_s(t) \sin \omega_c t$$

$$< x_c, x_s > = < X_c, X_s > = 0 \qquad < X_c, X_s > = \int_{-\infty}^{\infty} X_c(f) X_s^*(f) df$$

$$x_c(t) = \frac{1}{\sqrt{2}} (s_c(t) e^{j\omega_c t} + s_c(t) e^{-j\omega_c t}) \qquad X_c(f) = \frac{1}{\sqrt{2}} \left(S_c(f - f_c) + S_c(f + f_c) \right)$$

$$\cos \theta = \frac{1}{2} (e^{j\theta} + e^{-j\theta})$$

$$X_s(f) = \frac{1}{\sqrt{2} j} \left(S_s(f - f_c) - S_s(f + f_c) \right)$$

$$< X_c, X_s > = \frac{1}{2j} \int_{-\infty}^{\infty} \left((S_c(f - f_c) + S_c(f + f_c)) \times (S_s^*(f - f_c) - S_s^*(f + f_c)) \right) df$$

Orthogonality (Freq. Dom.)

- In the above integral, if the carrier frequency is larger than the signal bandwidth, then the frequency shifted signals do not overlap

$$S_c(f - f_c)S_s^*(f + f_c) \equiv 0$$

$$S_c(f + f_c)S_s^*(f - f_c) \equiv 0$$

$$< X_c, X_s >= \frac{1}{2j}\left[\int_{-\infty}^{\infty} S_c(f - f_c)S_s^*(f - f_c)df - \int_{-\infty}^{\infty} S_c(f + f_c)S_s^*(f + f_c)df\right]$$

$$< X_c, X_s >= \frac{1}{2j}\left[\int_{-\infty}^{\infty} S_c(f)S_s^*(f)df - \int_{-\infty}^{\infty} S_c(f)S_s^*(f)df\right] = 0$$

- Due to this orthogonality, we can double the bandwidth of our signal my modulating the I and Q independently. Also, we have

$$< u_p, v_p >=< u_c, v_c > + < u_s, v_s >= \mathrm{Re}\left(< u, v >\right)$$

Complex Baseband Spectrum

- Since the passband signal is real, it has a conjugate symmetric spectrum about the origin. Let's define the positive portion as follows:

$$S_p^+(f) = S_p(f)u(f)$$

- Then the spectrum of the passband and baseband complex signal are related by:

$$S(f) = \sqrt{2}S_p^+(f+f_c) \qquad S_p(f) = \frac{S(f-f_c) + S^*(-f-f_c)}{\sqrt{2}}$$

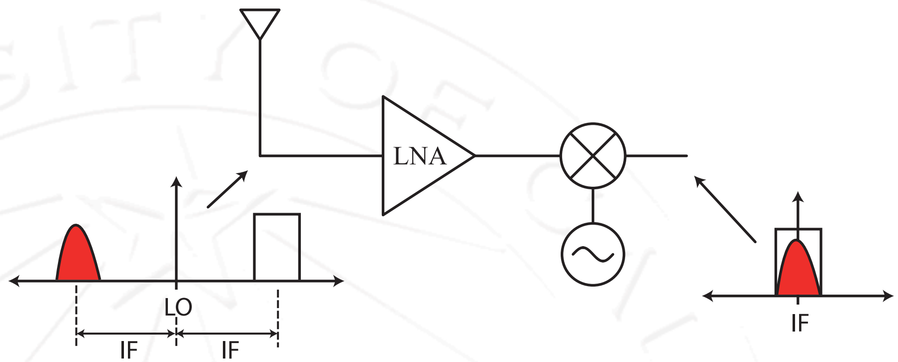
  - Since:

$$v(t) = \sqrt{2}s(t)e^{j\omega_c t} \qquad V(f) = \sqrt{2}S(f-f_c)$$

$$S_p(t) = \mathrm{Re}(v(t)) = \frac{v(t) + v(t)^*}{2}$$

$$S_p(f) = \frac{V(f) + V^*(-f)}{2} = \frac{S(f-f_c) + S^*(-f-f_c)}{\sqrt{2}}$$

The Image Problem



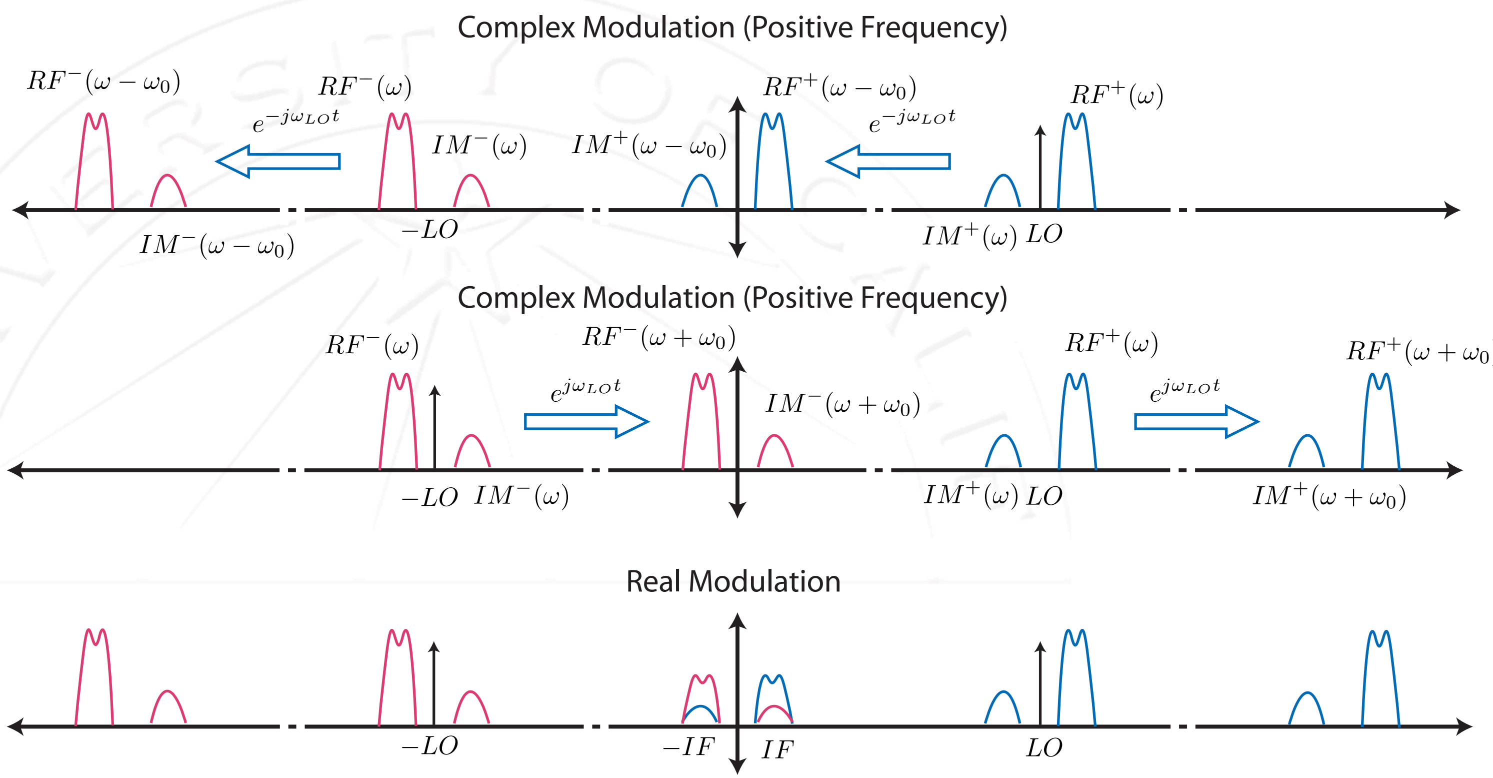
$$m_r(t)\cos(\omega_{LO}+\omega_{IF})t\times\cos(\omega_{LO})t = \frac{1}{2}m_r(t)\left(\cos(2\omega_{LO}+\omega_{IF})t+\cos(\omega_{IF})t\right)$$

$$m_i(t)\cos(\omega_{LO}-\omega_{IF})t\times\cos(\omega_{LO})t = \frac{1}{2}m_i(t)\left(\cos(2\omega_{LO}-\omega_{IF})t+\cos(\omega_{IF})t\right)$$

- After low-pass filtering the mixer output, the IF is given by

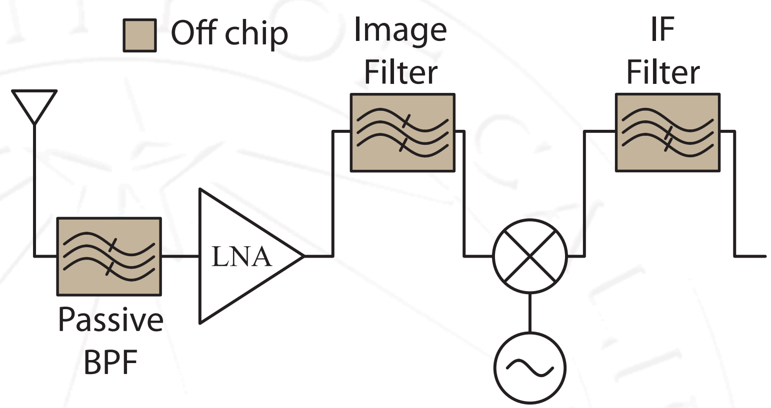
$$IF_{output} = \frac{1}{2}\left(m_i(t)+m_r(t)\right)\cos(\omega_{IF})t$$

Image Problem (Freq Dom)



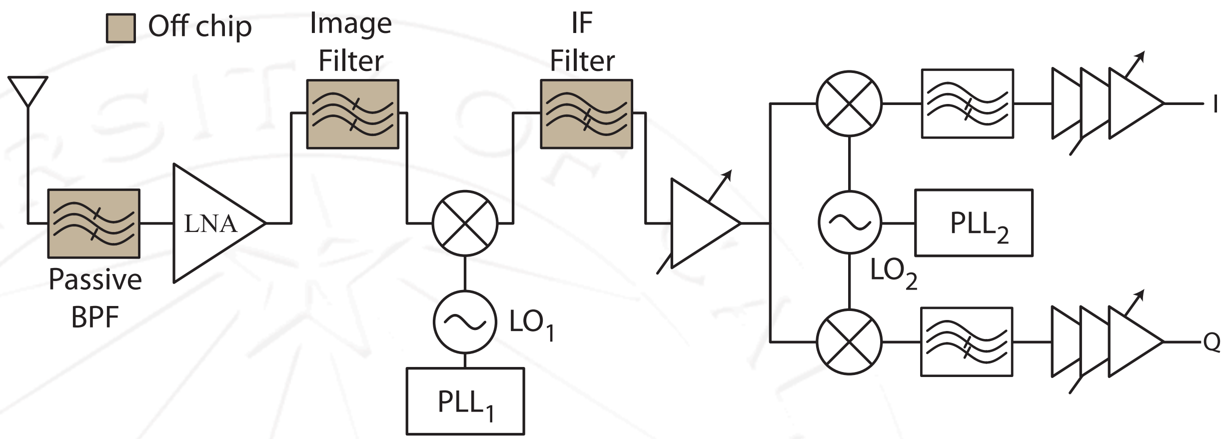
- Complex modulation shifts in only one direction … real modulation shifts up and down

Superheterodyne Architecture

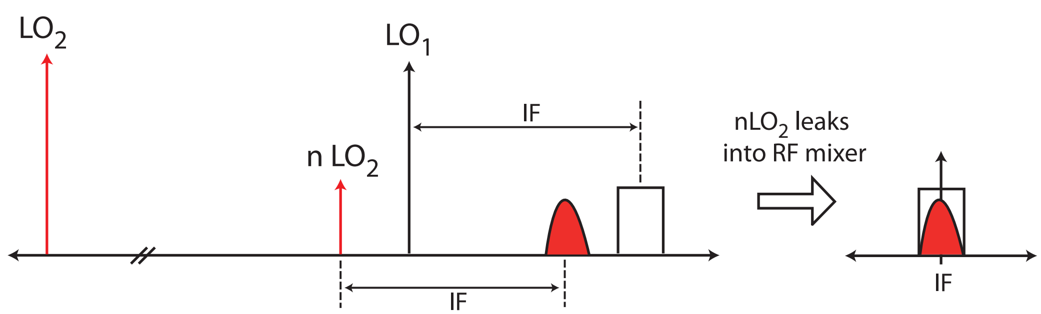


- The choice of the IF frequency dictated by:
  - If the IF is set too low, then we require a very high-Q image reject filter, which introduces more loss and therefore higher noise figure in the receiver (not to mention cost).
  - If the IF is set too high, then subsequent stages consume more power (VGA and filters)
  - Typical IF frequency is 100-200 MHz.

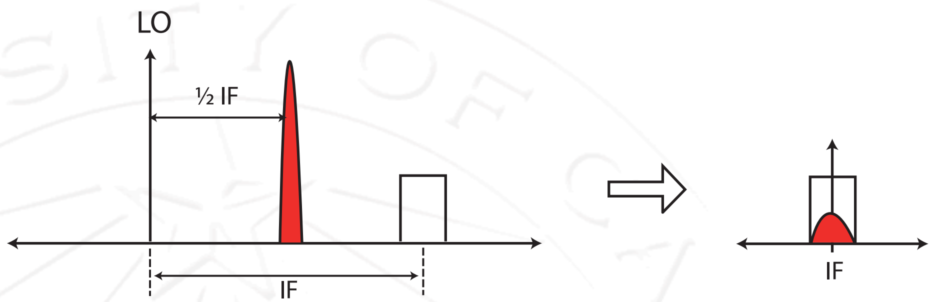
LO Planning in Superhet



- Two separate VCOs and synthesizers are used. The IF LO is fixed, while the RF LO is variable to down-convert the desired channel to the passband of the IF filter (SAW).
- This typically results in a 3-4 chip solution with many off-chip components.
- LO$_1$ should never be made close to an integer multiple of LO$_2$ for any channel. The Nth harmonic of the the fixed LO2 could leak into the RF mixer and cause unwanted mixing.



The ½ IF Problem



- Assume that there is a blocker half-way between the LO and the desired channel. Due to second-order non-linearity in the RF front-end:

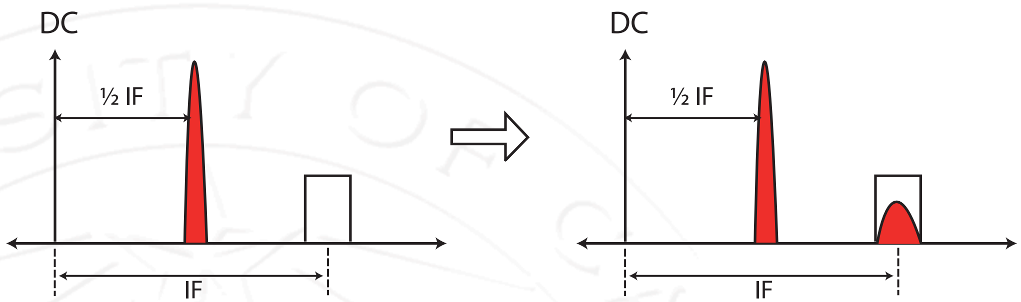
$$\left[m_{blocker}(t)\cos(\omega_{LO}+\frac{1}{2}\omega_{IF})t\right]^2 = (m_{blocker}(t))^2 + (m_{blocker}(t))^2)\cos(2\omega_{LO}+\omega_{IF})t$$

- If the LO has a second-order component, then this signal will fold right on top of the desired signal at IF:

$$\left[(m_{blocker}(t))^2)\cos(2\omega_{LO}+\omega_{IF})t\right]\cos(2\omega_{LO})t = (m_{blocker}(t))^2\cos(\omega_{IF})t + \cdots$$

Note: Bandwidth expansion of blocker due to squaring operation.

Half-IF Continued

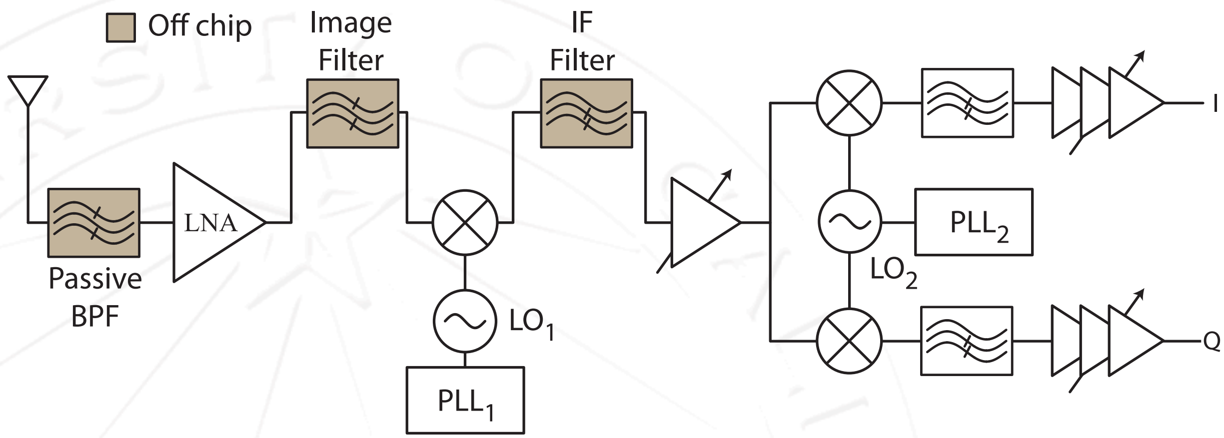


- If the IF stage has strong second-order non-linearity, then the half-IF problem occurs through this mechanism:

$$2\left[m_{blocker}(t)\cos(\frac{1}{2}\omega_{IF})t\right]^2 = (m_{blocker}(t))^2 + (m_{blocker}(t))^2\cos(\omega_{IF})t$$

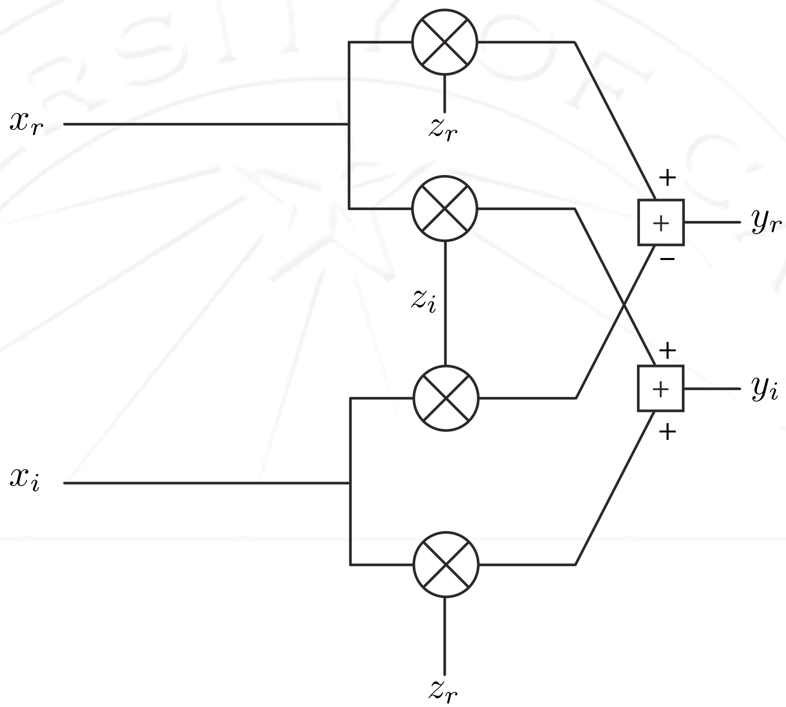
- This highlights the importance of frequency planning. One should select the IF by making sure that there is no strong half-IF blocker. If one exists, then the second-order non-linearity must be carefully managed.

Dual-Conversion Single-Quad



- Disadvantages:
  - Requires bulky off-chip SAW filters
  - As before, two synthesizers are required
  - Typically a three chip solution (RF, IF, and Synth)
- Advantages:
  - Robust. The clear choice for extremely high sensitivity radios
  - High dynamic range SAW filter reduces/relaxes burden on active circuits. This makes it much easier to design the active circuitry.
  - By the same token, the power consumption is lower (< 25mA)

Complex Mixer



$$y = x \cdot z = (x_r + jx_i) \cdot (z_r + jz_i)$$

$$y_r = x_r z_r - x_i z_i$$

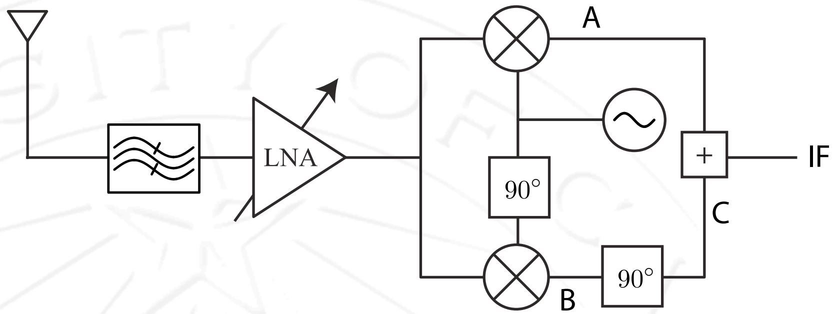
$$y_i = x_i z_r + x_r z_i$$

$$y(t) = e^{j\omega_0 t} x(t)$$

$$Y(\omega) = X(\omega - \omega_0)$$

- A complex mixer is derived by simple substitution.
- Note that a complex exponential only introduces a frequency shift in one direction (no image rejection problems).

Hilbert Architecture



- Image suppression by proper phase shifting.

$$RF = m_r(t)\cos(\omega_{LO} + \omega_{IF})t + m_i(t)\cos(\omega_{LO} - \omega_{IF})t$$

$$A = RF \times \cos(\omega_{LO} t) = \frac{1}{2} m_r(t)\left(\cos(2\omega_{LO} + \omega_{IF})t + \cos(\omega_{IF})t\right) + \frac{1}{2} m_i(t)\left(\cos(2\omega_{LO} - \omega_{IF})t + \cos(\omega_{IF})t\right)$$

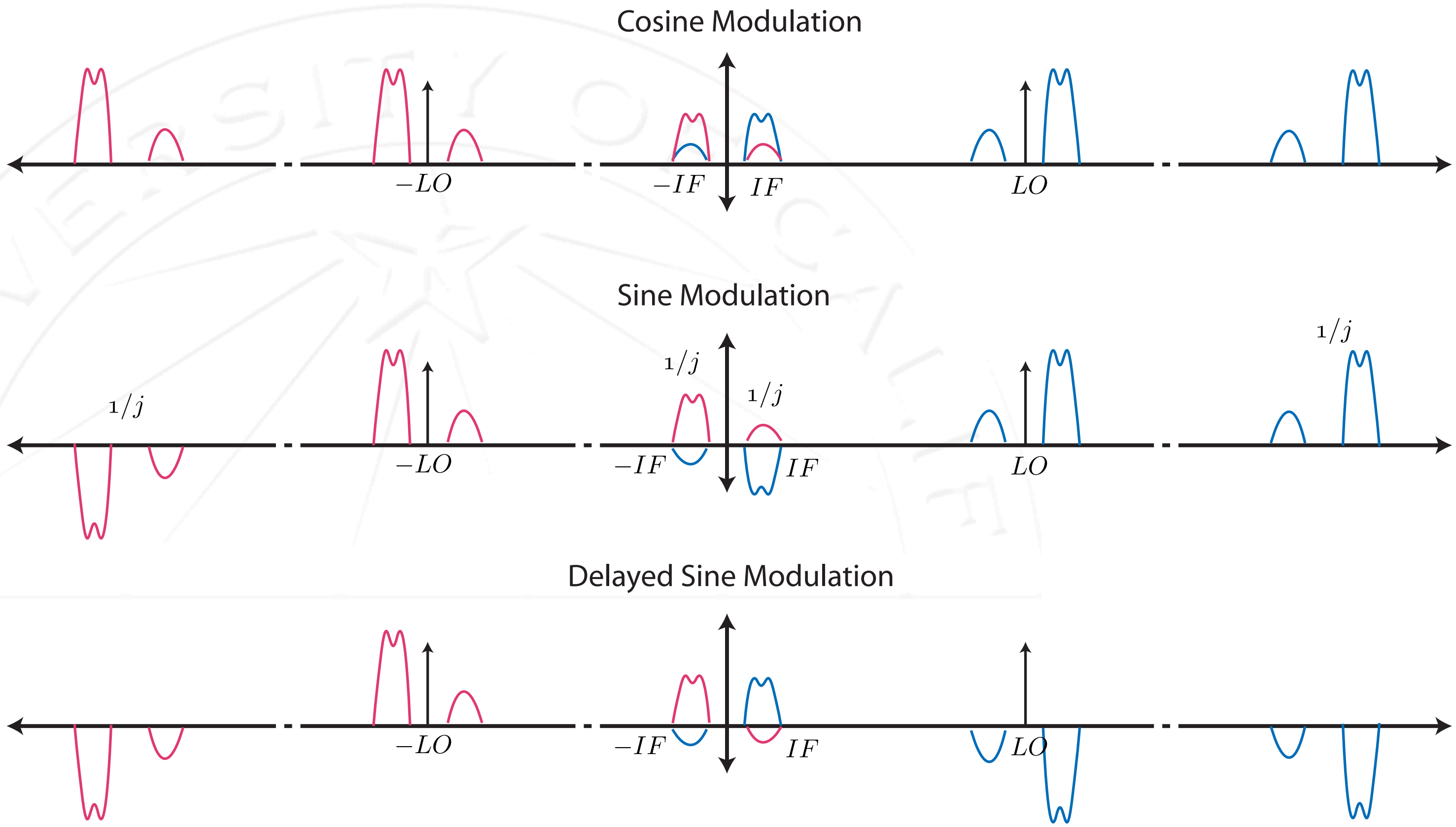
$$B = RF \times \sin(\omega_{LO} t) = \frac{1}{2} m_r(t)\left(\sin(2\omega_{LO} + \omega_{IF})t - \sin(\omega_{IF})t\right) + \frac{1}{2} m_i(t)\left(\sin(2\omega_{LO} - \omega_{IF})t + \sin(\omega_{IF})t\right)$$

$$C = \frac{1}{2} m_r(t)\left(-\cos(2\omega_{LO} + \omega_{IF})t + \cos(\omega_{IF})t\right) + \frac{1}{2} m_i(t)\left(-\cos(2\omega_{LO} - \omega_{IF})t - \cos(\omega_{IF})t\right)$$

$$IF^+ = A + C = m_r(t)\cos(\omega_{IF} t)$$

$$IF^- = A - C = m_i(t)\cos(\omega_{IF} t)$$

Sine/Cosine Together

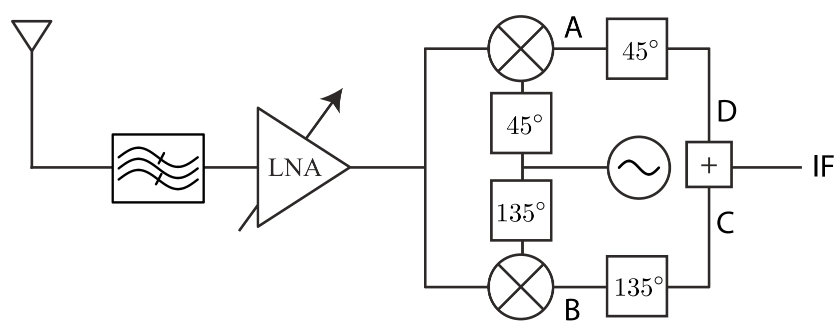


- Since the sine treats positive/negative frequencies differently (above/below LO), we can exploit this behavior
- A 90° phase shift is needed to eliminate the image
- 90° phase shift equivalent to multiply by –j sign(f)

Hilbert Implementation

- Advantages:
  - Remove the external image-reject SAW filter
  - Better integration
- Requires extremely good matching of components (paths gain/phase). Without trimming/calibration, only ~40dB image rejection is possible. Many applications require 60dB or more.
- Power hungry (more mixers and higher cap loading).

Note: A real implementation uses 45/135 degree phase shifters for better matching/ tracking.



Gain/Phase Imbalance

$$A = RF \times (1+\alpha)\cos(\omega_{LO}t + \frac{\phi}{2}) = \begin{array}{l} \frac{1}{2}m_r(t)(1+\alpha)\left(\cos(2\omega_{LO}t + \omega_{IF}t + \frac{\phi}{2}) + \cos(\omega_{IF}t - \frac{\phi}{2})\right) + \\ \frac{1}{2}m_i(t)(1+\alpha)\left(\cos(2\omega_{LO}t - \omega_{IF}t + \frac{\phi}{2}) + \cos(\omega_{IF}t + \frac{\phi}{2})\right) \end{array}$$

$$B = RF \times (1-\alpha)\cos(\omega_{LO}t - \frac{\phi}{2}) = \begin{array}{l} \frac{1}{2}m_r(t)(1-\alpha)\left(\sin(2\omega_{LO}t + \omega_{IF}t - \frac{\phi}{2}) - \sin(\omega_{IF}t - \frac{\phi}{2})\right) + \\ \frac{1}{2}m_i(t)(1-\alpha)\left(\sin(2\omega_{LO}t - \omega_{IF}t - \frac{\phi}{2}) + \sin(\omega_{IF}t - \frac{\phi}{2})\right) \end{array}$$

$$C = \begin{array}{l} \frac{1}{2}m_r(t)(1-\alpha)\left(-\cos(2\omega_{LO}t + \omega_{IF}t - \frac{\phi}{2}) + \cos(\omega_{IF}t - \frac{\phi}{2})\right) + \\ \frac{1}{2}m_i(t)(1-\alpha)\left(-\cos(2\omega_{LO}t - \omega_{IF}t - \frac{\phi}{2}) - \cos(\omega_{IF}t - \frac{\phi}{2})\right) \end{array}$$

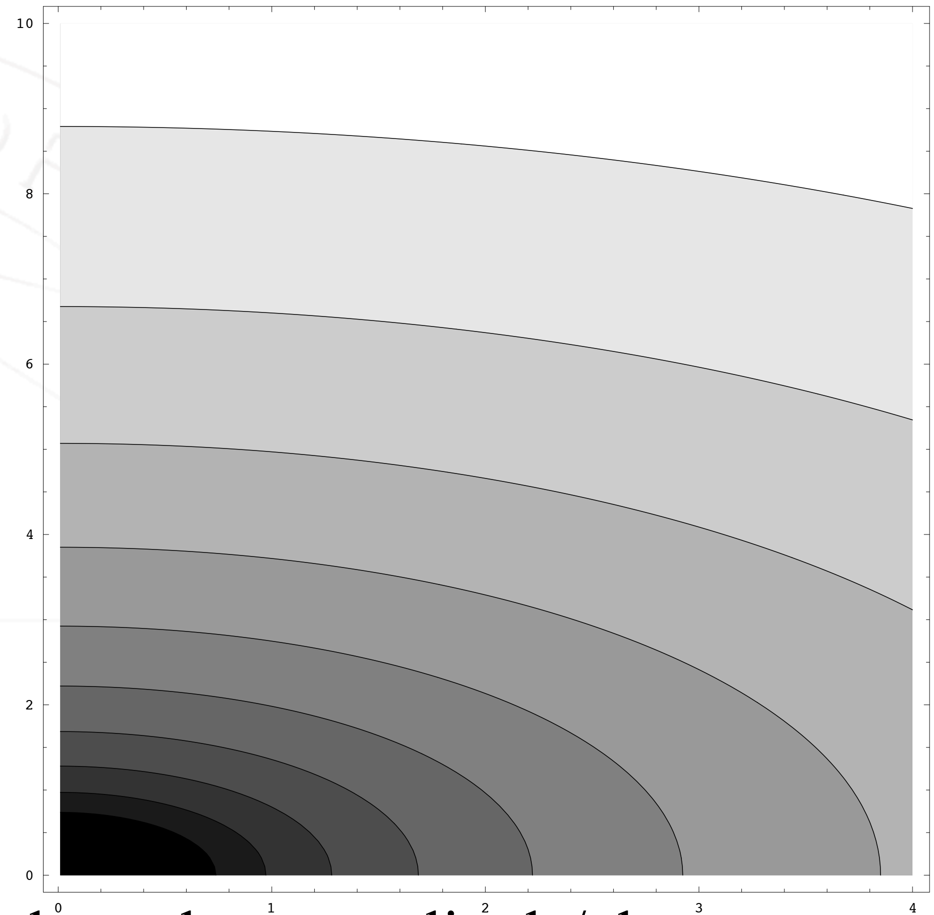
$$IF = A + C = \frac{m_r(t)}{2}\left((1+\alpha)\cos(\omega_{IF}t - \frac{\phi}{2}) + (1-\alpha)\cos(\omega_{IF}t + \frac{\phi}{2})\right) + \frac{m_i(t)}{2}\left((1+\alpha)\cos(\omega_{IF}t + \frac{\phi}{2}) - (1-\alpha)\cos(\omega_{IF}t - \frac{\phi}{2})\right)$$

$$IF = m_r(t)\left[\cos(\omega_{IF}t)\cos(\frac{\phi}{2}) - \alpha\sin(\omega_{IF}t)\sin(\frac{\phi}{2})\right] + m_i(t)\left[\alpha\cos(\omega_{IF}t)\cos(\frac{\phi}{2}) - \sin(\omega_{IF}t)\sin(\frac{\phi}{2})\right]$$

Image-Reject Ratio

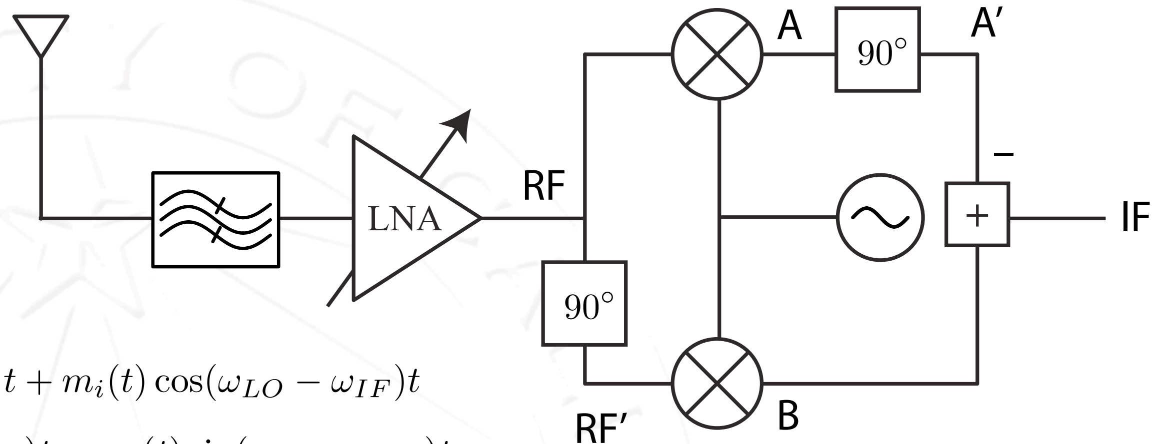
$$\mathrm{IR(dB)} = 10\log\left(\frac{\cos\frac{\phi}{2} - \alpha\sin\frac{\phi}{2}}{\alpha\cos\frac{\phi}{2} + \sin\frac{\phi}{2}}\right)^2$$

$$IR \approx= \frac{\alpha^2 + \phi^2}{4}$$



- Level of image rejection depends on amplitude/phase mismatch
- Typical op-chip values of 30-40 dB achieved (< 5°, < 0.6 dB)

RF/IF Phase Shift, Fixed LO



$$RF = m_r(t)\cos(\omega_{LO} + \omega_{IF})t + m_i(t)\cos(\omega_{LO} - \omega_{IF})t$$

$$RF' = -m_r(t)\sin(\omega_{LO} + \omega_{IF})t - m_i(t)\sin(\omega_{LO} - \omega_{IF})t$$

$$A = RF \times \cos(\omega_{LO}t) = \frac{1}{2}m_r(t)\left(\cos(2\omega_{LO} + \omega_{IF})t + \cos(\omega_{IF})t\right) + \frac{1}{2}m_i(t)\left(\cos(2\omega_{LO} - \omega_{IF})t + \cos(\omega_{IF})t\right)$$

$$A'_{LPF} = -m_r(t)\sin(\omega_{IF}t) - m_i(t)\sin(\omega_{IF}t)$$

$$B = RF' \times \cos(\omega_{LO}t) = \frac{1}{2}m_r(t)\left(-\sin(2\omega_{LO} + \omega_{IF})t + \sin(\omega_{IF})t\right) + \frac{1}{2}m_i(t)\left(-\sin(2\omega_{LO} - \omega_{IF})t - \sin(\omega_{IF})t\right.$$

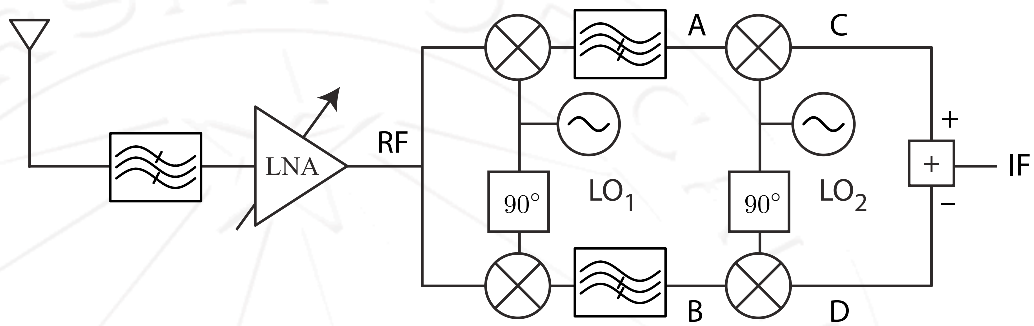
$$IF^+ = B - A' = m_r(t)\sin(\omega_{IF}t)$$

- This requires a 90 degree phase shift across the band. It's much easier to shift the phase of a single frequency (LO).
- Polyphase filters can be used to do this, but a broadband implementation requires many stages (high loss)

Weaver Architecture

$RF = m_r(t)\cos(\omega_{LO_1} + \omega_{IF_1})t + m_i(t)\cos(\omega_{LO_1} - \omega_{IF_1})t$

$IF_1 = LO_1 - RF$



$$IF = LO_2 - IF_1 = LO_2 - LO_1 + RF = RF - (LO_1 - LO_2)$$

- Eliminates the need for a phase shift in the signal path. Easier to implement phase shift in the LO path.
- Can use a pair of quadrature VCOs. Requires 4X mixers!
- Sensitive to second image.

$$A_{LPF} = \cos\omega_{LO_1}t \times RF = \frac{m_r}{2}\cos(\omega_{IF_1})t + \frac{m_i}{2}\cos(\omega_{IF_1})t$$

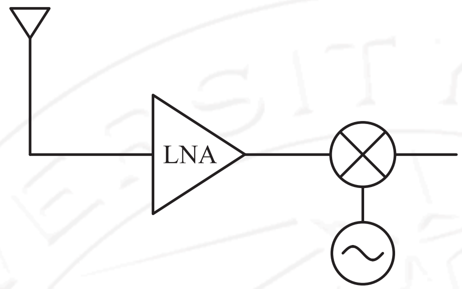
$$B_{LPF} = \sin\omega_{LO_1}t \times RF = -\frac{m_r}{2}\sin(\omega_{IF_1})t + \frac{m_i}{2}\sin(\omega_{IF_1})t$$

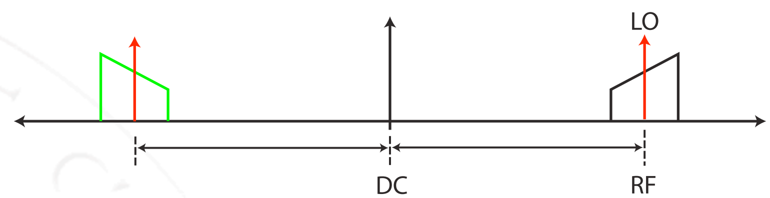
$$C_{LPF} = A \times \cos\omega_{LO2}t = \frac{m_r}{4}\cos(\omega_{IF})t + \frac{m_i}{4}\cos(\omega_{IF})t$$

$$D_{LPF} = B \times \sin\omega_{LO2}t = -\frac{m_r}{4}\cos(\omega_{IF})t + \frac{m_i}{4}\cos(\omega_{IF})t$$

$$IF = C - D = \frac{m_r}{2}\cos\omega_{IF}t$$

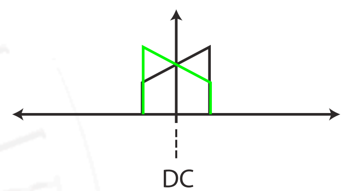
Direct Conversion (Zero-IF)





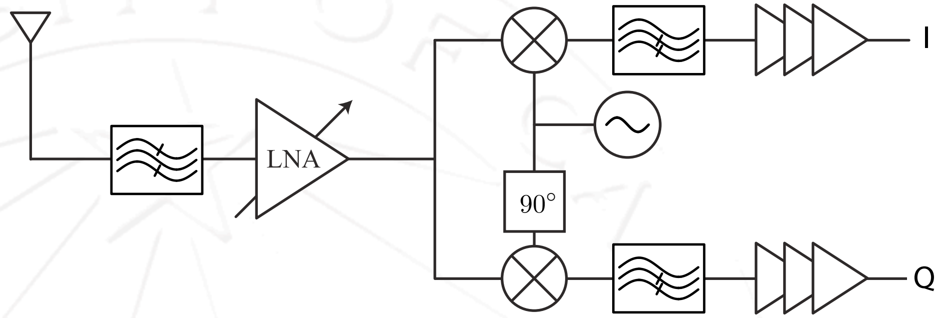
Let $\omega_{RF} = \omega_{LO} = \omega_0$

$$m_r(t)\cos(\omega_{RF})t \times \cos(\omega_{LO})t = \frac{1}{2}m_r(t)\left(1 + \cos(2\omega_0)t\right)$$



- The most obvious choice of LO is the RF frequency, right? IF = LO – RF = DC?
- Why not?
- Even though the signal is its own image, if a complex modulation is used, the complex envelope is asymmetric and thus there is a “mangling” of the signal

Direct Conversion (cont)

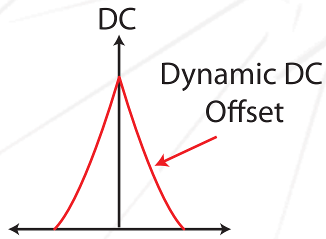


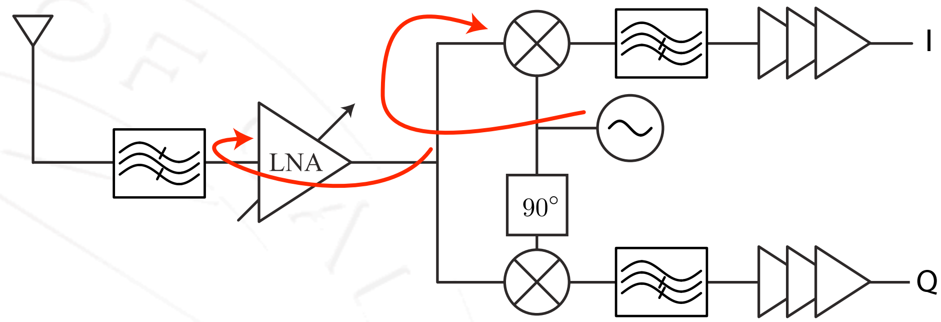
- Use orthogonal mixing to prevent signal folding and retain both I and Q for complex demodulation (e.g. QPSK or QAM)
- Since the image and the signal are the same, the image-reject requirements are relaxed (it's an SNR hit, so typically 20-25 dB is adequate)

Problems with Zero-IF

$$LO = p(t)\cos(\omega_{LO}t + \phi(t))$$

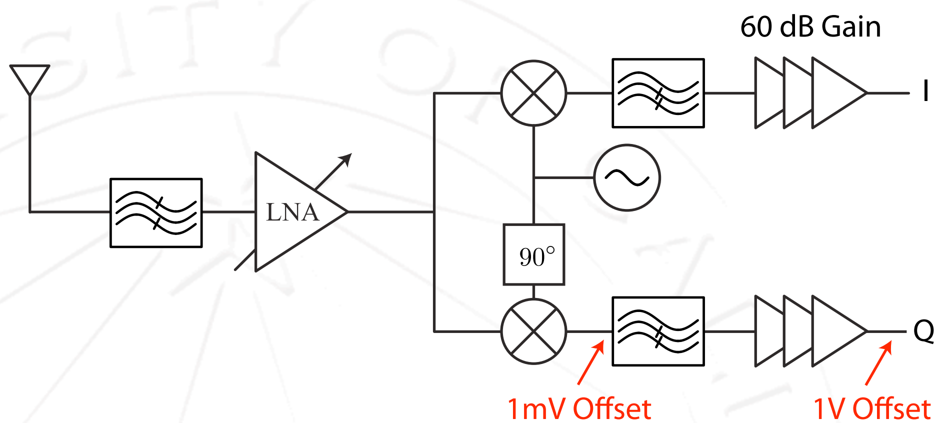
$$LO \times LO = p(t)^2 + p(t)^2 \cos(2\omega_{LO}t + 2\phi(t))$$





- Self-mixing of the LO signal is a big concern.
- LO self-mixing degrades the SNR. The signal that reflects from the antenna and is gained up appears at the input of the mixer and mixes down to DC.
- If the reflected signal varies in time, say due to a changing VSWR on the antenna, then the DC offset is time-varying

DC Offset



- DC offsets that appear at the baseband experience a large gain.   This signal can easily saturate out the receive chain.
- A large AC coupling capacitor or a programmable DC-offset cancellation loop is required. The HPF corner should be low (kHz), which requires a large capacitor.
- Any transients require a large settling time as a result.

Sensitivity to 2nd Order Disto

- Assume two jammers have a frequency separation of Δf:

$$s_1 = m_1(t)\cos(\omega_1 t)$$

$$s_2 = m_2(t)\cos(\omega_1 t + \Delta\omega t)$$

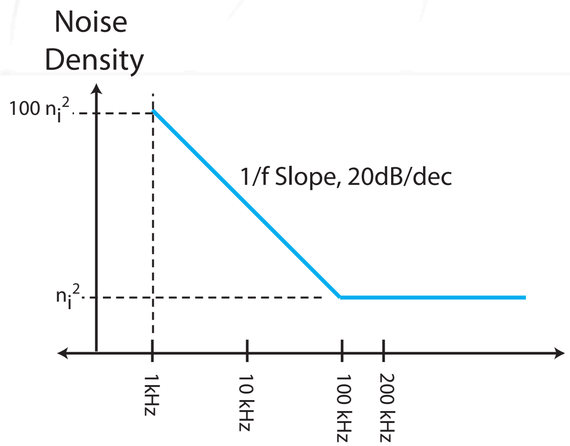
$$(s_1 + s_2)^2 = (m_1(t)\cos\omega_1 t)^2 + (m_2(t)\cos\omega_2 t)^2 + 2m_1(t)m_2(t)\cos(\omega_1 t)\cos(\omega_1 + \Delta\omega)t$$

$$\text{LPF}\{(s_1 + s_2)^2\} = m_1(t)^2 + m_2(t)^2 + m_1(t)m_2(t)\cos(\Delta\omega)t$$

- The two produce distortion at DC. The modulation of the jammers gets doubled in bandwidth.
- If the jammers are close together, then their inter-modulation can also fall into the band of the receiver.
- Even if it is out of band, it may be large enough to saturate the receiver.

Sensitivity to 1/f Noise

- Since the IF is at DC, any low frequency noise, such as 1/f noise, is particularly harmful.
- CMOS has much higher 1/f noise, which requires careful device sizing to ensure good operation.
- Many cellular systems are narrowband and the entire baseband may fall into the 1/f regime!



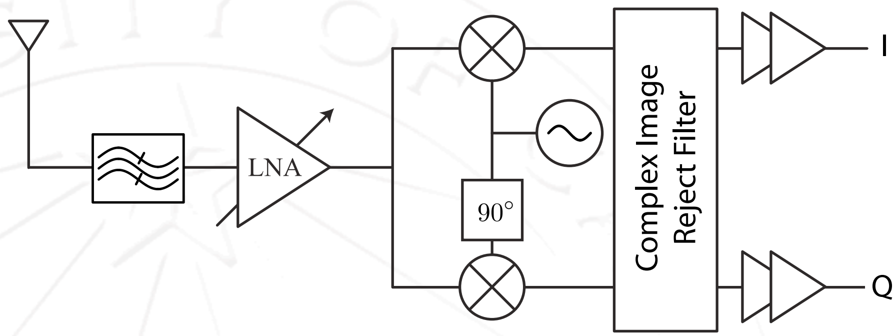
Example: GSM has a 200 kHz bandwidth. Suppose that the flicker corner frequency is 100 kHz. The in-band noise degradation is thus:

$$\overline{v_{ave}^2} = \frac{1}{200\text{kHz}} \int_{1\text{kHz}}^{100\text{kHz}} \frac{a}{f} df + \int_{100\text{kHz}}^{200\text{kHz}} b df$$

$$a = 1\text{kHz} \cdot 100 \cdot \overline{v_i^2} \qquad b = \overline{v_i^2}$$

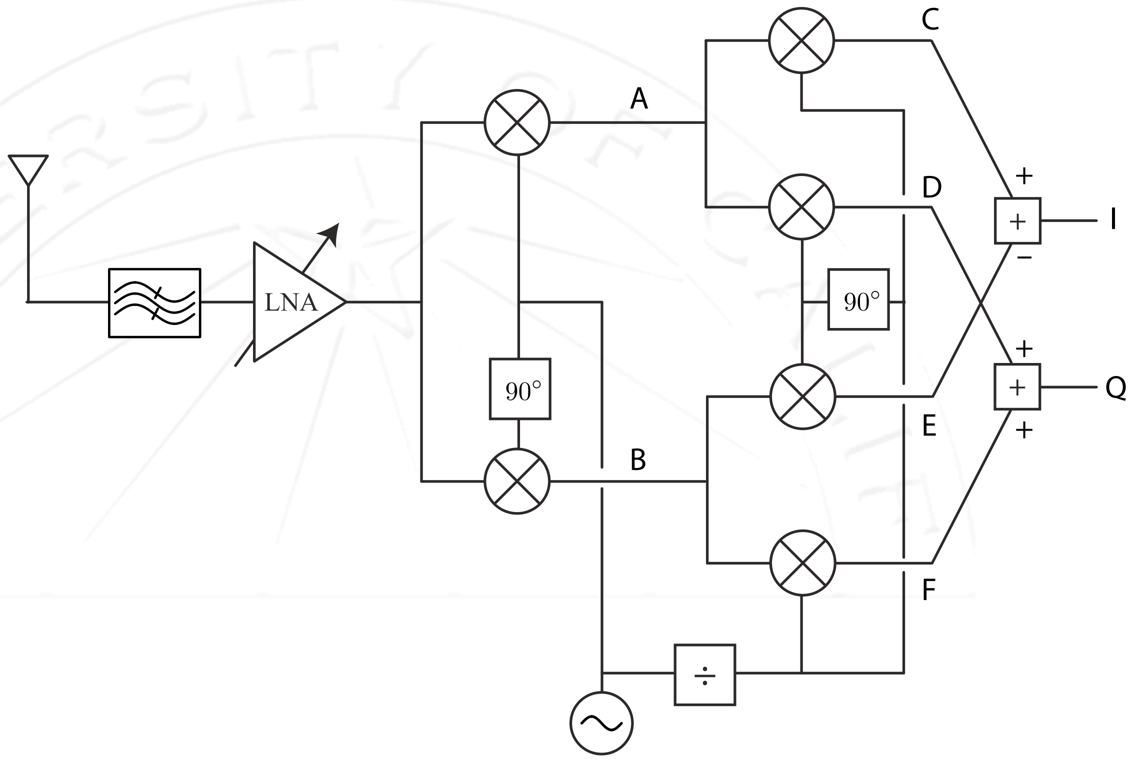
$$\overline{v_{ave}^2} = \frac{1}{200\text{kHz}} \left[a \ln \frac{100\text{k}}{1\text{k}} + b(200\text{k} - 100\text{k}) \right] = \frac{1}{200\text{kHz}} (11.5a + 100\text{k}b) = 6.25\overline{v_i^2}$$

Low IF Architecture



- Instead of going to DC, go a low IF, low enough so that the IF circuitry and filters can be implemented on-chip, yet high enough to avoid problems around DC (flicker noise, offsets, etc). Typical IF is twice the signal bandwidth.
- The image is rejected through a complex filter.

Double-Conversion Double-Quad



- The dual-conversion double-quad architecture has the advantage of de-sensitizing the receiver gain and phase imbalance of the I and Q paths.

Analysis of Double/Double

- Assuming ideal quadrature and no gain errors:

$$RF = m_r(t)\cos(\omega_{LO1} + \omega_{LO2} + \omega_{IF})t + m_i(t)\cos(\omega_{LO1} + \omega_{LO2} - \omega_{IF})t +$$

$$A = \mathrm{LPF}\{RF \times \cos(\omega_{LO1}t)\} = \frac{1}{2}\left\{\begin{array}{c} m_r(t)\cos(\omega_{LO2} + \omega_{IF})t + \\ m_i(t)\cos(\omega_{LO2} - \omega_{IF})t \end{array}\right.$$

$$B = \mathrm{LPF}\{RF \times \sin(\omega_{LO1}t)\} = \frac{1}{2}\left\{\begin{array}{c} -m_r(t)\sin(\omega_{LO2} + \omega_{IF})t \\ -m_i(t)\sin(\omega_{LO2} - \omega_{IF})t \end{array}\right.$$

$$C = \mathrm{LPF}\{A \times \cos(\omega_{LO2}t)\} = \frac{1}{2}\left\{\begin{array}{c} m_r(t)\cos(\omega_{IF})t + \\ m_i(t)\cos(\omega_{IF})t \end{array}\right.$$

$$D = \mathrm{LPF}\{A \times \sin(\omega_{LO2}t)\} = \frac{1}{2}\left\{\begin{array}{c} m_r(t)\sin(\omega_{IF})t + \\ -m_i(t)\sin(\omega_{IF})t \end{array}\right.$$

$$E = \mathrm{LPF}\{B \times \cos(\omega_{LO2}t)\} = \frac{1}{2}\left\{\begin{array}{c} m_r(t)\sin(\omega_{IF})t + \\ -m_i(t)\sin(\omega_{IF})t \end{array}\right.$$

$$F = \mathrm{LPF}\{B \times \sin(\omega_{LO2}t)\} = \frac{1}{2}\left\{\begin{array}{c} -m_r(t)\cos(\omega_{IF})t + \\ -m_i(t)\cos(\omega_{IF})t \end{array}\right.$$

$$I = C - F = (m_r(t) + m_i(t))\cos(\omega_{IF})t$$

$$Q = D + E = (m_r(t) - m_i(t))\sin(\omega_{IF})t$$

Gain Error Analysis

$$RF = m_r(t)\cos(\omega_{LO1}+\omega_{LO2}+\omega_{IF})t + m_i(t)\cos(\omega_{LO1}+\omega_{LO2}-\omega_{IF})t +$$

$$A = \text{LPF}\{RF \times \left(1+\frac{\Delta a_1}{2}\right)\cos(\omega_{LO1}t)\} = \frac{1}{2}\left(1+\frac{\Delta a_1}{2}\right)\begin{cases} m_r(t)\cos(\omega_{LO2}+\omega_{IF})t + \\ m_i(t)\cos(\omega_{LO2}-\omega_{IF})t \end{cases}$$

$$B = \text{LPF}\{RF \times \left(1-\frac{\Delta a_1}{2}\right)\sin(\omega_{LO1}t)\} = \frac{1}{2}\left(1-\frac{\Delta a_1}{2}\right)\begin{cases} -m_r(t)\sin(\omega_{LO2}+\omega_{IF})t \\ -m_i(t)\sin(\omega_{LO2}-\omega_{IF})t \end{cases}$$

$$C = \text{LPF}\{A \times \left(1+\frac{\Delta a_2}{2}\right)\cos(\omega_{LO2}t)\} = \frac{1}{2}\left(1+\frac{\Delta a_1}{2}\right)\left(1+\frac{\Delta a_2}{2}\right)\begin{cases} m_r(t)\cos(\omega_{IF})t + \\ m_i(t)\cos(\omega_{IF})t \end{cases}$$

$$D = \text{LPF}\{A \times \left(1-\frac{\Delta a_2}{2}\right)\sin(\omega_{LO2}t)\} = \frac{1}{2}\left(1+\frac{\Delta a_1}{2}\right)\left(1-\frac{\Delta a_2}{2}\right)\begin{cases} m_r(t)\sin(\omega_{IF})t + \\ -m_i(t)\sin(\omega_{IF})t \end{cases}$$

$$E = \text{LPF}\{B \times \left(1+\frac{\Delta a_2}{2}\right)\cos(\omega_{LO2}t)\} = \frac{1}{2}\left(1-\frac{\Delta a_1}{2}\right)\left(1+\frac{\Delta a_2}{2}\right)\begin{cases} m_r(t)\sin(\omega_{IF})t + \\ -m_i(t)\sin(\omega_{IF})t \end{cases}$$

$$F = \text{LPF}\{B \times \left(1-\frac{\Delta a_2}{2}\right)\sin(\omega_{LO2}t)\} = \frac{1}{2}\left(1-\frac{\Delta a_1}{2}\right)\left(1-\frac{\Delta a_2}{2}\right)\begin{cases} -m_r(t)\cos(\omega_{IF})t + \\ -m_i(t)\cos(\omega_{IF})t \end{cases}$$

$$I = C - F = (1+\Delta a_1\Delta a_2)(m_r(t)+m_i(t))\cos(\omega_{IF})t$$

$$Q = D + E = (1-\Delta a_1\Delta a_2)(m_r(t)-m_i(t))\sin(\omega_{IF})t$$

- The gain mismatch is reduced since due to the product of two small numbers (amplitude errors).

Phase Error Analysis

$$RF = m_r(t)\cos(\omega_{LO1} + \omega_{LO2} + \omega_{IF})t + m_i(t)\cos(\omega_{LO1} + \omega_{LO2} - \omega_{IF})t +$$

$$A = \text{LPF}\{RF \times \cos(\omega_{LO1}t + \phi_1)\} = \frac{1}{2}\left\{\begin{array}{c} m_r(t)\cos(\omega_{LO2} + \omega_{IF} + \phi_1)t + \\ m_i(t)\cos(\omega_{LO2} - \omega_{IF} + \phi_1)t \end{array}\right.$$

$$B = \text{LPF}\{RF \times \sin(\omega_{LO1}t - \phi_1)\} = \frac{1}{2}\left\{\begin{array}{c} -m_r(t)\sin(\omega_{LO2} + \omega_{IF} - \phi_1)t \\ -m_i(t)\sin(\omega_{LO2} - \omega_{IF} - +\phi_1)t \end{array}\right.$$

$$C = \text{LPF}\{A \times \cos(\omega_{LO2}t + \phi_2)\} = \frac{1}{2}\left\{\begin{array}{c} m_r(t)\cos(\omega_{IF} + \phi_1 + \phi_2)t + \\ m_i(t)\cos(\omega_{IF} + \phi_1 + \phi_2)t \end{array}\right.$$

$$D = \text{LPF}\{A \times \sin(\omega_{LO2}t - \phi_2)\} = \frac{1}{2}\left\{\begin{array}{c} m_r(t)\sin(\omega_{IF} - \phi_1 - \phi_2)t + \\ -m_i(t)\sin(\omega_{IF} - \phi_1 - \phi_2)t \end{array}\right.$$

$$E = \text{LPF}\{B \times \cos(\omega_{LO2}t + \phi_2)\} = \frac{1}{2}\left\{\begin{array}{c} m_r(t)\sin(\omega_{IF} + \phi_1 + \phi_2)t + \\ -m_i(t)\sin(\omega_{IF} + \phi_1 + \phi_2)t \end{array}\right.$$

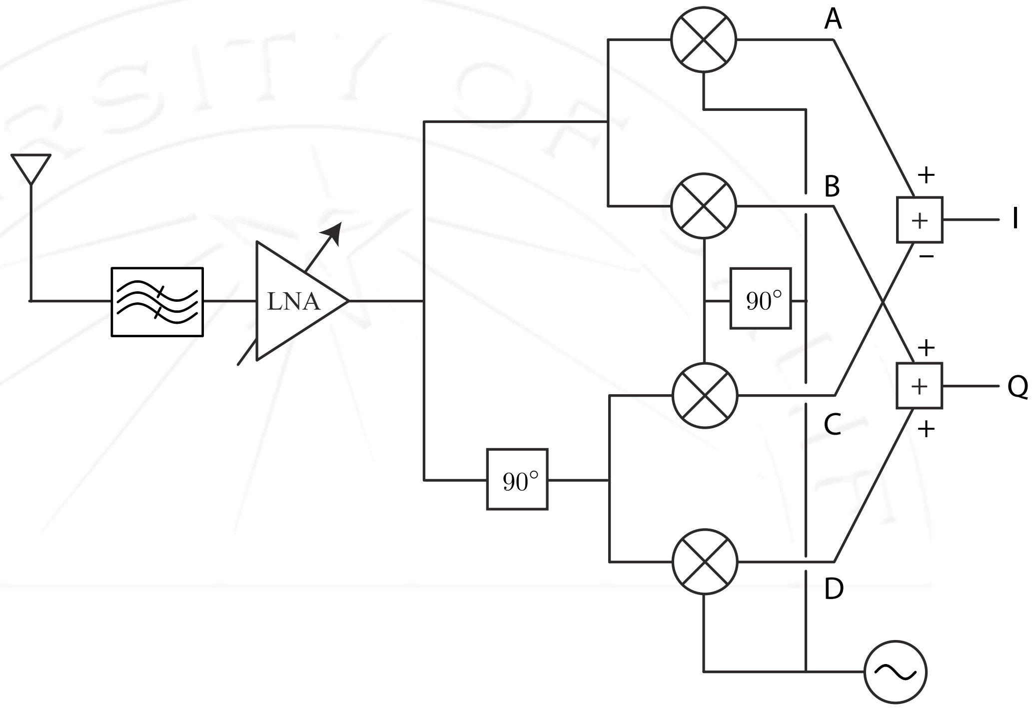
$$F = \text{LPF}\{B \times \sin(\omega_{LO2}t - \phi_2)\} = \frac{1}{2}\left\{\begin{array}{c} -m_r(t)\cos(\omega_{IF} - \phi_1 - \phi_2)t + \\ -m_i(t)\cos(\omega_{IF} - \phi_1 - \phi_2)t \end{array}\right.$$

$$I = C - F = (m_r(t) + m_i(t))\cos(\phi_1 + \phi_2)\cos(\omega_{IF}t)$$

$$Q = D + E = (m_r(t) - m_i(t))\cos(\phi_1 + \phi_2)\sin(\omega_{IF})t$$

- The phase error impacts the I/Q channels in the same way, and as long as the phase errors are small, it has a minimal impact on the gain of the I/Q channels.

Double-Quad Low-IF



- Essentially a complex mixer topology. Mix RF I/Q with LO I/Q to form baseband I/Q
- Improved image rejection due to desensitization to quadrature gain and phase error.

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