EECS 242:
RF Mixers

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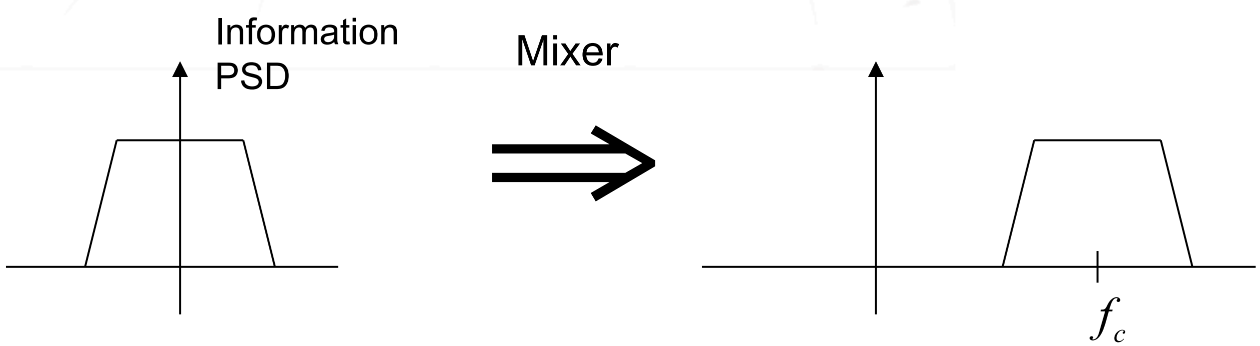
Mixers

The Mixer is a critical component in communication circuits. It translates information content to a new frequency.



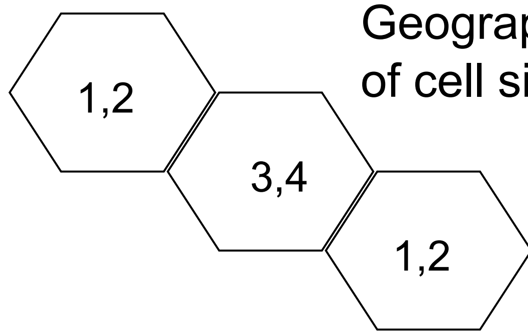
Why use a mixer (transmit side)?

1) Translate information to a frequency appropriate for transmission

   Example: Antennas smaller and more efficient at high frequencies

2) Spectrum sharing: Move information into separate channels in order to share spectrum and allow simultaneous use

3) Interference resiliance

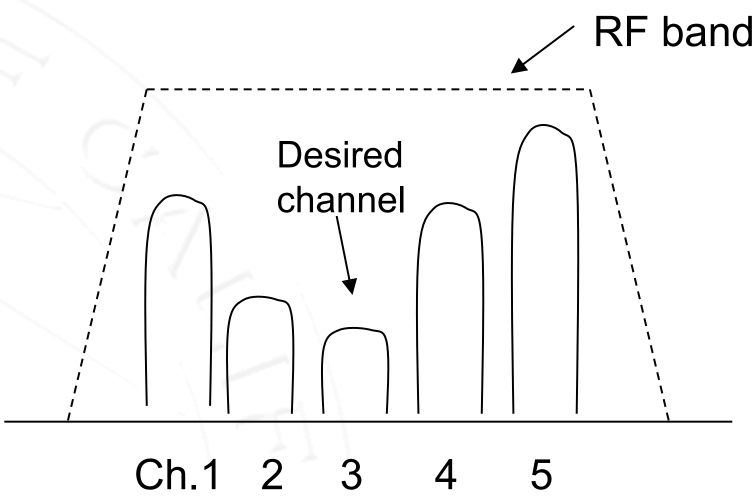


Geographic map of cell sites

Why use mixer in the receiver?

$$Q \sim \frac{\omega_o}{2\Delta\omega}$$ Q of filter

Bandpass filter at ω_o requires a high-Q for narrowband signals



$\Delta f \sim 200$ kHz (GSM)

$$f_o \sim 1GHz$$

$$Q = \frac{10^9}{2 \times 200 \times 10^6} = \frac{1000}{0.4} = 2500$$ High Q

Mixers in Receivers (cont)

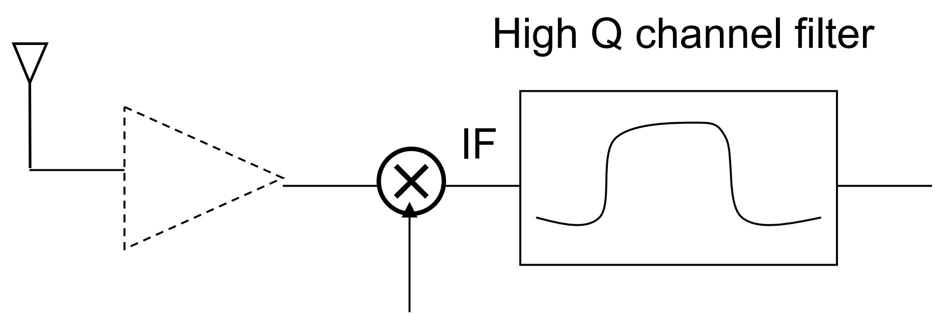
High Q ⇒ Insertion Loss

Filter center frequency must change to select a given channel ⇒ tunable filter difficult to implement

Mixing has big advantage! Translate information down to a fixed (intermediate frequency) or IF.

1 GHz ⇒ 10 MHz: 100x decrease in Q required

Don't need a tunable filter



Superheterodyne receiver architecture

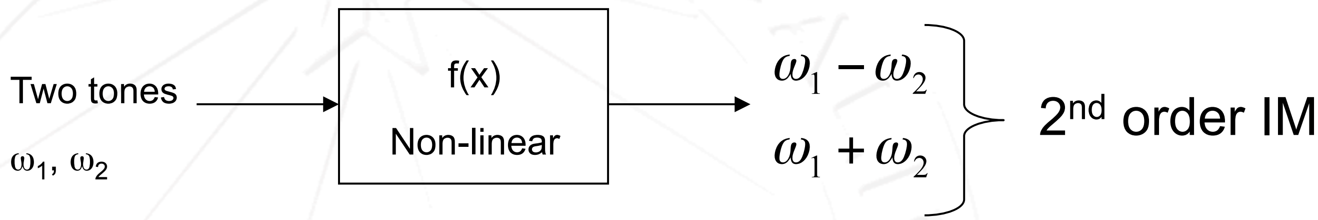
Issue: Mixer has high noise factor

Mixers Specifications

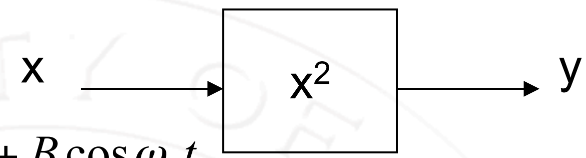
- Conversion Gain: Ratio of voltage (power) at output frequency to input voltage (power) at input frequency
  - Downconversion: RF power / IF power
  - Up-conversion: IF power / RF power
- Noise Figure
  - DSB versus SSB
- Linearity
- Image Rejection
- LO Feedthrough
  - Input
  - Output
- RF Feedthrough

Mixer Implementation

We know that any non-linear circuit acts like a mixer



Squarer Example

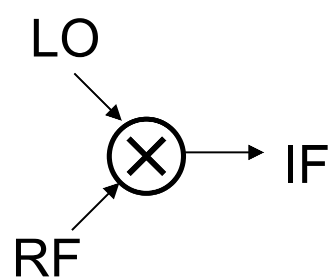


$$A\cos\omega_1 t + B\cos\omega_2 t$$

$$y = \underbrace{A^2\cos^2\omega_1 t + B^2\cos^2\omega_2 t}_{\text{DC \& second harmonic}} + \underbrace{2AB\cos\omega_1 t\cos\omega_2 t}_{\text{Desired mixing}}$$

Product component: $\frac{2AB}{2}\{\cos(\omega_1+\omega_2)t + \cos(\omega_1-\omega_2)t\}$

What we would prefer:

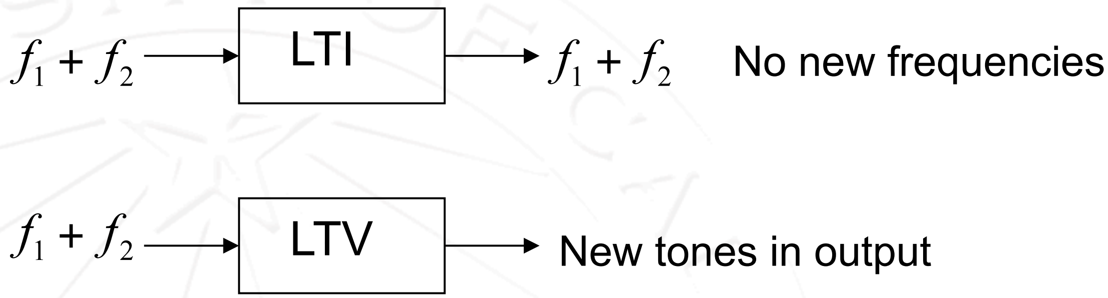


$$v_{IF} = v_{LO}\cdot v_{RF}\cos(\omega_1 \pm \omega_2)t$$

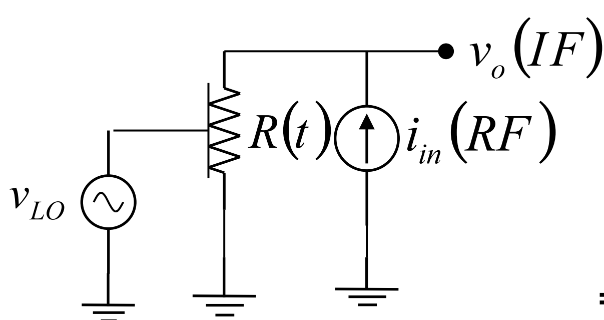
$$v_{RF} = v_{RF}\cos\omega_1 t$$

A true quadrant multiplier with good dynamic range is difficult to fabricate

LTV Mixer



Example: Suppose the resistance of an element is modulated harmonically



$$
\begin{aligned}
v_o &= i_{in} \cdot R(t) \\
&= I_o \cos(\omega_{RF} t) \cdot R_o \cos(\omega_{LO} t) \\
&= \frac{I_o R_o}{2} \{ \cos(\omega_{RF} + \omega_{LO})t + \cos(\omega_{RF} - \omega_{LO})t \}
\end{aligned}
$$

Time Varying Systems

In general, any periodically time varying system can achieve frequency translation

$$v(t) = p(t)v_i(t) \qquad p(t+T) = p(t)$$

$$= \sum_{n=-\infty}^{\infty} c_n e^{j\omega_o n t} v_i(t)$$

$$c_n = \frac{1}{T}\int_0^T p(t) e^{-j\omega_o n t} dt \qquad v_i(t) = A(t)\cos\omega_1 t = A(t)\left(\frac{e^{j\omega_1 t} + e^{-j\omega_1 t}}{2}\right)$$

$$v_o(t) = A(t)\sum_{-\infty}^{\infty} c_n \frac{e^{j(\omega_o n t + \omega_1 t)} + e^{+j(\omega_o n t - \omega_1 t)}}{2}$$

consider n=1 plus n=-1

Desired Mixing Product

$$c_1 = c_{-1}$$

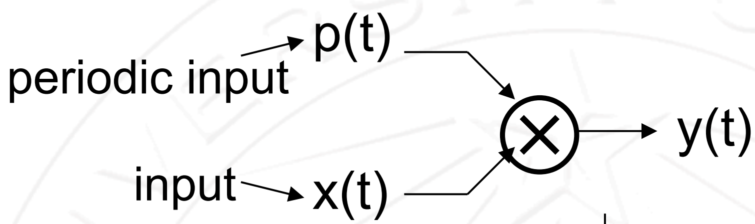
$$v_o(t) = \frac{c_1}{2} e^{j(\omega_o t - \omega_1 t)} + \frac{c_{-1}}{2} e^{-j(\omega_o t + \omega_1 t)}$$

$$= c_1 \cos(\omega_o t - \omega_1 t)$$

Output contains desired signal (plus a lot of other signals)
→ filter out undesired components

Convolution in Frequency

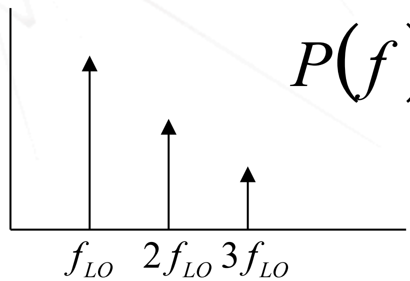
Ideal multiplier mixer:



$$y(t) = p(t)x(t)$$

$$Y(f) = X(f) * P(f)$$

$$P(f) = \sum_{-\infty}^{\infty} c_n \delta(f - n f_{LO})$$

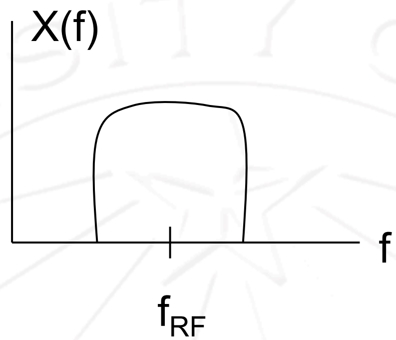


$$Y(f) = \int_{-\infty}^{\infty} \sum_{-\infty}^{\infty} c_n \delta(\sigma - n f_{LO}) X(f - \sigma) d\sigma$$

$$= \sum_{-\infty}^{\infty} c_n \left(\int_{-\infty}^{\infty} \delta(\sigma - n f_{LO}) X(f - \sigma) d\sigma \right)$$

$$= \sum_{-\infty}^{\infty} c_n X(f - n f_{LO})$$

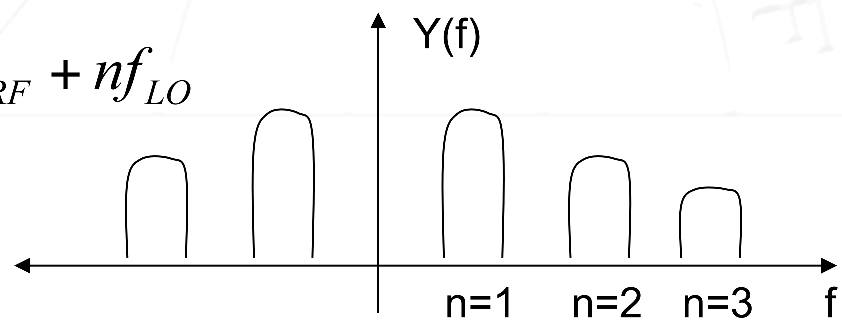
Convolution in Frequency (cont)



X(f) peaks at f_{RF}

Translated spectrum peaks: $f - nf_{LO} = f_{RF}$

$f = f_{RF} + nf_{LO}$

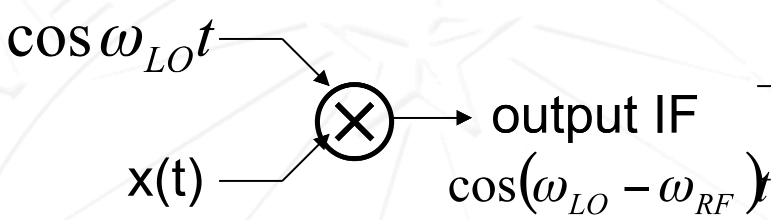


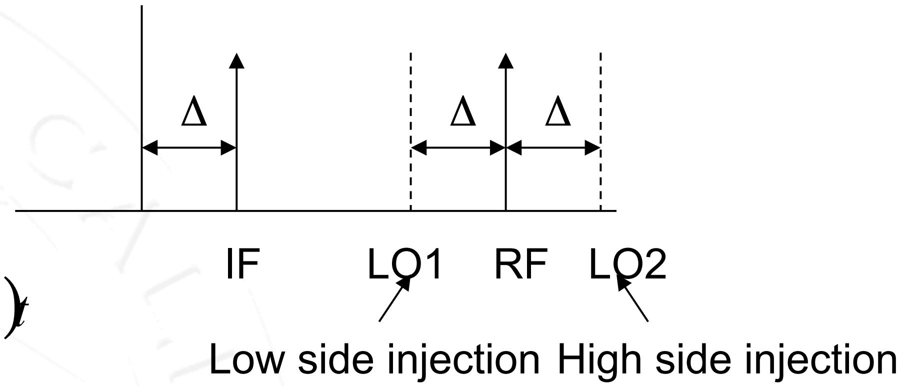
Input spectrum is translated into multiple "sidebands" or "image" frequencies

⇒ Also, the output at a particular frequency originates from multiple input frequency bands

How Low can you LO?

Take the simplest mixer:





Side note:

Which LO frequency to pick? LO1 or LO2?

$$f_{LO} = f_{LO}' + \frac{n\Delta f}{N}$$

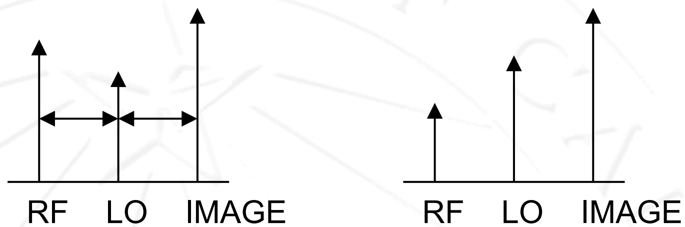
Δf ← Channel spacing

N ← No. of channels

Tuning range: $\frac{\Delta f}{f_{LO}} \Rightarrow$ f_{LO} larger implies smaller tuning range

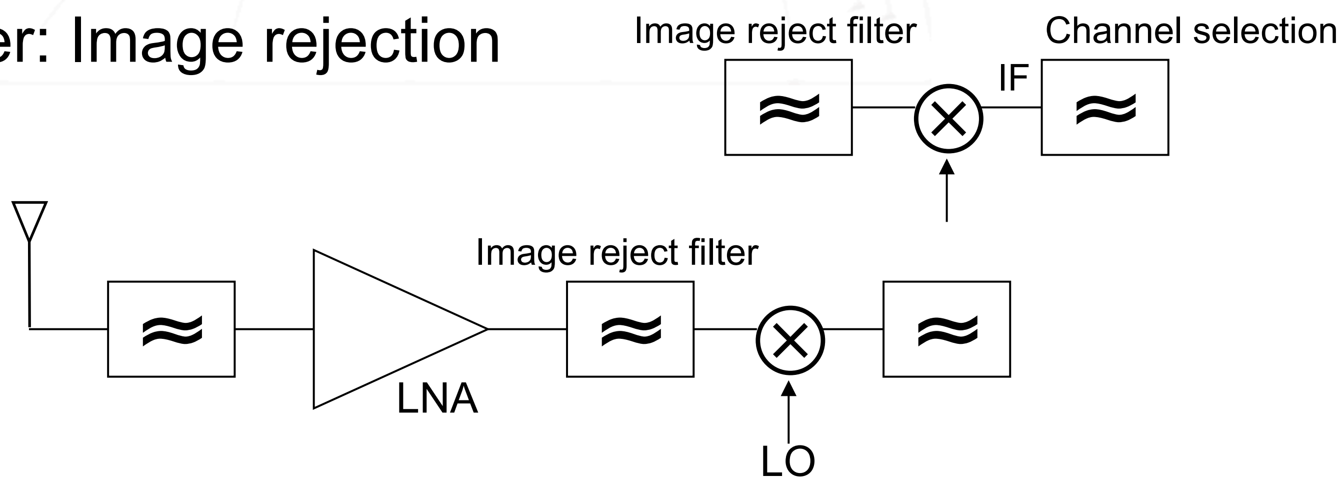
Image Problem

<u>Back to the original problem</u>:



Question: Why filter before mixer in spectrum analyzer?

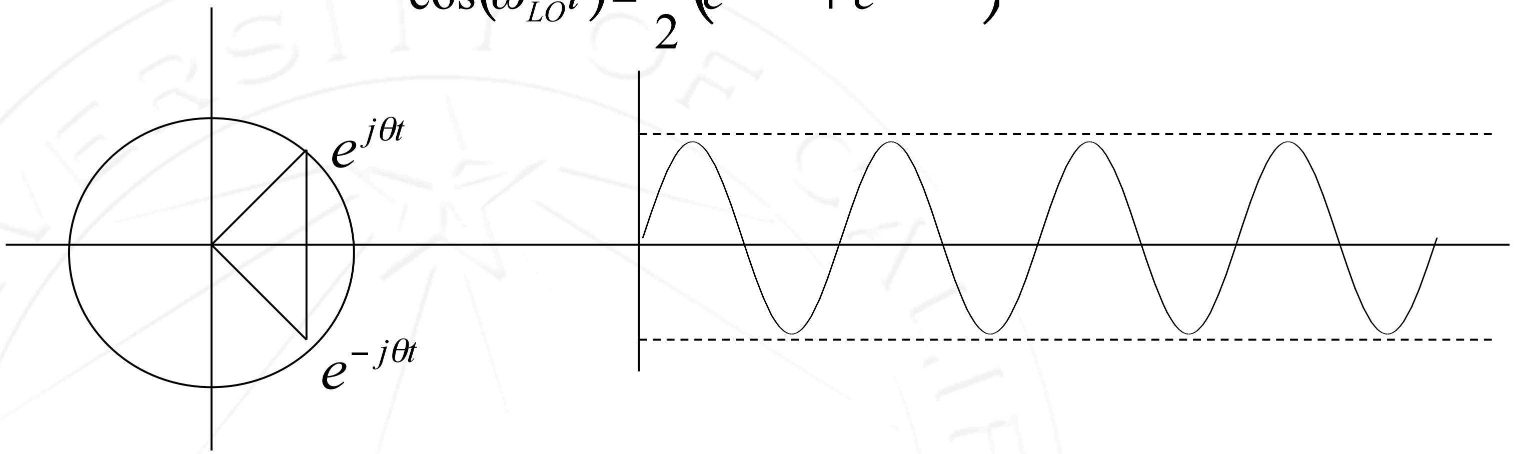
Answer: Image rejection



Receiver architecture is getting complicated…

Origin of Image Problem

$$\cos(\omega_{LO} t) = \frac{1}{2}\left(e^{j\omega_{LO} t} + e^{-j\omega_{LO} t}\right)$$



If we could multiply by a complex exponential, then image problem goes away…

$$e^{j\theta_{LO}} \cos(\omega_{RF} t) = e^{j(\theta_{LO}+\theta_{RF})t} + \underbrace{e^{+j(\theta_{LO}-\theta_{RF})}}_{e^{j\theta_{IF}}}$$

$$e^{j\theta_{RF}} + e^{-j\theta_{RF}}$$

$e^{j\theta_{IF}}$ IF frequency

$\theta_{RF} = \theta_{LO} - \theta_{IF}$ High side injection

$\theta_{IM} = \theta_{LO} + \theta_{IF}$ (Low side injection) Image Freq.

Review of Linear Systems and PSD

Average response of LTI system:

$$y_1(t) = H_1[x(t)] = \int_{-\infty}^{\infty} h_1(t)\,x(t-\tau)\,d\tau$$

$$\overline{y_1(t)} = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} y_1(t)\,dt$$

$$= \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T}\left(\int_{-\infty}^{\infty} h_1(\tau)\,x(t-\tau)\,d\tau\right)dt$$

$$= \int_{-\infty}^{\infty}\left(\underbrace{\lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x(t-\tau)\,dt}_{\overline{x(t)}}\right)h_1(\tau)\,dt$$

Average Value Property

$$\overline{y_1(t)} = \overline{x(t)} \int_{-\infty}^{\infty} h_1(t)\,dt$$

$$H_1(j\omega) = \int_{-\infty}^{\infty} h_1(t) e^{-j\omega t}\,dt$$

$$\overline{y_1(t)} = \overline{x(t)} H_1(0)$$

"DC gain"

Output RMS Statistics

$$\overline{y_1^2(t)} = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \left(\int_{-\infty}^{\infty} h_1(\tau_1) x(t-\tau_1) d\tau_1 \right) \left(\int_{-\infty}^{\infty} h_1(\tau_2) x(t-\tau_2) d\tau_2 \right) dt$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h_1(\tau_1) h_1(\tau_2) \left(\lim_{T\to\infty} \frac{1}{2\pi} \int_{-T}^{T} x(t-\tau_1) x(t-\tau_2) dt \right) d\tau_1 d\tau_2$$

Recall the definition for the autocorrelation function

$$\phi_{xx}(t) = \overline{x(t)x(t+\tau)}$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x(t) x(t+\tau) dt$$

Autocorrelation Function

$$\overline{y_1^{\,2}(t)} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h_1(\tau_1) h_2(\tau_2) \phi_{xx}(\tau_1 - \tau_2) d\tau_1 d\tau_2$$

$$\phi_{xx}(j\omega) = \int_{-\infty}^{\infty} \phi_{xx}(\tau) e^{-j\omega\tau} d\tau$$

$$\phi_{xx}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \phi_{xx}(j\omega) e^{j\omega\tau} d\omega$$

$\phi_{xx}(j\omega)$ is a real and even function of ω

since $\phi_{xx}(\tau)$ is a real and even function of τ

Autocorrelation Function (2)

$$\overline{y_1^{\,2}(t)} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h_1(\tau_1)h_1(\tau_2)\frac{1}{2\pi}\int_{-\infty}^{\infty}\phi_{xx}(j\omega)e^{j\omega(\tau_1-\tau_2)}d\omega d\tau_1 d\tau_2$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\phi_{xx}(j\omega)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h_1(\tau_1)h_1(\tau_2)e^{j\omega(\tau_1-\tau_2)}d\tau_1 d\tau_2$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\phi_{xx}(j\omega)\left(\int_{-\infty}^{\infty} h_1(\tau_1)e^{+j\omega\tau_1}d\tau_1\right)\left(\int_{-\infty}^{\infty} h_1(\tau_2)e^{-j\omega\tau_2}d\tau_2\right)d\omega$$

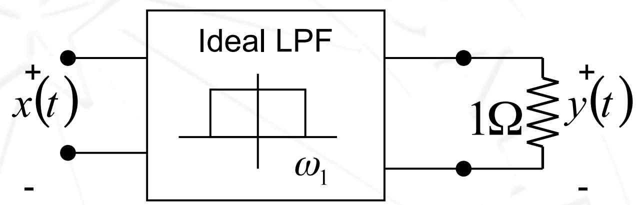
$$H_1^{*}(j\omega) = \left(\int_{-\infty}^{\infty} h_1(\tau)e^{-j\omega\tau}d\tau\right)^{*} =$$

$$\overline{y_1^{\,2}(t)} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\phi_{xx}(j\omega)H_1(j\omega)H_1^{*}(j\omega)d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\phi_{xx}(j\omega)\left|H_1(j\omega)\right|^2 d\omega$$

Average Power in X(t)

Consider x(t) as a voltage waveform with total average power $\overline{x^2(t)}$. Let's measure the power in x(t) in the band $0<\omega<\omega_1$.



The average power in the frequency range $0<\omega<\omega_1$ is now

$$\overline{y_1^{\,2}(t)} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \phi_{xx}(j\omega)\left|H_1(j\omega)\right|^2 d\omega$$

$$= \frac{1}{2\pi}\int_{-\omega_1}^{\omega_1} \underbrace{\phi_{xx}(j\omega)}_{\text{W/radian}} d\omega$$

$$= \int_{-f_1}^{f_1} \underbrace{\phi_{xx}(j2\pi f)}_{\text{W/Hz}} df$$

Average Power in X(t) (2)

$$= 2\int_0^{f_1} \phi_{xx}(j2\pi f)\,df$$

Generalize: To measure the power in any frequency range apply an ideal bandpass filter with passband $\omega_1 < \omega < \omega_2$

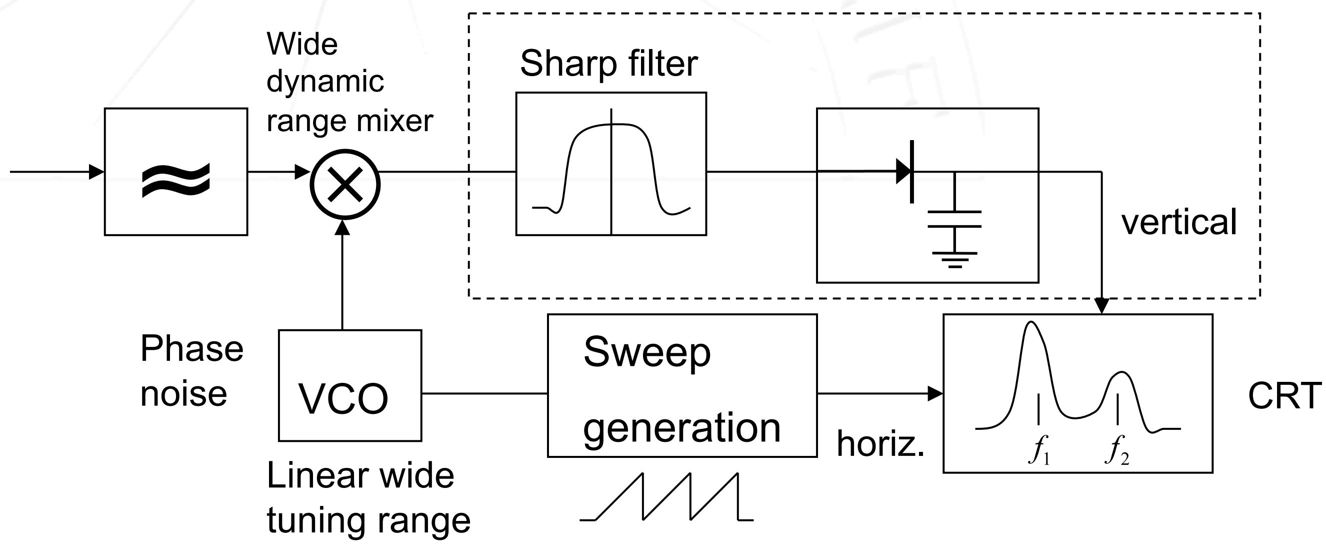
$$\overline{y_1^{\,2}(t)} = 2\int_{f_1}^{f_2} \phi_{xx}(j2\pi f)\,df$$

The interpretation of ϕ_{xx} as the “power spectral density” (PSD) is clear

Spectrum Analyzer

A spectrum analyzer measures the PSD of a signal

Poor man's spectrum analyzer:



EECS 242:
Current Commutating Active Mixers

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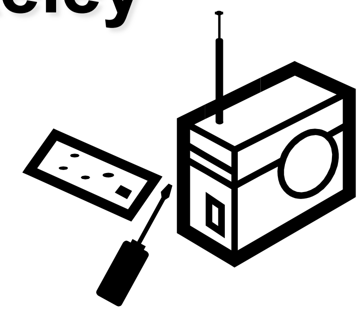


Balanced Mixer

- An unbalanced mixer has a transfer function:

$$y(t) = x(t)\times s(t) = (1 + A(t)\cos(\omega_{RF}t))\times\begin{Bmatrix}0\\1\end{Bmatrix}$$

Has "DC"

- which contains both RF, LO, and IF
- For a single balanced mixer, the LO signal is "balanced" (bipolar) so we have

Has "DC" No "DC"

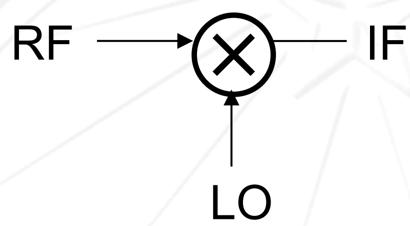
$$y(t) = x(t)\times s(t) = (1 + A(t)\cos(\omega_{RF}t))\times\begin{Bmatrix}+1\\-1\end{Bmatrix}$$

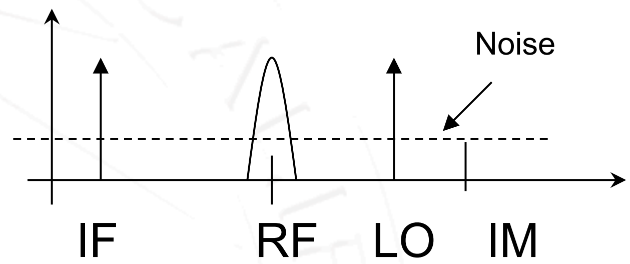
- As a result, the output contacts LO but no RF component
- For a double balanced mixer, the LO and RF are balanced so there is no LO or RF leakage

$$y(t) = x(t)\times s(t) = A(t)\cos(\omega_{RF}t)\times\begin{Bmatrix}+1\\-1\end{Bmatrix}$$

Noise in an Ideal Mixers

Consider the simplest ideal multiplying mixer:





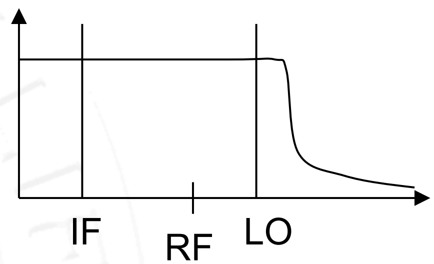
- What's the noise figure for the conversion process?
- Input noise power due to source is kTB where B is the bandwidth of the input signal
- Input signal has power P_s at either the lower or upper sideband

$$SNR_i = \frac{P_s}{kTB}$$

Noise in Ideal Mixers

- At the IF frequency, we have the down-converted signal $G \cdot P_s$ and down-converted noise from <u>two</u> sidebands, LO - IF and LO + IF

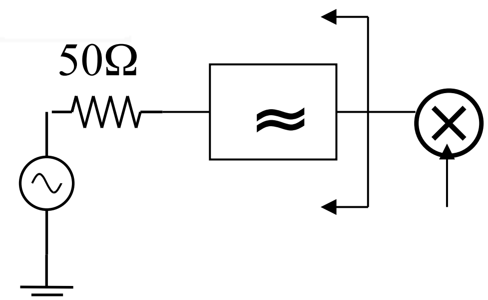
$$SNR_o = \frac{G \cdot P_s}{(G' + G'')kTB}$$



For ideal mixer, G=G'=G"

$$F = \frac{SNR_i}{SNR_o} = \frac{P_s}{kTB}\frac{2kTB}{P_s} = 2$$

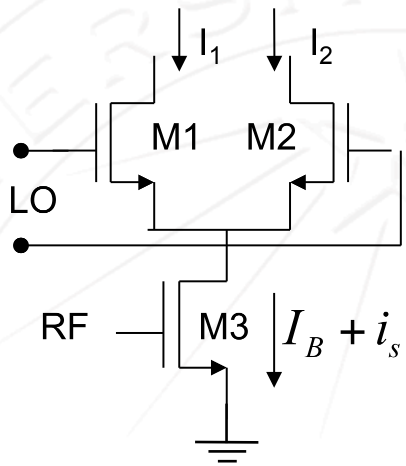
$$NF = 3\text{dB}$$



For a real mixer, noise from multiple sidebands can fold into IF frequency & degrade NF

Noise in CMOS Current Commutating Mixer

(After Terrovitis, JSSC)

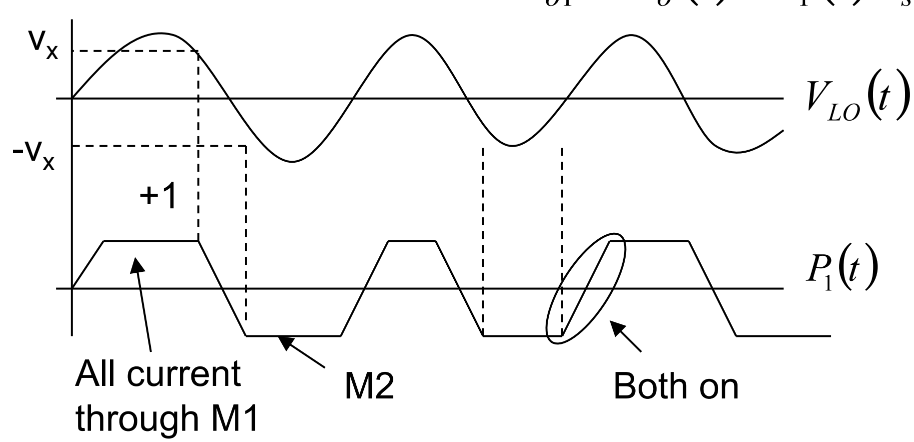


$$I_{o1} = I_1 - I_2 = F\left(V_{LO}(t), I_B + i_s\right)$$

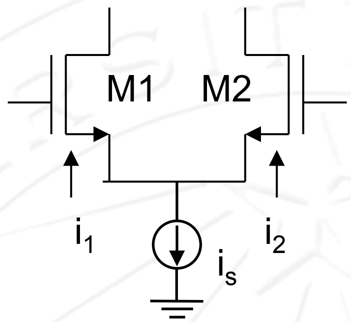
Assume i_s is small relative to I_B and perform Taylor series expansion

$$I_{o1} \approx F\left(V_{LO}(t), I_B\right) + \frac{\delta F}{\delta I_B}\left(V_{LO}(t), I_B\right) \cdot i_s + \ldots$$

$$I_{o1} = P_o(t) + P_1(t) \cdot i_s$$



Noise in Current Commutating Mixers



$$\frac{i_1}{i_s} = \frac{\frac{1}{g_{m2}}}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}} \qquad \frac{i_2}{i_s} = \frac{\frac{1}{g_{m1}}}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}}$$

$$p_1(t) = \frac{g_{m1}(t) - g_{m2}(t)}{g_{m1}(t) + g_{m2}(t)} \left(= \frac{i_1 - i_2}{i_s} \right)$$

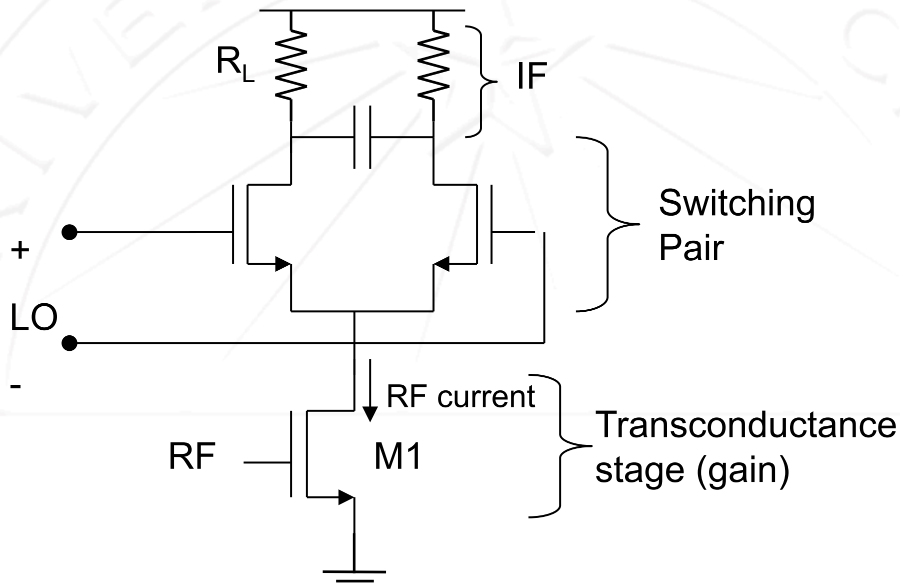
Note that with good device matching $p_1(t) = -p_1\left(t + \frac{T_o}{2}\right)$

Expand $p_1(t)$ into a Fourier series:

$$p_{1,2k} = \frac{1}{T_{LO}} \int_0^{T_{LO}} p_1(t) e^{-j2\pi 2kt/T_{LO}} dt = \int_0^{T_{LO}/2} + \int_{T_{LO}/2}^{T_{LO}} = 0$$

Only odd coefficients of $p_{1,n}$ non-zero

Single Balanced Mixer



Assume LO signal <u>strong</u> so that current (RF) is alternatively sent to either M_2 or M_3. This is equivalent to multiplying i_{RF} by ±1.

$$v_{IF} \approx sign(V_{LO}) g_m R_L v_{RF} = g(t) g_m R_L v_{RF}$$

Period waveform with period = T_{LO}

Current Commutating Mixer (2)

g(t) = square wave = $\frac{4}{\pi}\left(\cos\omega_{LO}t - \cos 3\omega_{LO}t + \ldots\right)$

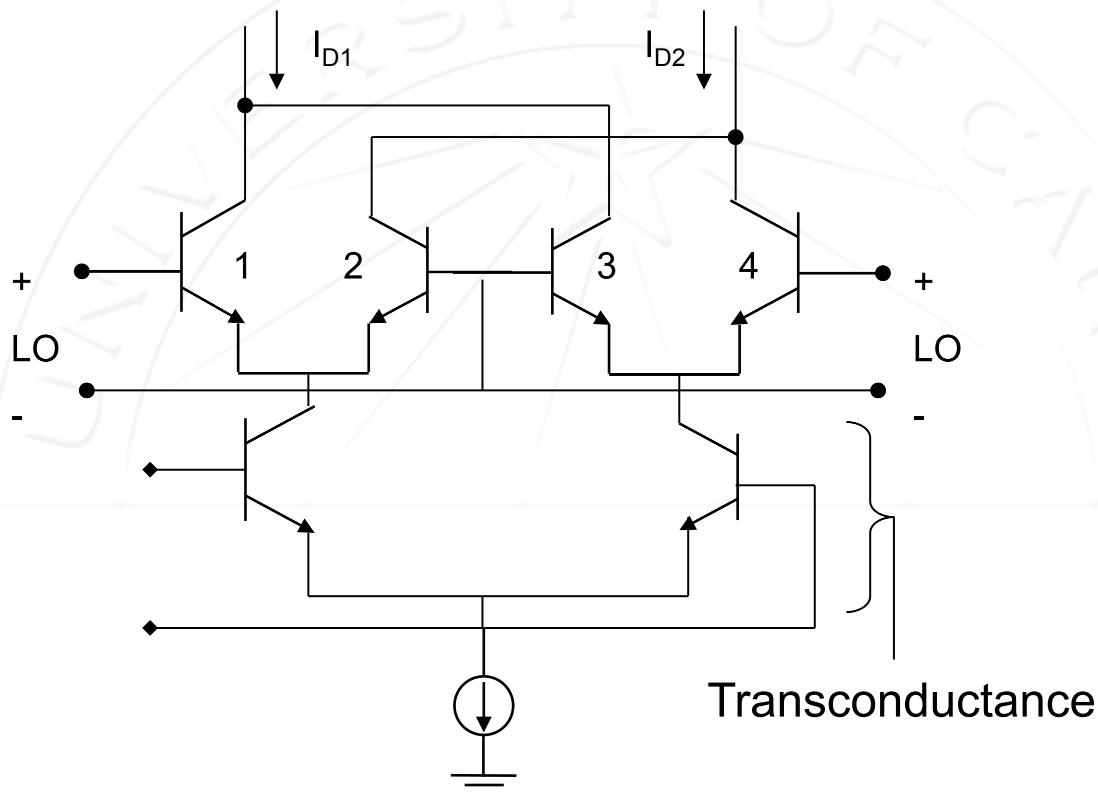
Let $v_{RF} = A\cos\omega_{RF}t$

$$LPF(v_{IF}) = \frac{4}{\pi}\frac{1}{2}\cos(\omega_{RF} - \omega_{LO})t \cdot g_m R_L \cdot A$$

$$A_v = \frac{\tilde{v}_{IF}}{A} = \frac{2}{\pi} g_m R_L$$ gain

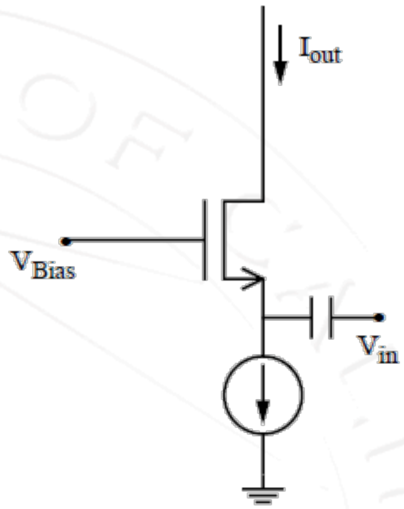
LO-RF isolation good, but LO signal appears in output (just a diff pair amp).
Strong LO might desensitize (limit) IF stage (even after filtering).

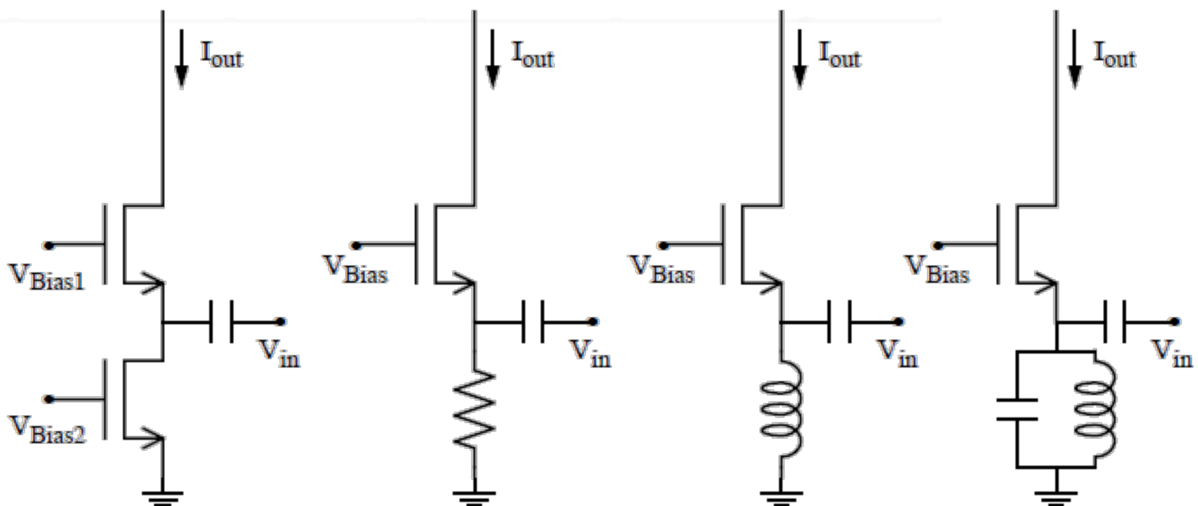
Double Balanced Mixer



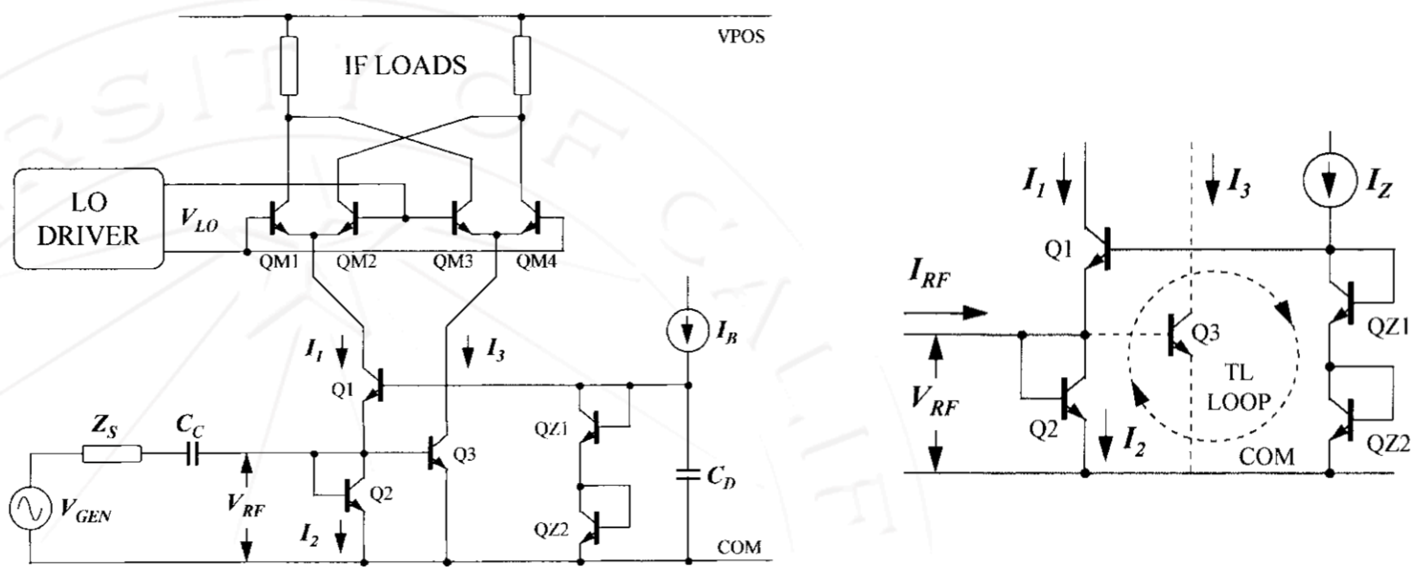
- LO signal is rejected up to matching constraints
- Differential output removes even order non-linearities
- Linearity is improved: Half of signal is processed by each side
- Noise higher than single balanced mixer since no cancellation occurs

Common Gate Input Stage



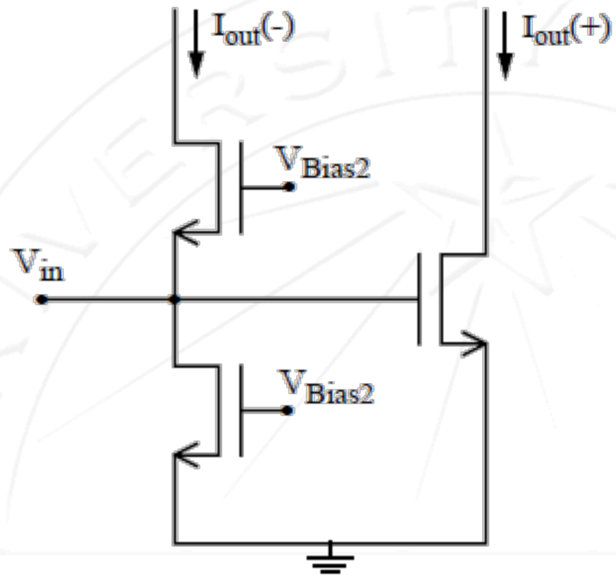


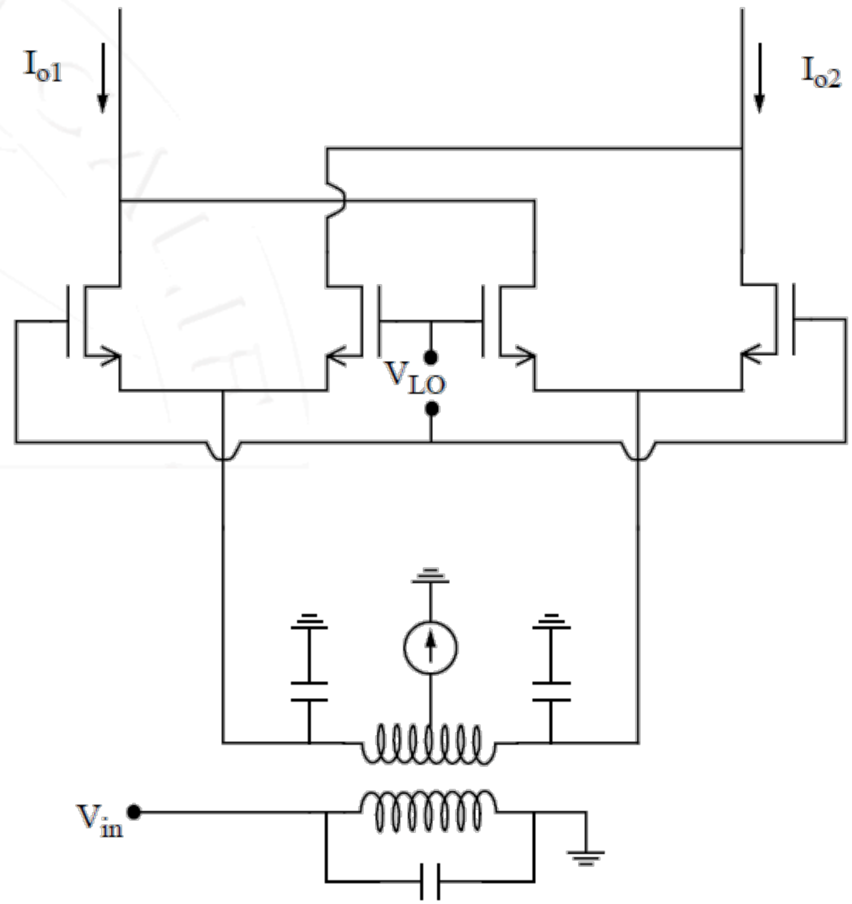
Gilbert Micromixer

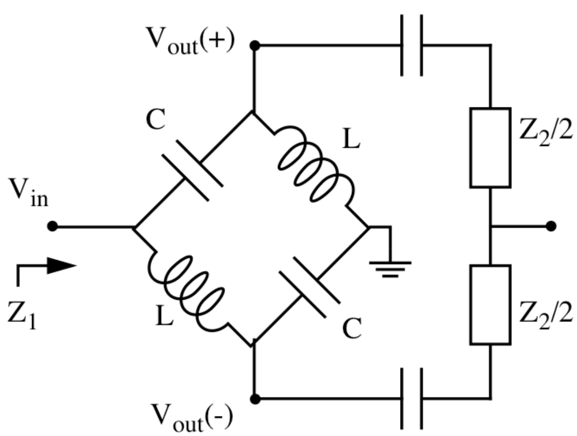


- The LNA output is often single-ended. A good balanced RF signal is required to minimize the feedthrough to the output. LC bridge circuits can be used, but the bandwidth is limited. A transformer is a good choice for this, but bulky and bandwidth is still limited.
- A broadband single-ended to differential conversion stage is used to generate highly balanced signals. Gm stage is Class AB.

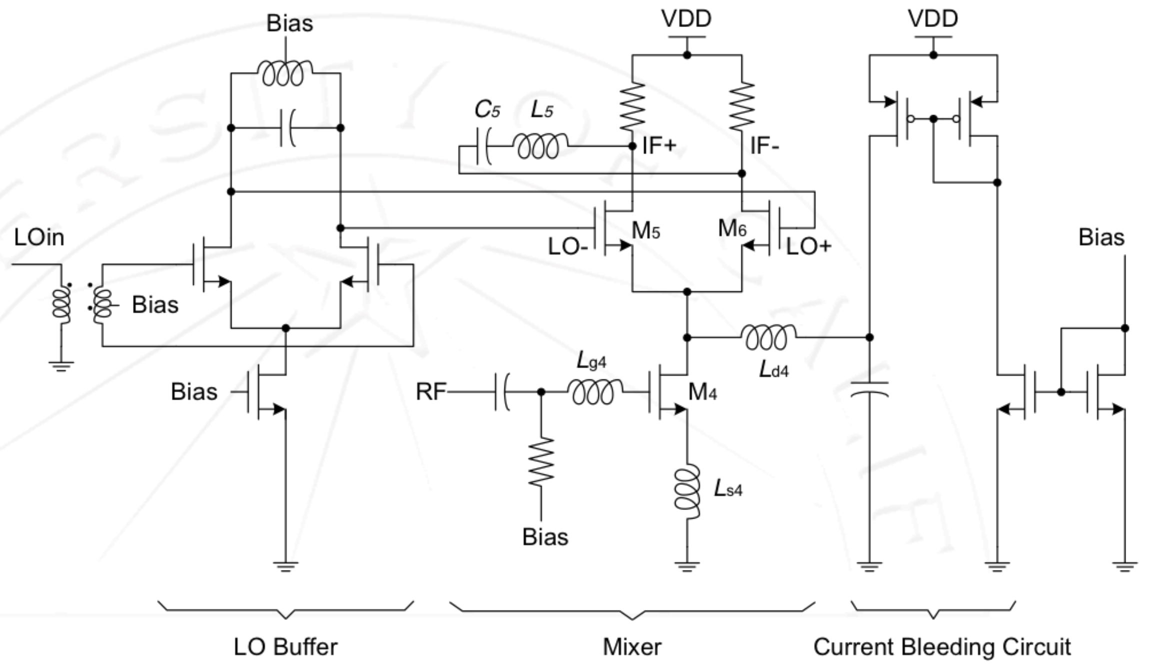
Active and Passive Balun







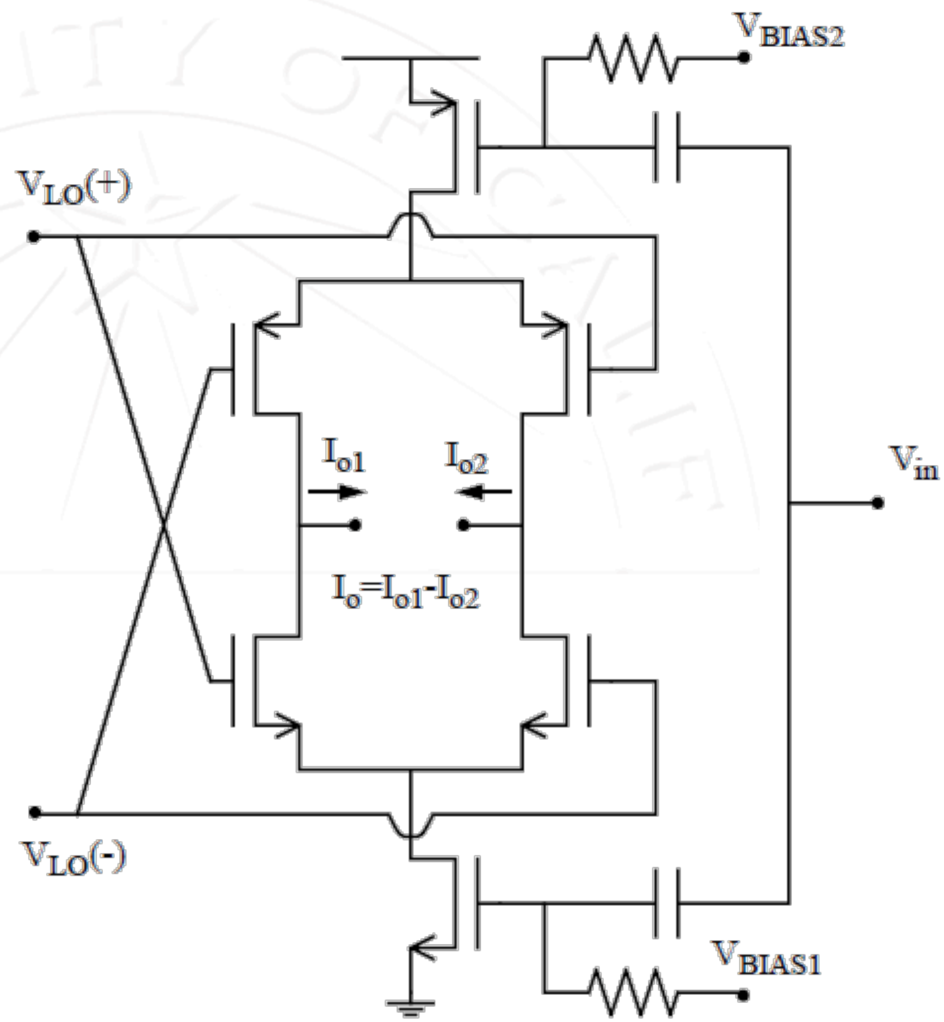
Bleeding the Switching Core



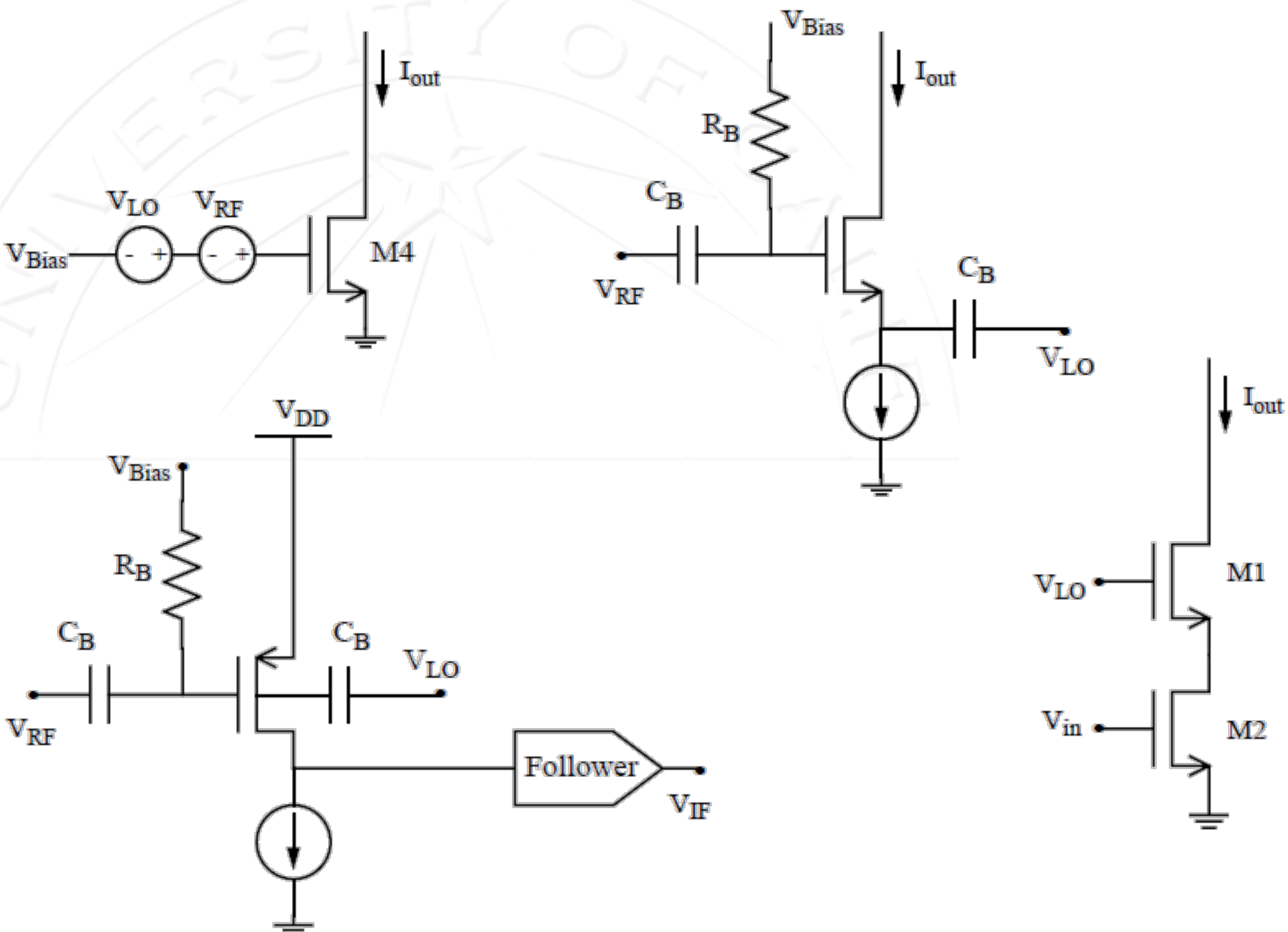
- Large currents are good for the gm stage (noise, conversion gain), but require large devices in the switching core → hard to switch due to capacitance or requires a large LO (large Vgs-Vt)
- A current source can be used to feed the Gm stage with extra current.

[3] J. Park, C.-H. Lee, B.-S. Kim, J. Laskar, "Design and Analysis of Low Flicker Noise CMOS Mixers for Direct-Conversion Receivers," *IEEE Trans. Microwave Theory Tech.*, vol.54, no.12, pp. 4372-4380, December 2006

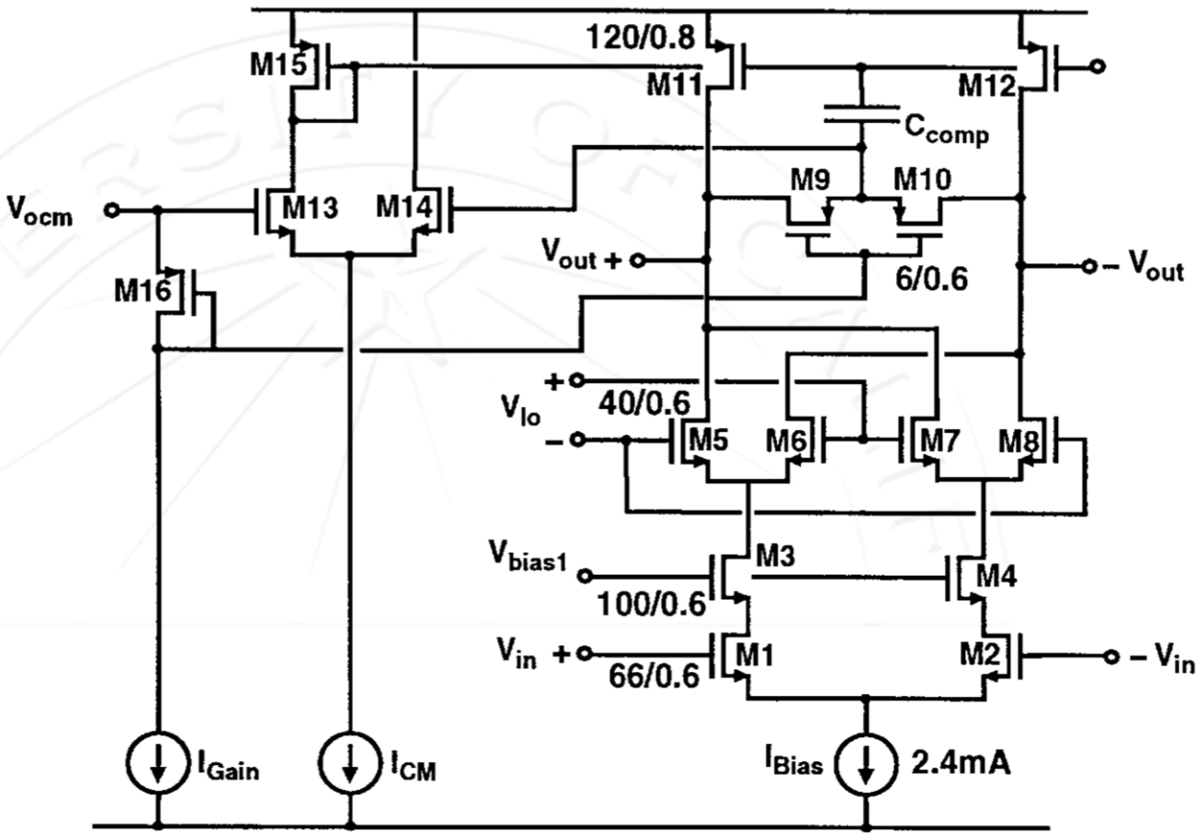
Current Re-Use Gm Stage



Single, Dual, and Back Gate



Rudell CMOS Mixer



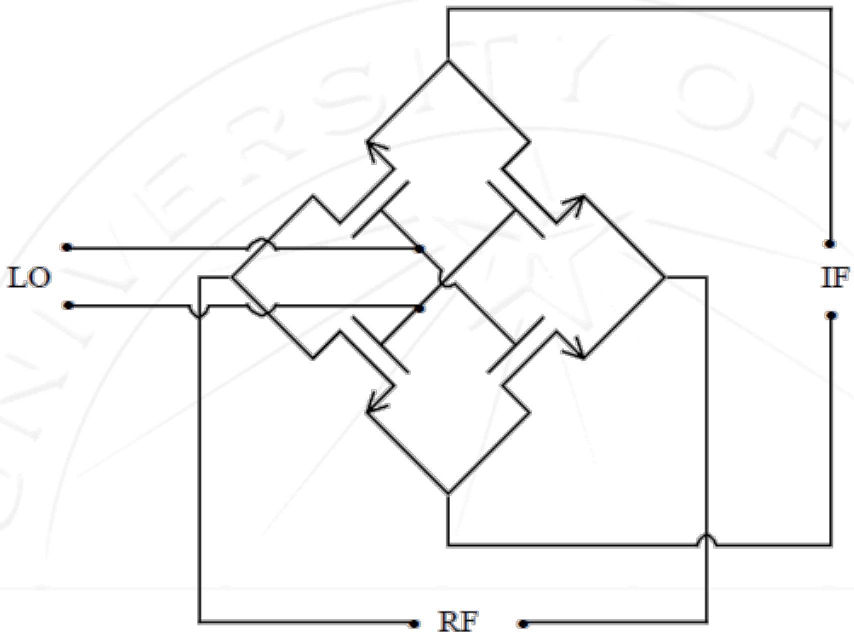
IEEE JOURNAL OF SOLID-STATE CIRCUITS, VOL. 32, NO. 12, DECEMBER 1997 2071

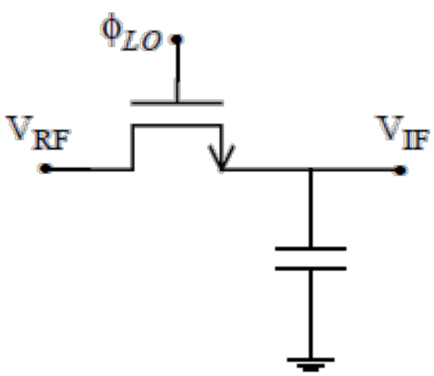
A 1.9-GHz Wide-Band IF Double Conversion CMOS Receiver for Cordless Telephone Applications

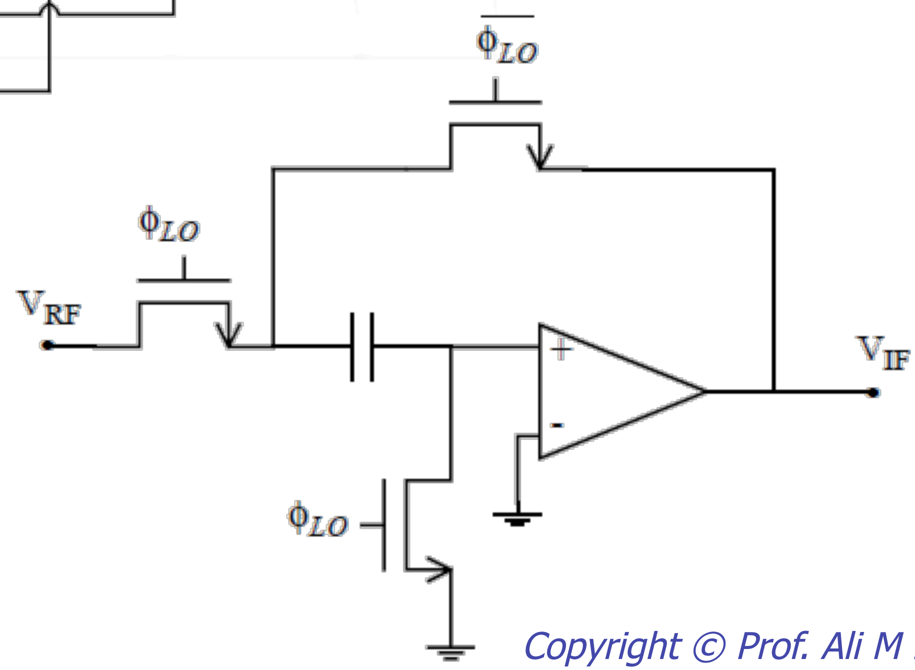
Jacques C. Rudell, *Student Member, IEEE*, Jia-Jiunn Ou, *Student Member, IEEE*, Thomas Byunghak Cho, *Member, IEEE*, George Chien, *Student Member, IEEE*, Francesco Brianti, Jeffrey A. Weldon, *Student Member, IEEE*, and Paul R. Gray, *Fellow, IEEE*

- Gain programmed using current through M16 (set by resistance of triode region devices M9/M10)
- Common mode feedback to set output point
- Cascode improves isolation (LO to RF)

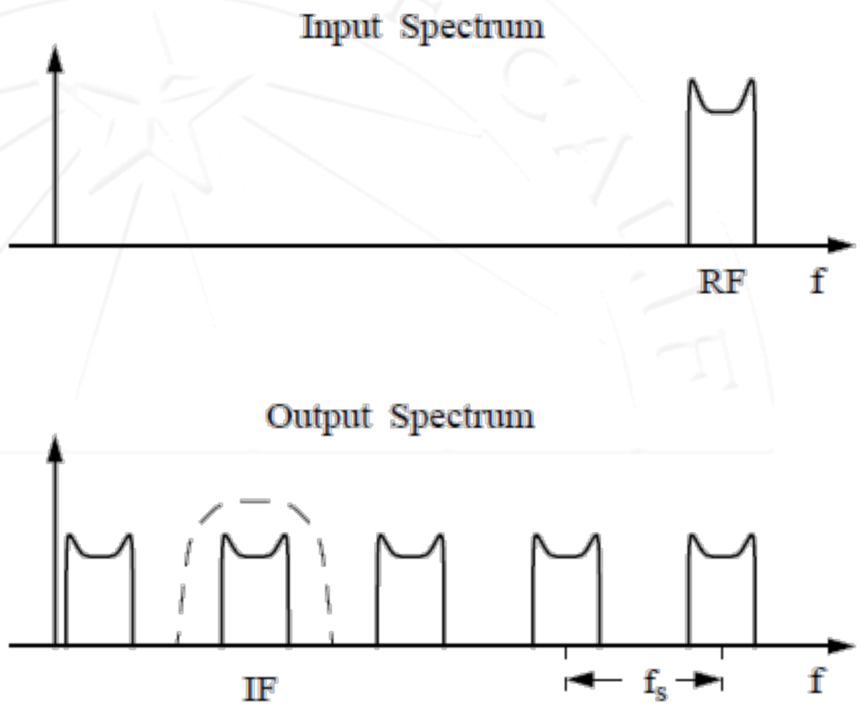
Passive Mixers/Sampling



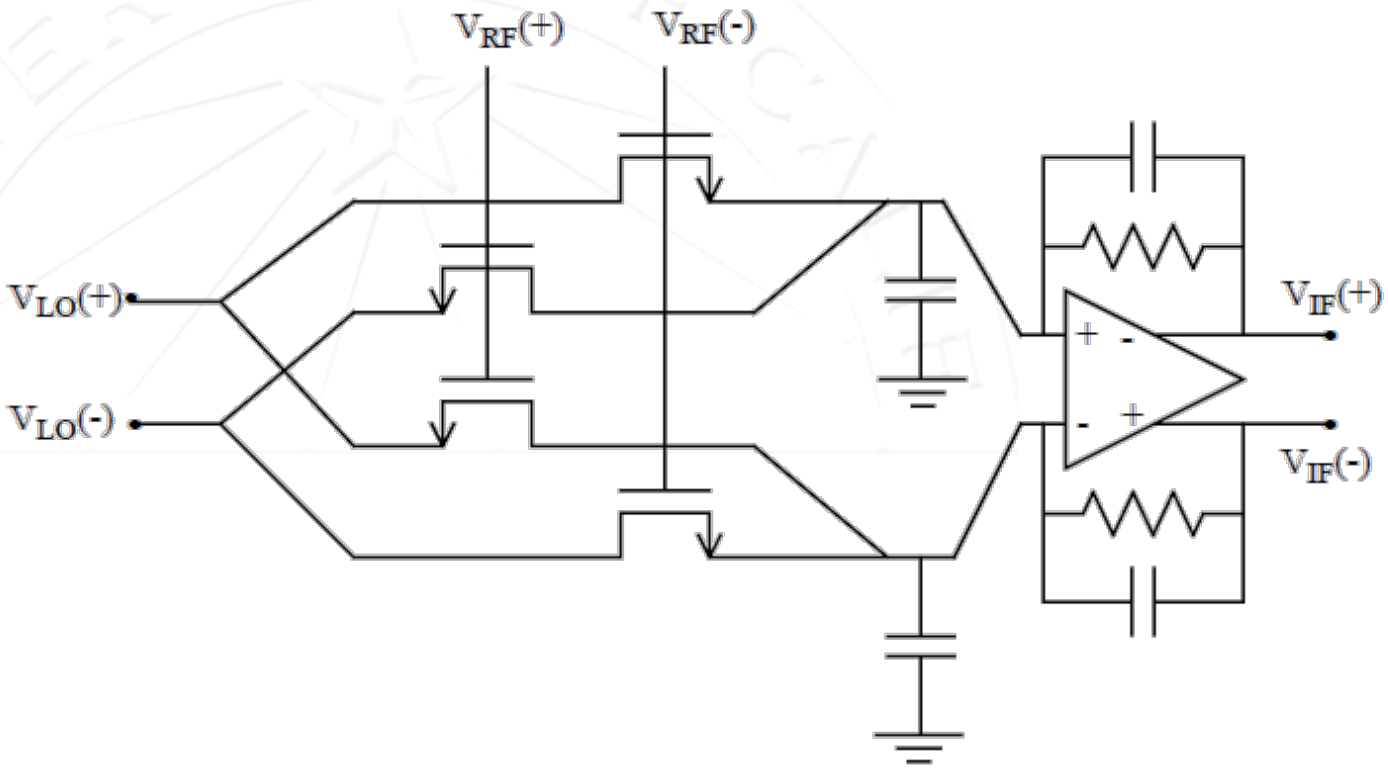




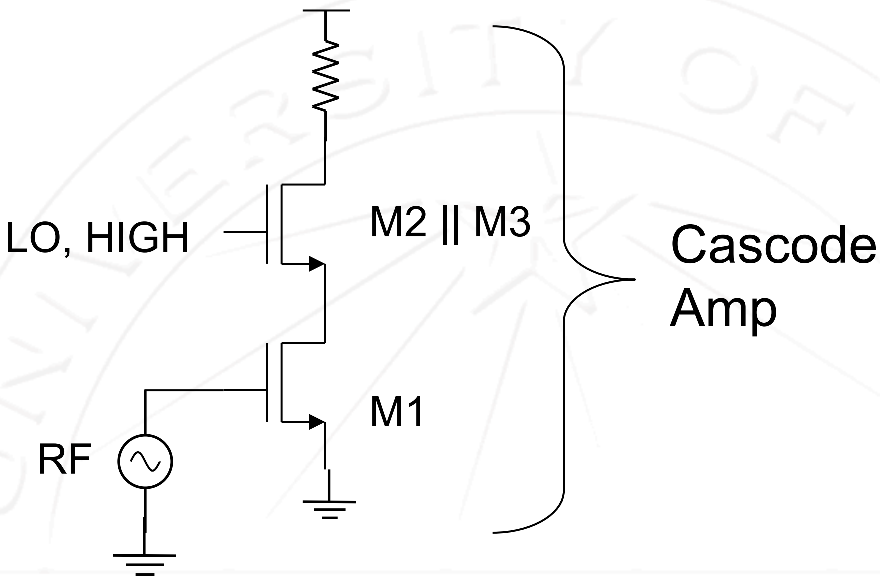
Sub-Sampling Mixers



Triode Region Mixer



Improved Linearity

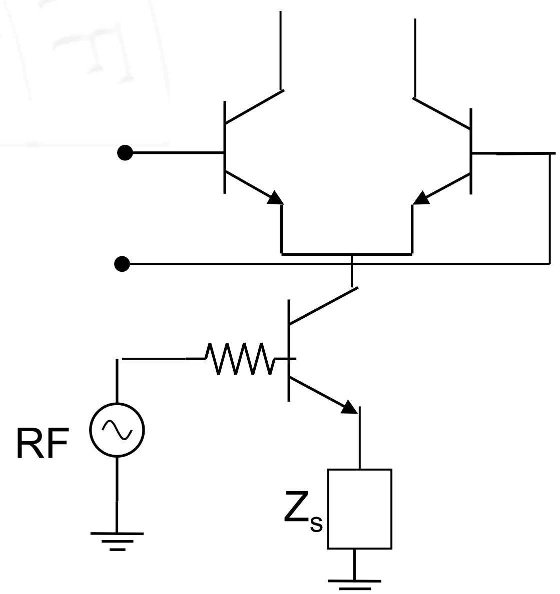


To improve M_1, apply local series feedback

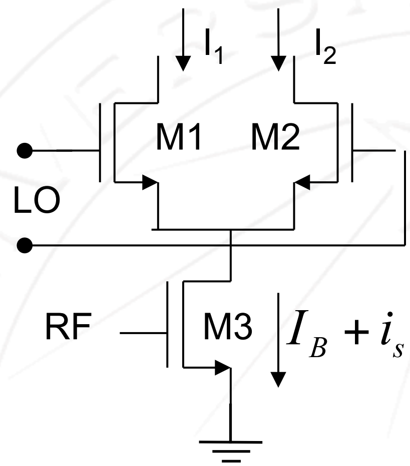
$$Z_s = j\omega L$$

Provide input matching and feedback

$\Rightarrow$ No DC headroom sacrificed



Recap: CMOS Mixer Operation



$$I_{o1} = I_1 - I_2 = F\left(V_{LO}(t), I_B + i_s\right)$$

$$\approx F\left(V_{LO}(t), I_B\right) + \frac{\partial F}{\partial I_B}\left(V_{LO}(t), I_B\right)\cdot i_s + \ldots$$

$$= P_o(t) + P_1(t)\cdot i_s + \ldots$$

$$P_1(t) = \frac{g_{m1}(t) - g_{m2}(t)}{g_{m1}(t) + g_{m2}(t)}$$

Periodic

Fourier Series expansion

$$P_{1,2k} \equiv 0 \qquad P_1(t) = -P_1\left(t + \frac{T_{LO}}{2}\right)$$

References

- **Noise in current-commutating CMOS mixers**
  Terrovitis, M.T.; Meyer, R.G.;
  Solid-State Circuits, IEEE Journal of
  Volume 34, Issue 6, June 1999 Page(s):772 - 783
- **Intermodulation distortion in current-commutating CMOS mixers** Terrovitis, M.T.; Meyer, R.G.;
  Solid-State Circuits, IEEE Journal of
  Volume 35, Issue 10, Oct. 2000 Page(s):1461 – 1473
- **A systematic approach to the analysis of noise in mixers**
  Hull, C.D.; Meyer, R.G.;
  Circuits and Systems I: Fundamental Theory and Applications, IEEE Transactions on [see also Circuits and Systems I: Regular Papers, IEEE Transactions on]
  Volume 40, Issue 12, Dec. 1993 Page(s):909 - 919