EECS 242:
MOS High Frequency Distortion

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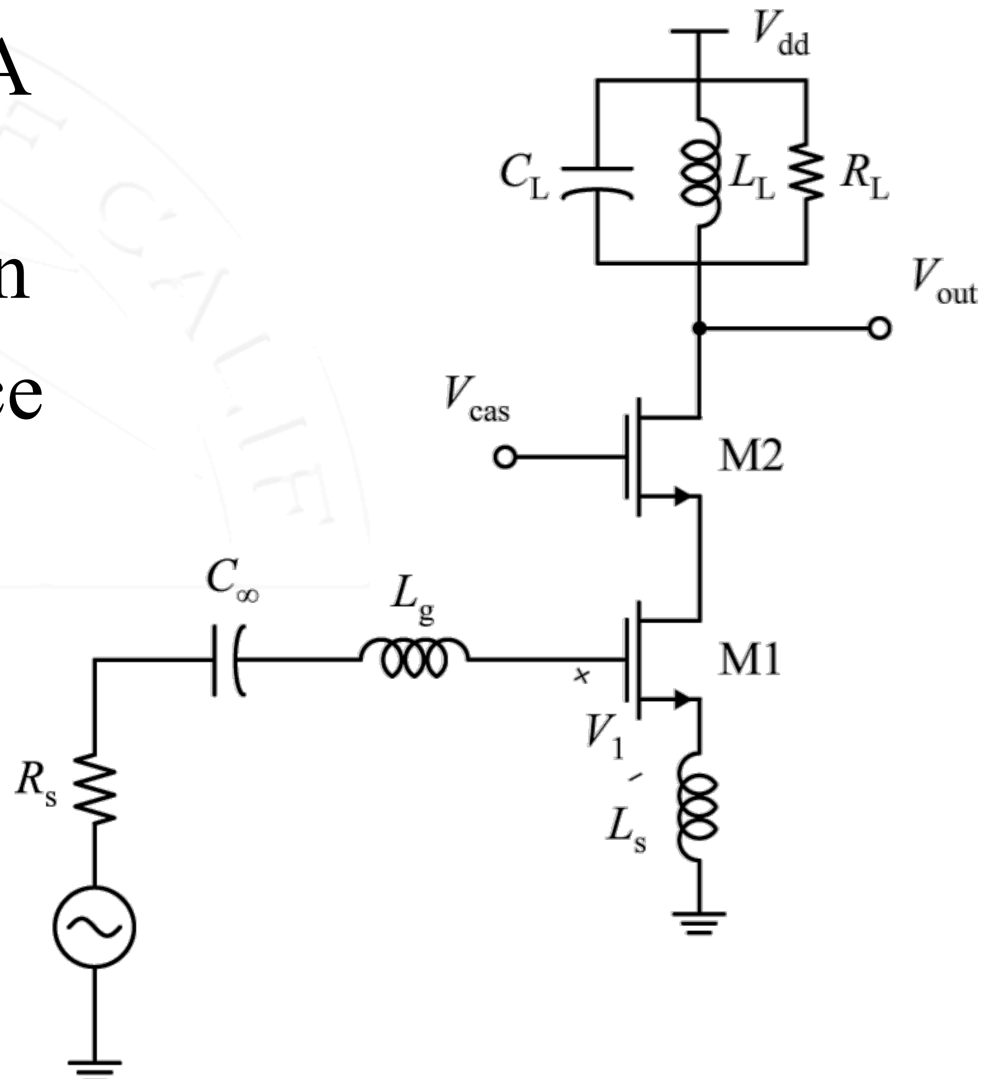
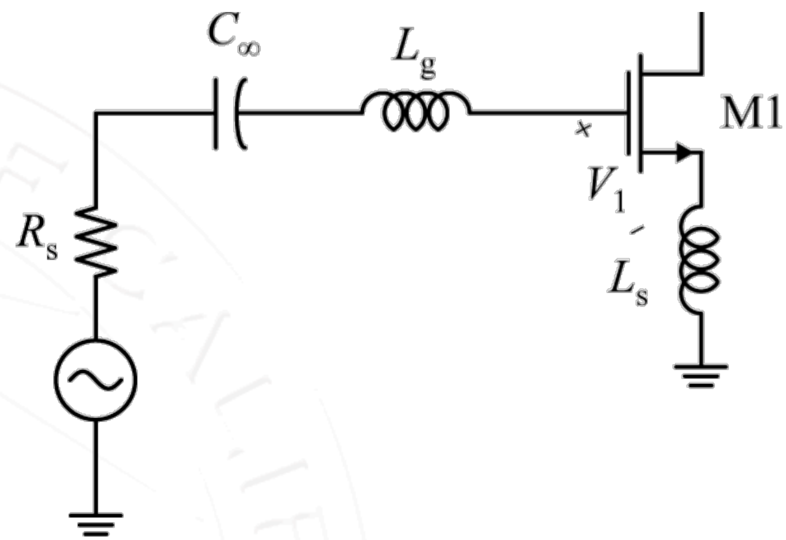
MOS LNA Distortion

- Workhorse MOS LNA is a cascode with inductive degeneration
- Assume that the device is square law
- Neglect body effect (source tied to body)



Governing Differential Eq



$$v_S = v_{R_s} + v_{L_g} + v_1 + v_{L_s}$$

$$i_i = C_{gs}\frac{dv_1}{dt}$$

$$v_{R_s} = R_s C_{gs}\frac{dv_1}{dt}$$

$$v_{L_g} = L_g \frac{d}{dt}\left(C_{gs}\frac{dv_1}{dt}\right) = L_g C_{gs}\frac{d^2 v_1}{dt^2}$$

$$v_{L_s} = L_s \frac{d}{dt}(i_i + i_d) = L_s C_{gs}\frac{d^2 v_1}{dt^2} + L_s \frac{d}{dt}\left(g_{m1}v_1 + g_{m2}v_1^2 + \cdots\right)$$

$$v_S = R_s C_{gs}\frac{dv_1}{dt} + L_g C_{gs}\frac{d^2 v_1}{dt^2} + v_1 + L_s C_{gs}\frac{d^2 v_1}{dt^2} + L_s \frac{d}{dt}\left(g_{m1}v_1 + g_{m2}v_1^2 + \cdots\right)$$

Linear Analysis

- Let $v_1 = A_1(\omega_1)\circ v_s + A_2(\omega_1,\omega_2)\circ v_s^2 + A_3(\omega_1,\omega_2,\omega_3)\circ v_s^3 + \cdots$
- Equating linear terms

$$1 = C_{gs}R_s j\omega_1 A_1 - L_g C_{gs}\omega_1^2 + A_1 + L_s j\omega_1 g_{m1} A_1 - L_s C_{gs}\omega_1^2 A_1$$

$$A_1 = \frac{1}{1-(L_g+L_s)C_{gs}\omega_1^2 + j\omega_1(C_{gs}R_s + g_{m1}L_s)}$$

- By design, at resonance we have

$$A_1 = \frac{1}{\underbrace{1-(L_g+L_s)C_{gs}\omega_0^2}_{0} + j\omega_0 C_{gs}(R_s + \underbrace{\omega_T L_s}_{R_s})} = \frac{1}{j\omega_0 2C_{gs}R_s} = -jQ_{net}$$

Relevant Time Constants

$$\tau_1 = R_s C_{gs} \qquad \omega_0^2 = \frac{1}{(L_s + L_g)C_{gs}}$$

$$\tau_2 = L_s g_{m1} = \frac{g_{m1}}{C_{gs}} C_{gs} L_s = \omega_T L_s C_{gs} = R_s C_{gs} = \tau_1$$

$$Q = \frac{1}{2R_s \omega_0 C_{gs}} = \frac{1}{2\tau_1 \omega_0} \qquad \tau_1 = \frac{1}{2Q\omega_0}$$

- Using these relations we simplify A_1 as

$$A_1 = \frac{1}{1 - \left(\frac{\omega}{\omega_0}\right)^2 + j\frac{\omega_1}{\omega_0}\frac{1}{Q}} = \frac{1}{D(\omega)}$$

Second Order Terms

- Equate second-order terms

$$0 = C_{gs} R_s j(\omega_1 + \omega_2) A_2 - (L_g + L_s) C_{gs} (\omega_1 + \omega_2)^2 A_2 + A_2 + L_s j(\omega_1 + \omega_2)\left(g_{m1} A_2 + g_{m2} A_1(\omega_1) A_1(\omega_2)\right)$$

$$A_2(\omega_1, \omega_2) = \frac{-A_1(\omega_1) A_1(\omega_2) j(\omega_1 + \omega_2) g_{m1} L_s}{1 - (\omega_1 + \omega_2)^2 (L_s + L_g) C_{gs} + j(\omega_1 + \omega_2)(C_{gs} R_s + L_s g_{m1})} \frac{1}{2(V_{gs} - V_t)}$$

$$A_2(\omega_1, \omega_2) = \frac{-j(\omega_1 + \omega_2)\tau_1}{D(\omega_1) D(\omega_2) D(\omega_1 + \omega_2)} \frac{1}{2(V_{gs} - V_t)}$$

Third Order Terms

- If the MOS is truly square law, then there are no A_3 terms … but when we calculate the output current, we generate third order terms
- These are generated by the action of the feedback.

$$i_d = g_{m1}\left(A_1(\omega_1)\circ v_s + A_2(\omega_1,\omega_2)\circ v_s^2\right) + g_{m2}\left(A_1(\omega_1)\circ v_s + A_2(\omega_1,\omega_2)\circ v_s^2\right)^2$$

$$i_d = B_1(\omega_1)\circ v_s + B_2(\omega_1,\omega_2)\circ v_s^2 + \cdots$$

Output Current Volterra Series

- Using Volterra algebra, we have

$$B_1(\omega_1) = g_{m1}A_1 = \frac{g_{m1}}{D(\omega_1)}$$

$$B_2(\omega_1,\omega_2) = g_{m1}A_2 + g_{m2}A_1A_1 = \left(\frac{-j(\omega_1+\omega_2)\tau_1}{D(\omega_1)D(\omega_2)D(\omega_1+\omega_2)} + \frac{1}{D(\omega_1)D(\omega_2)}\right)g_{m2}$$

$$= \left(1 - \frac{j(\omega_1+\omega_2)\tau_1}{D(\omega_1+\omega_2)}\right)\frac{g_{m1}}{D(\omega_1)D(\omega_2)2(V_{gs}-V_t)}$$

$$B_3(\omega_1,\omega_2,\omega_3) = g_{m2}2A_1(\omega_1)A_2(\omega_1,\omega_2) =$$

$$\left(\frac{-j(\omega_1+\omega_2)}{D(\omega_1+\omega_2)} + \frac{-j(\omega_1+\omega_3)}{D(\omega_1+\omega_3)} + \frac{-j(\omega_3+\omega_2)}{D(\omega_3+\omega_2)}\right)\frac{2g_{m1}\tau_1}{2(V_{gs}-V_t)D(\omega_1)D(\omega_2)D(\omega_3)}$$

MOS HD2/IIP2

$$B_1(\omega_1) = \frac{g_{m1}}{D(\omega_1)} \qquad B_2(\omega_1,-\omega_2) = \left(1 - \frac{j(\omega_1-\omega_2)\tau_1}{D(\omega_1-\omega_2)}\right)\frac{g_{m1}}{D(\omega_1)D(-\omega_2)2(V_{gs}-V_t)}$$

$$\frac{B_2(\omega_1,-\omega_2)}{B_1(\omega_1)} = \left(1 - \frac{j(\omega_1-\omega_2)\tau_1}{D(\omega_1-\omega_2)}\right)\frac{1}{D(-\omega_2)2(V_{gs}-V_t)}$$

$$V_{IIP2} = \frac{B_1(\omega_1)}{B_2(\omega_1,-\omega_2)} = \frac{D(-\omega_2)2(V_{gs}-V_t)}{\left(1 - \frac{j(\omega_1-\omega_2)\tau_1}{D(\omega_1-\omega_2)}\right)} = \frac{D(-\omega_2)D(\omega_1-\omega_2)}{D(\omega_1-\omega_2) - j(\omega_1-\omega_2)\tau}2(V_{gs}-V_t)$$

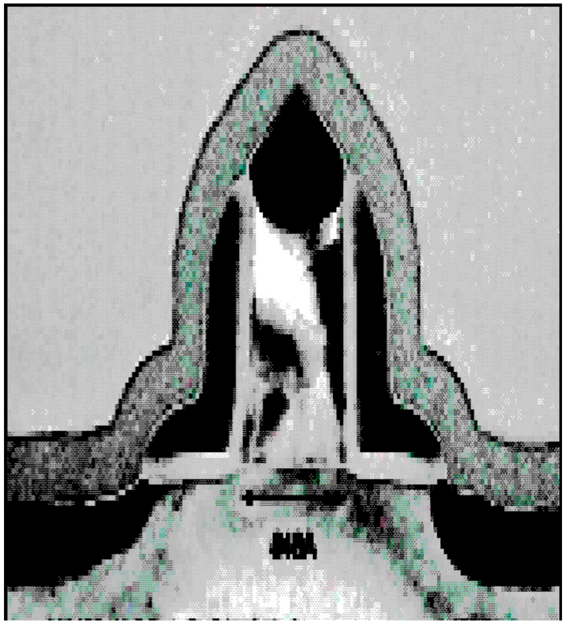
- The expression for IIP2 is now very simple! For a zero-IF system, we care about distortion at DC:

$$V_{IIP2} = \left|\frac{B_1(\omega_1)}{B_2(\omega_1,-\omega_2)}\right| = \left|\frac{D(-\omega_0)D(\omega_0-\omega_0)}{D(\omega_0-\omega_0) - j(\omega_0-\omega_0)\tau}\right|2(V_{gs}-V_t)$$

$$= \left|D(-\omega_0)\right|2(V_{gs}-V_t) = 2(V_{gs}-V_t)\cdot Q$$

Compact Model Physics vs. Reality

- Even the most advanced *compact models* have very humble physical origins
- Essentially a 1D core transistor model due to GCA (gradual channel approximation)
- Quantum effects necessarily ignored in core model
- Small dimension and 2D effects treated in a perturbational manner as corrections to 1D core
- Conclusion: Core model is important but any claims of "physical accuracy" should be taken with a grain of salt!
- Physical behavior and computational efficiency are therefore the key attributes
- Need a simple design model to capture important effects!



Building on a strong foundation: BSIM4

Technology

- Short/Narrow Channel Effects on Threshold Voltage
- Non-Uniform Vertical and Lateral Doping Effects
- Mobility Reduction Due to Vertical Field
- Quantum Mechanic Effective Gate Oxide Thickness Model

Saturation

- Carrier Velocity Saturation
- Channel Length Modulation (CLM)
- Substrate Current Induced Body Effect (SCBE)
- Unified current saturation model(velocity saturation, velocity overshoot, source end velocity limit)

Leakage

- Gate Dielectric Tunneling Current Model
- Gate Induced Drain Current Model (GIDL)
- Trap assisted tunneling and recombination current model

RF

- RF Model (Gate & Substrate Resistance Model)
- Unified Flicker Noise Model
- Holistic Thermal Noise Model

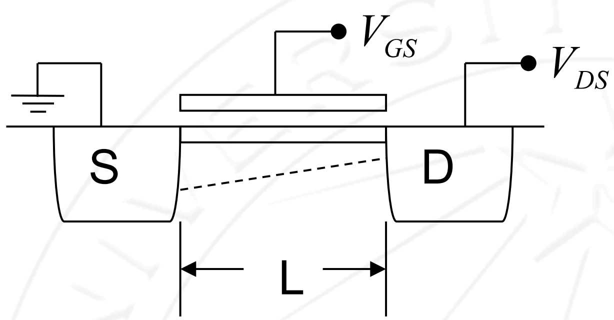
Parasitic

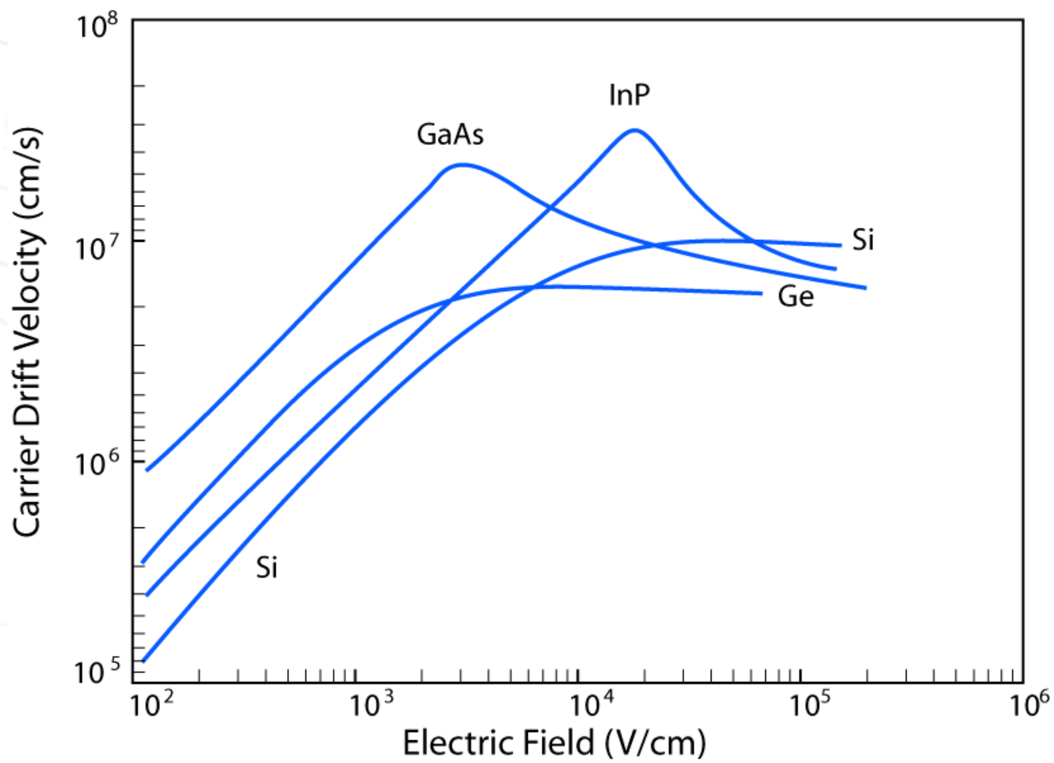
- Asymmetric Layout-Dependent Parasitic Model
- Scalable Stress Effect Model

Important MOS Non-Linearity

- Square law → short channel effects
- Mobility degradation
  - Universal mobility curve
  - Velocity saturation
- Body effect
- Output impedance non-linearity
- C_{gs} for high input swings

Velocity Saturation





- In triode, the average electric field across the channel is: $E = \dfrac{V_{DS}}{L}$
- The carrier velocity is proportional to the field for low field conditions $v = \mu E = \mu \dfrac{V_{DS}}{L}$

Mobility Reduction

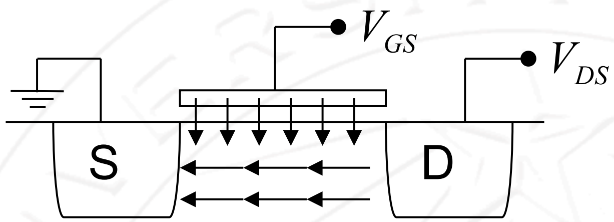
- As we decrease the channel length from say 10 μm to 0.1 μm and keep V_{DS}=1V, we see that the average electric field increases from 10^5 V/m to 10^7 V/m. For large field strengths, the carrier velocity approaches the scattering-limited velocity.
- This behavior follows the following curve-fit approximation:

$$v_d = \frac{\mu_n E}{1 + \frac{E}{E_c}}$$

For $E \ll E_c$, the linear behavior dominates

For $E \gg E_c$, $v_d \rightarrow \mu_n E_c = v_{scl}$

Vertical Mobility Degradation

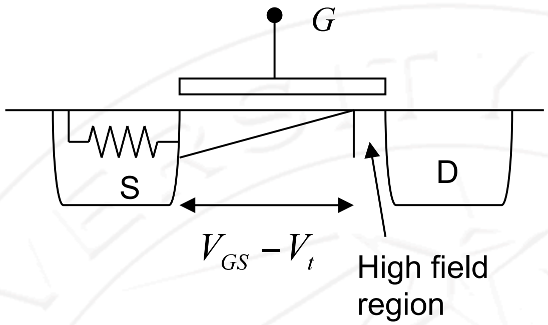


$$\mu_{eff} = \frac{\mu_n}{1+\theta\left(V_{GS}-V_t\right)}$$

$$\theta \propto \frac{1}{T_{ox}} \qquad T_{ox} = 100\text{Å}, \qquad \theta \cong 0.1V^{-1} \text{ to } 0.4V^{-1}$$

- Experimentally, it is observed that μ is degraded due to the presence of a vertical field. A physical explanation for this is that a stronger V_{GS} forces more of the carriers in the channel towards the surface where imperfections impede their movement.

Unified Mobility Degradation



$$I_D = \frac{\mu C_{ox}}{2}\left(\frac{W}{L}\right)(V_{GS}-V_t)^2$$

$$= \frac{\mu_{LF} C_{ox}}{2}\left(\frac{W}{L}\right)\frac{1}{1+\frac{E}{E_c}}(V_{GS}-V_t)^2$$

$$E = \frac{V_{DS}}{L} \approx \frac{V_{GS}-V_t}{L}$$

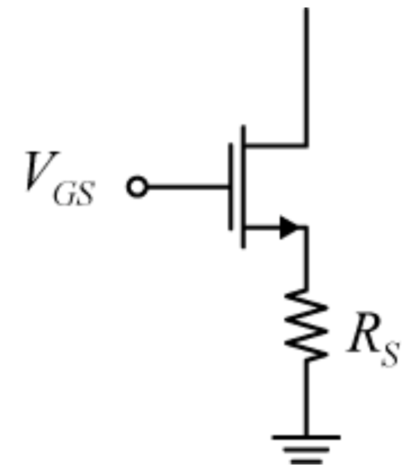
$$I_D = \frac{\mu C_{ox}}{2}\frac{1}{1+\theta(V_{GS}-V_t)}\left(\frac{W}{L}\right)(V_{GS}-V_t)^2$$

- Model horizontal mobility degradation in saturation, modify low field equation to include velocity saturation.
- Result looks like vertical field degradation.

Source Degeneration

$$I_D = \frac{\mu C_{ox}}{2}\left(\frac{W}{L}\right)\left(V'_{GS} - V_t\right)^2$$

$$V_{GS} = V'_{GS} + I_D R_S$$



$$I_D = \frac{\mu C_{ox}}{2}\left(\frac{W}{L}\right)\left(V_{GS} - I_D R_S - V_t\right)^2$$

$$= \frac{\mu C_{ox}}{2}\left(\frac{W}{L}\right)\left(\left(V_{GS} - V_t\right)^2 - 2 I_D R_S\left(V_{GS} - V_t\right) + \underbrace{\left(I_D R_S\right)^2}_{\text{neglect}}\right)$$

$$= \frac{\mu C_{ox}}{2}\left(\frac{W}{L}\right)\left(V_{GS} - V_t\right)^2 \frac{1}{1 + \underbrace{\mu C_{ox}\left(\frac{W}{L}\right)R_S}_{\theta}\left(V_{GS} - V_t\right)}$$

- Equation has approximately the same form.

Simple Short Channel Model

- The short channel I-V model can be approximated by

$$I_D = \frac{\mu C_{ox}}{2}\left(\frac{W}{L}\right)\frac{(V_{GS}-V_t)^2}{1+\theta(V_{GS}-V_t)}$$

- θ models:
  - Rsx parasitic
  - Vertical Field Mobility Degradation
  - Horizontal Field Mobility Degradation

$$\theta = f(T_{ox}, L, R_{sx})$$

Power Series Expansion

- Device is no longer square law…

$$I_D = a_1(V_{GS} - V_t) + a_2(V_{GS} - V_t)^2 + a_3(V_{GS} - V_t)^3 + \ldots$$

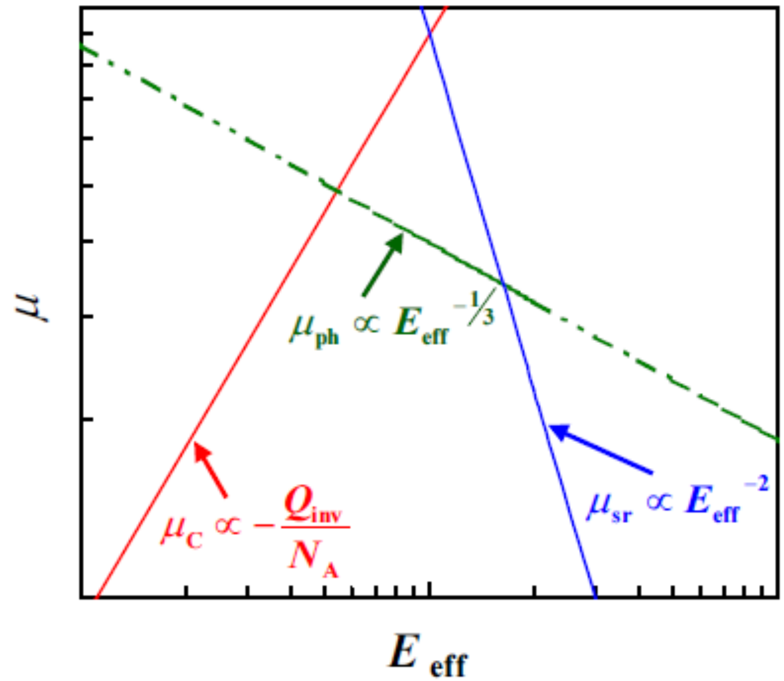
$$a_1 = \frac{dI_D}{d(V_{GS} - V_t)} = \frac{\mu C_{ox}}{2}\left(\frac{W}{L}\right)\frac{(1+\theta(V_{GS}-V_t))2(V_{GS}-V_t) - \theta(V_{GS}-V_t)}{(1+\theta(V_{GS}-V_t))^2}$$

$$= \frac{\mu C_{ox}}{2}\left(\frac{W}{L}\right)(V_{GS}-V_t)\frac{2+\theta(V_{GS}-V_t)}{(1+\theta(V_{GS}-V_t))^2}$$

$$a_2 = \frac{\mu C_{ox}}{2}\left(\frac{W}{L}\right)\frac{1}{(1+\theta(V_{GS}-V_t))^3}$$

$$a_3 = \frac{\mu C_{ox}}{2}\left(\frac{W}{L}\right)\frac{-\theta}{(1+\theta(V_{GS}-V_t))^4}$$

Coulomb Scattering

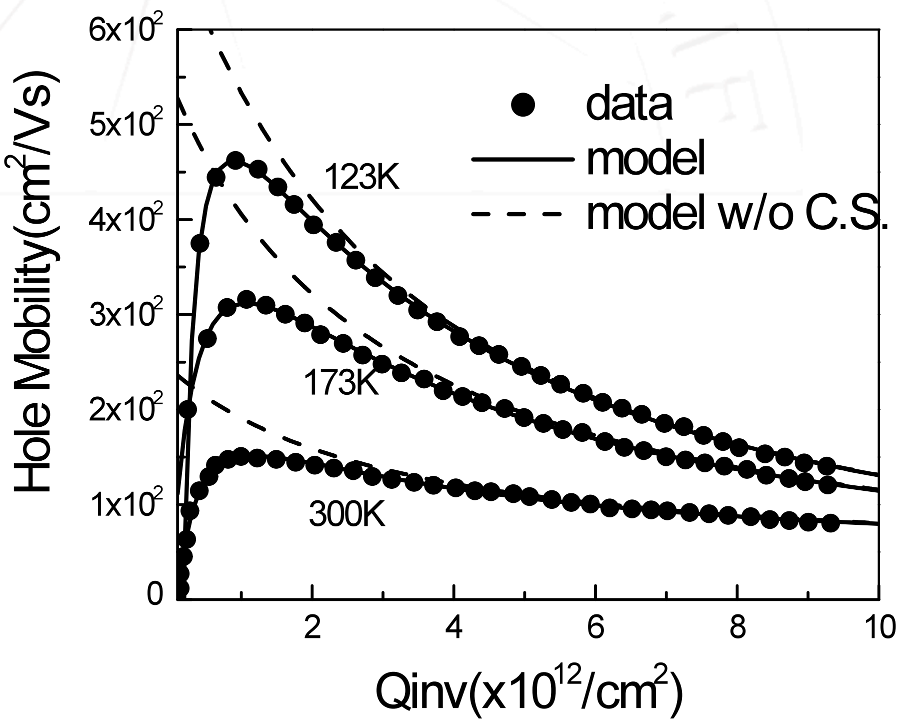


Source: Physical Background of MOS Model 11 (Level 1101), R. van Langevelde, A.J. Scholten and D.B.M. Klaassen

- A more complete model should include the effects of low field mobility.
- At low fields, the low level of inversion exposes "Coulomb Scattering" sites (due to shielding effect of inversion layer)

Mobility with Coulomb Scattering

$$\mu_{eff} = \frac{\mu_0}{1 + UA \cdot E_{eff} + UB \cdot E_{eff}^2 + UC \cdot \frac{\overline{Q_b}}{\overline{Q_b} + \overline{Q_{in}}}}$$



Need for Single Equation I-V

- Our models up to now only include strong inversion. In many applications, we would like a model to capture the entire I-V range from weak inversion to moderate inversion to strong inversion
  - Hard switching transistor
  - Power amplifiers, mixers, VCOs
- Surface potential models do this in a natural way but are implicit equations and more appropriate for numerical techniques

Smoothing Equation

- Since we have good models for weak/strong inversion, but missing moderate inversion, we can "smooth" between these regions and hope we capture the region in between (BSIM 3/4 do this too)

$$I_{DS} = f(V_{GS} - V_T) = K \frac{X^2}{1 + \theta X}$$

$$X = 2\eta \frac{kT}{q} \ln\left(1 + e^{\frac{q(V_{GS} - V_T)}{2\eta kT}} \right)$$

- As before θ models short-channel effects and η models the weak-inversion slope

Limiting Behavior

$$I_{DS} = f(V_{GS} - V_T) = K \frac{X^2}{1 + \theta X}$$

- In strong inversion, the exponential dominates giving us square law

$$X = 2\eta \frac{kT}{q} \ln\left(1 + e^{\frac{q(V_{GS}-V_T)}{2\eta kT}}\right) \approx V_{GS} - V_T$$

- In weak inversion, we expand the ln function

$$\ln(1 + x) \approx x \qquad X \approx 2\eta \frac{kT}{q} e^{\frac{q(V_{GS}-V_T)}{2\eta kT}}$$

$$I_{DS} = K \frac{X^2}{1 + \theta X} \approx K X^2 = K\left(2\eta \frac{kT}{q}\right)^2 e^{\frac{q(V_{GS}-V_T)}{\eta kT}}$$

I-V Derivatives

Use chain rule to differentiate the I-V relation:

$$I_{DS} = f(V) = K\frac{X^2}{1+\theta X}$$

$$f_V = f_X \cdot X_V \qquad f_{VV} = f_{XX} \cdot X_V^2 + f_X \cdot X_{VV}$$

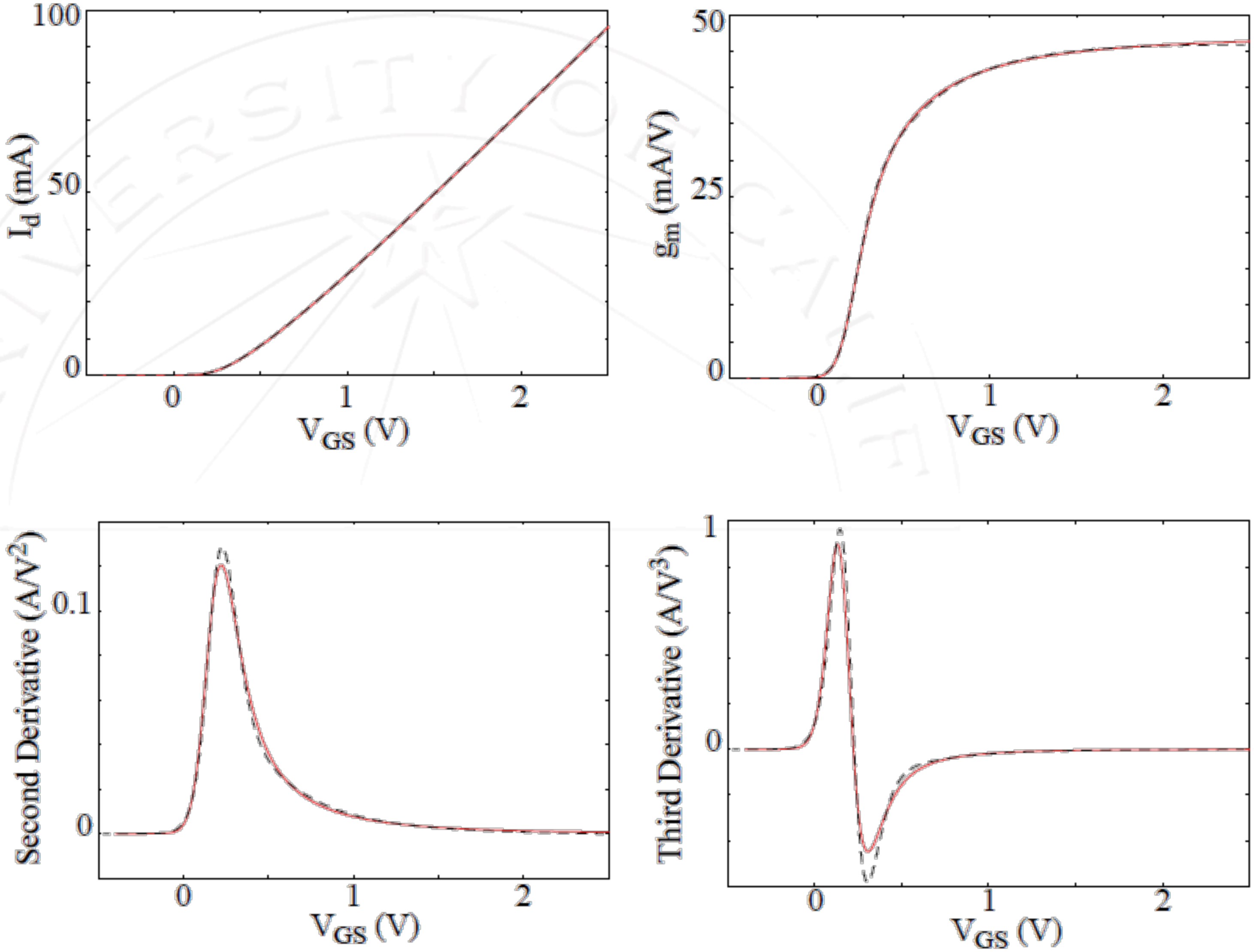
$$f_{VVV} = f_{XXX} \cdot X_V^3 + 3\left(f_{XX} \cdot X_V \cdot X_{VV}\right) + f_X \cdot X_{VVV}$$

$$f_X = K\frac{X \cdot (2+\theta X)}{(1+\theta X)^2} \qquad X_V = \frac{1}{1+s^{-2}}$$

$$f_{XX} = K\frac{2}{(1+\theta X)^3} \qquad X_{VV} = \frac{q}{2\eta kT}\frac{1}{\left(s+s^{-1}\right)^2}$$

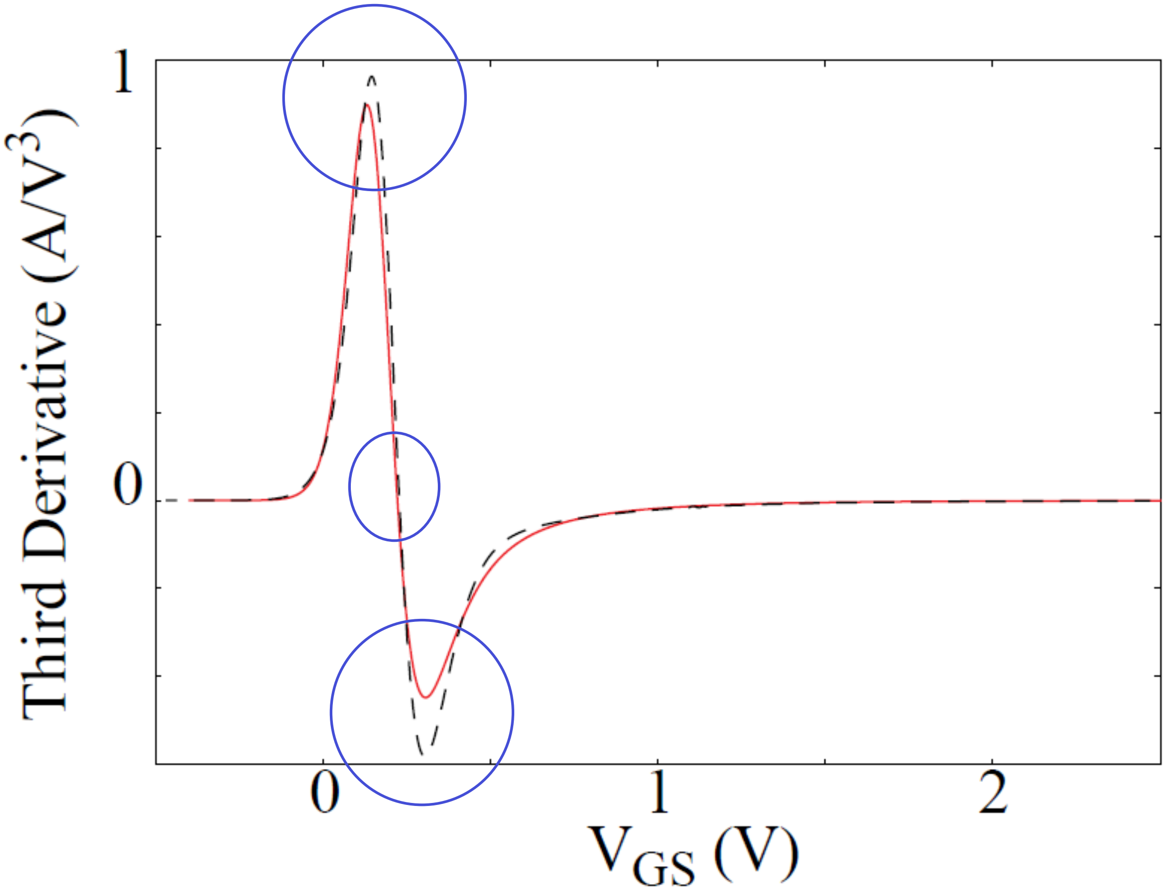
$$f_{XXX} = K\frac{-6\theta}{(1+\theta X)^4} \qquad X_{VVV} = -\frac{q^2}{(2\eta kT)^2}\frac{\left(s-s^{-1}\right)}{\left(s+s^{-1}\right)^3}$$

Single Equation Distortion



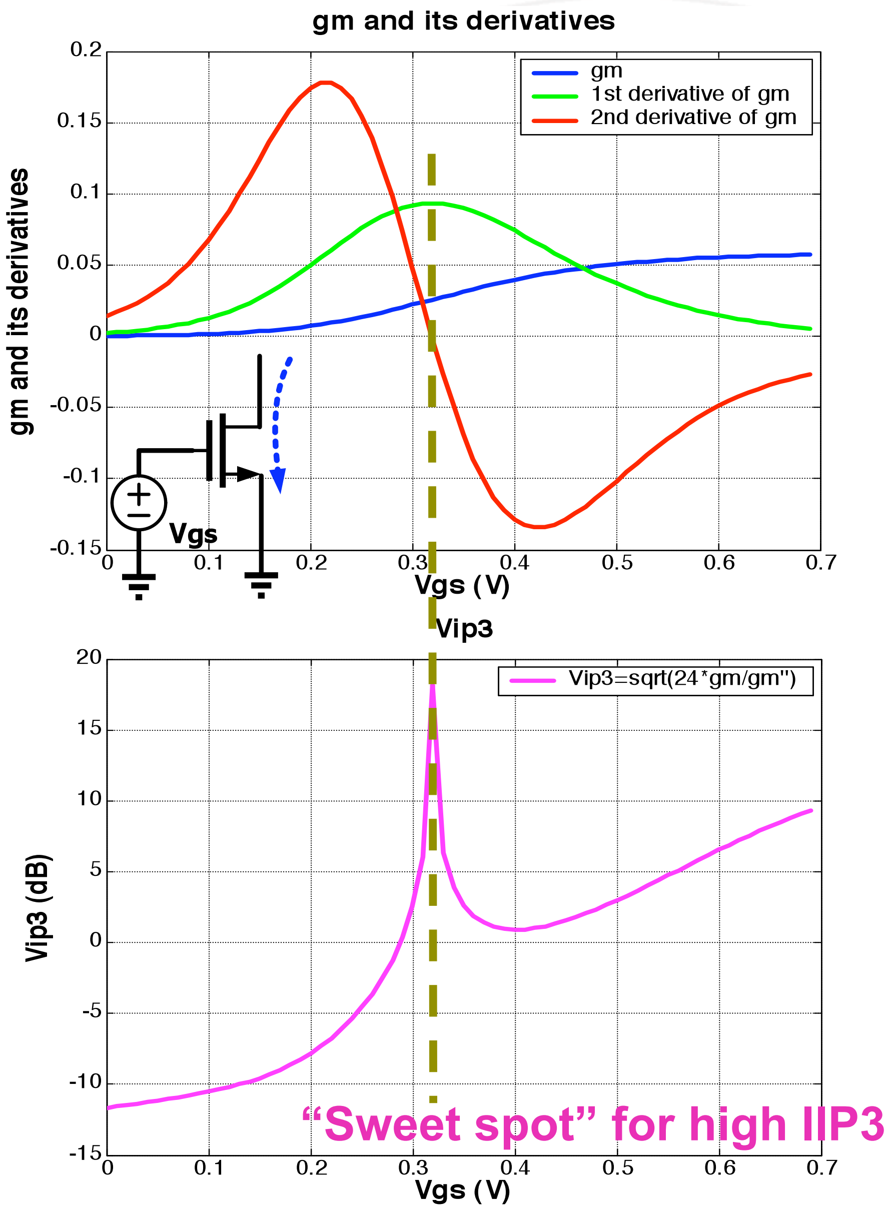
Source: Manolis Terrovitis **Analysis and Design of Current-Commutating CMOS Mixers**

IP3 "Sweet Spot"



- Notice "sweet spot" where $g_{m3} = 0$
- Notice the sign change in the third derivative … can we exploit this?

Bias Point for High Linearity



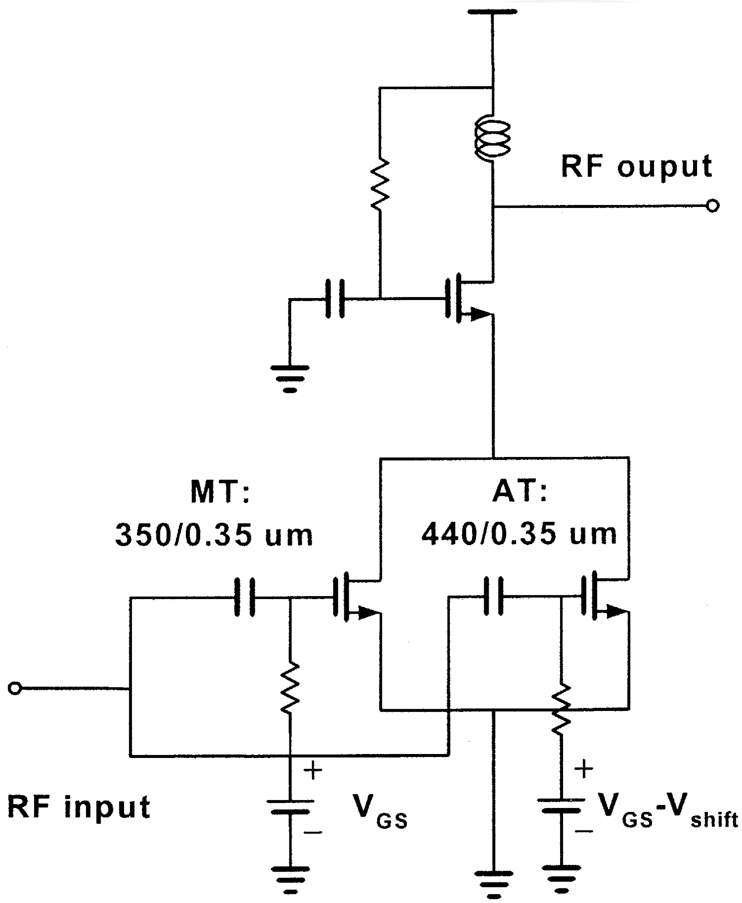
$$i_{out} \approx g_m \times v_{gs} + g_m' \times v_{gs}^2 + g_m'' \times v_{gs}^3$$

- Assume g_m nonlinearity dominates (current-mode operation and low output impedance)

$$V_{IP3} \cong \sqrt{24 g_m / g_m''}$$

- Sweet IIP3 point, $g_m'' \approx 0$, exists where the transistor transits from weak inversion (exponential law), moderate inversion (square law) to velocity saturation

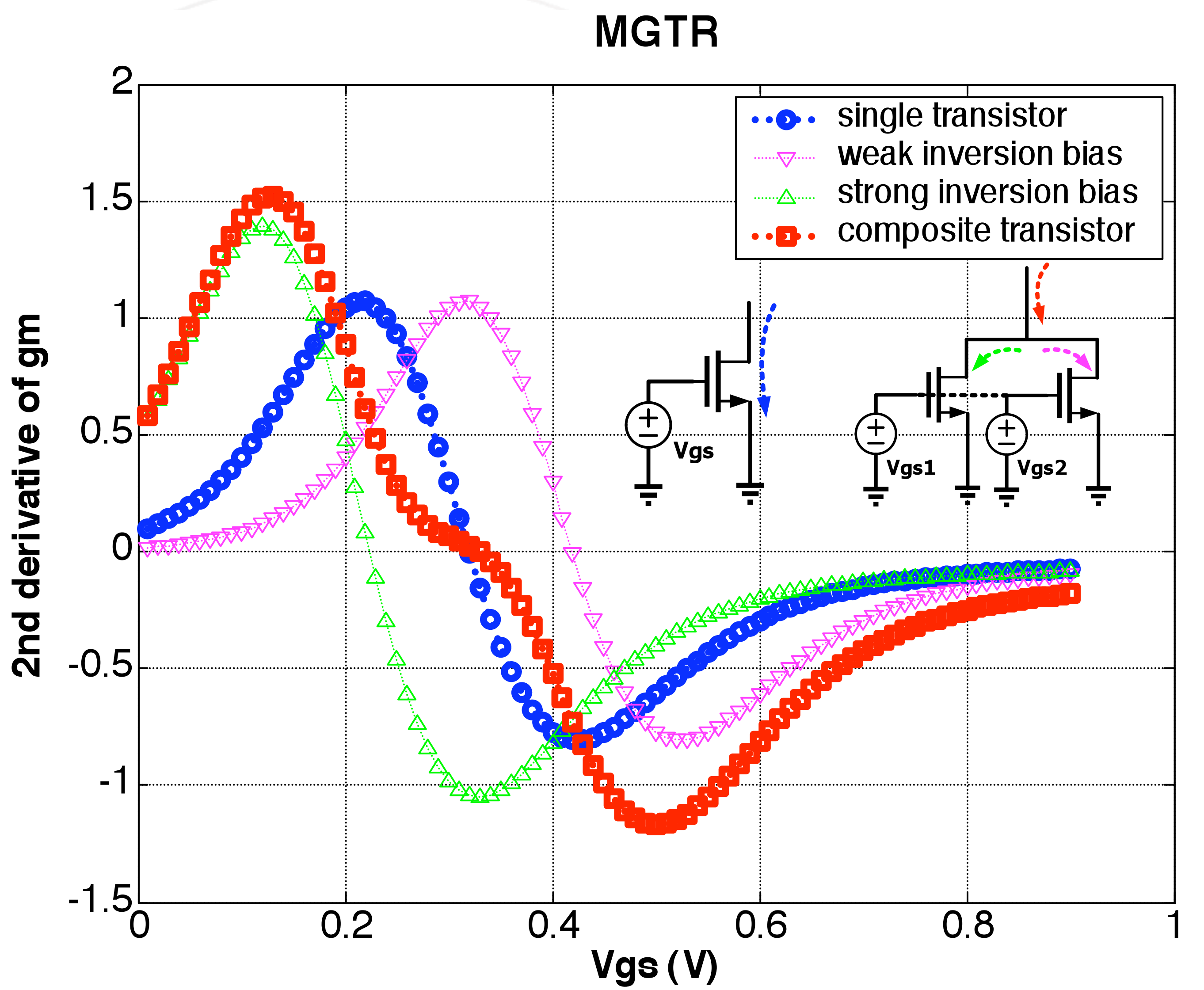
"Multiple Gated Transistors"



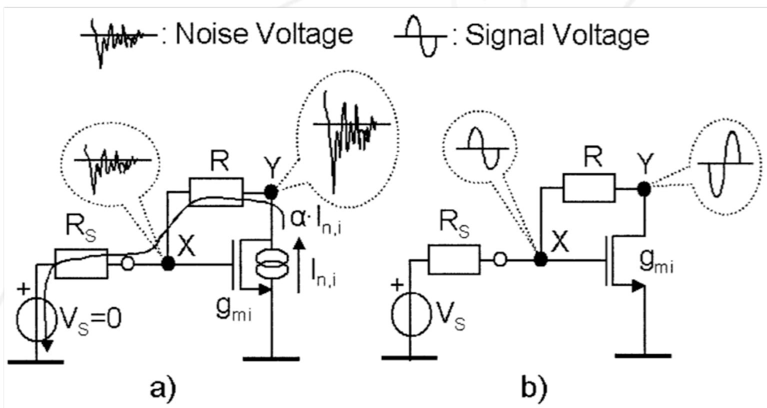
- Notice that we can use two parallel MOS devices, one biased in weak inversion (positive g_{m3}) and one in strong inversion (negative g_{m3}).
- The composite transistor has zero g_{m3} at bias point!

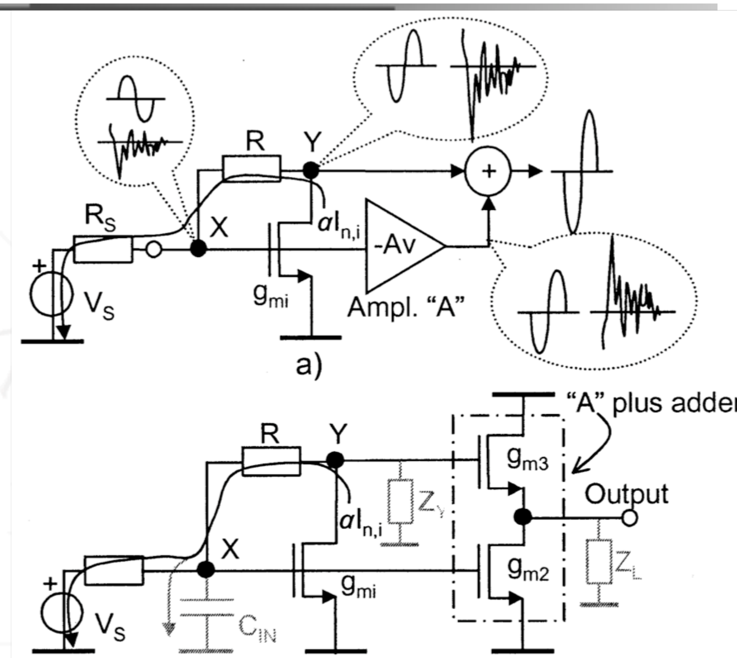
Source: A Low-Power Highly Linear Cascoded Multiple-Gated Transistor CMOS RF Amplifier With 10 dB IP3 Improvement, Tae Wook Kim et al., *IEEE MWCL*, Vol. 13, NO. 6, JUNE 2003

MGTR (Multi-Gated Transistor)



Wideband Noise Cancellation



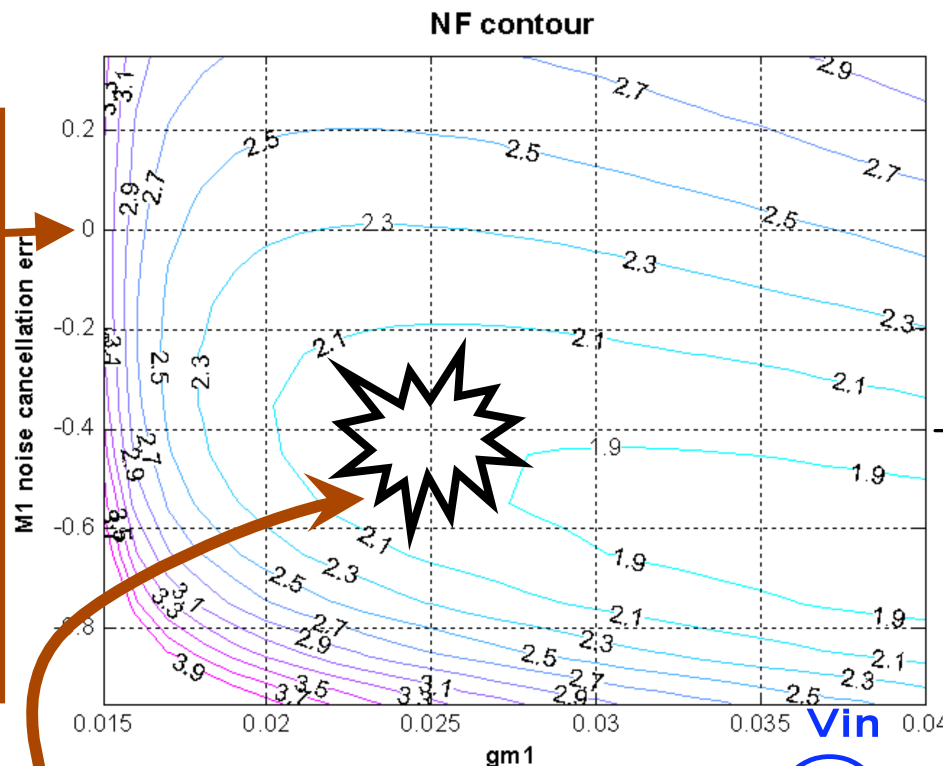


- Take advantage of amplifier topologies where the output thermal noise flows into the input (CG amplifier, shunt feedback amplifer, etc).
- Cancel thermal noise using a second feedforward path. Can we also cancel the distortion?
- Source: F. Bruccoleri, E. A. M. Klumperink, B. Nauta, “Wide-Band CMOS Low-Noise Amplifier Exploiting Thermal Noise Canceling,” *JSSC,* vol. 39, Feb. 2004.

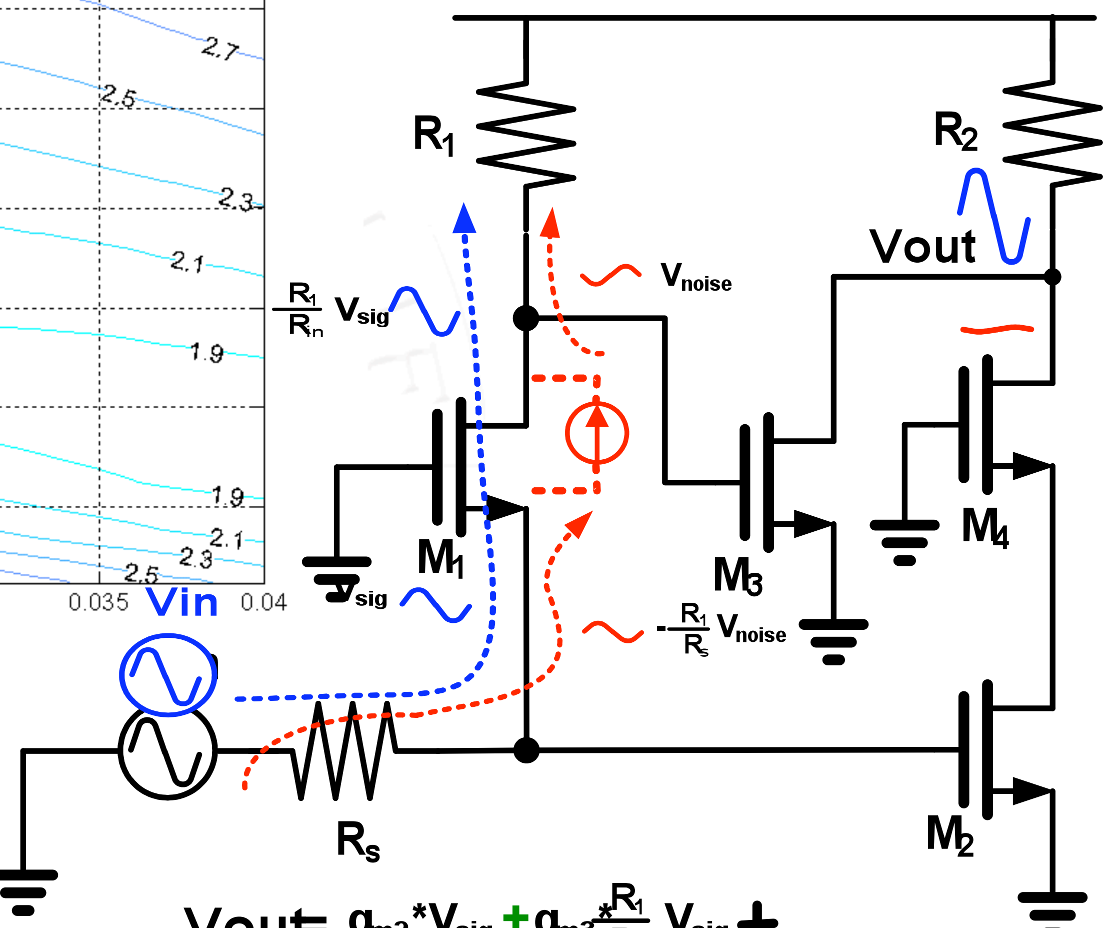
Noise Cancellation LNA

Motivated by [Bruccoleri, *et al.*, ISSCC02]

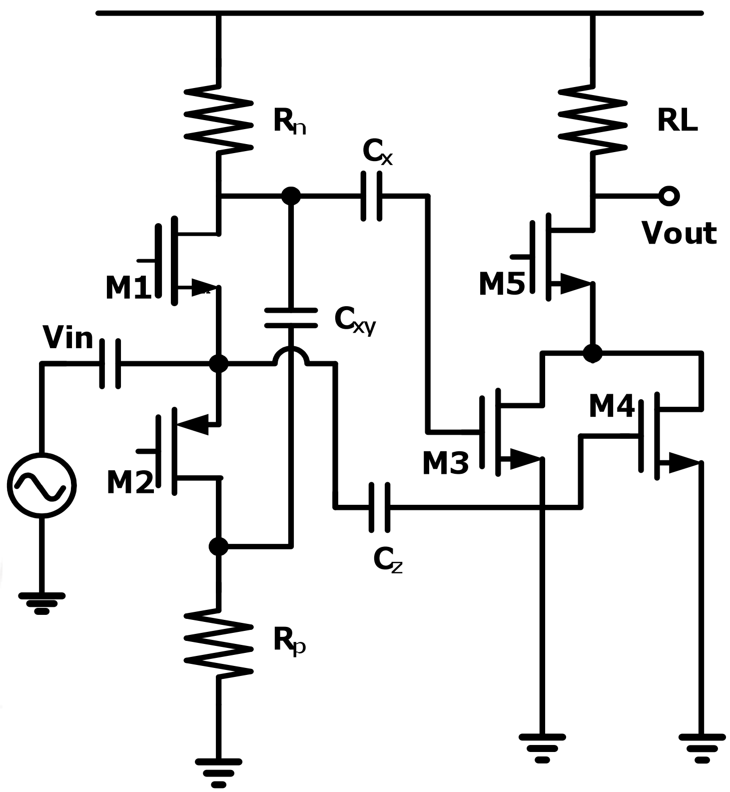
M1 noise full cancellation

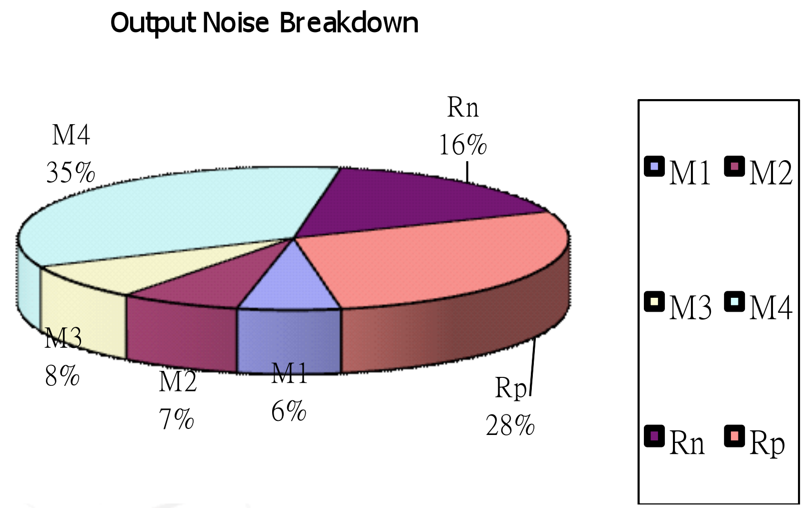


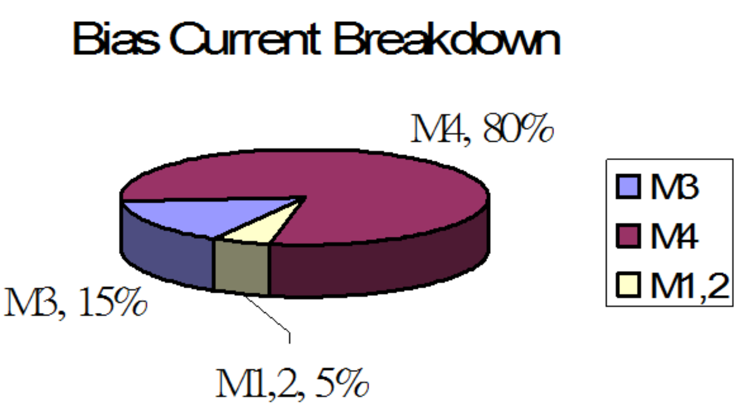
Optimal choice subject to fewer design parameter variations

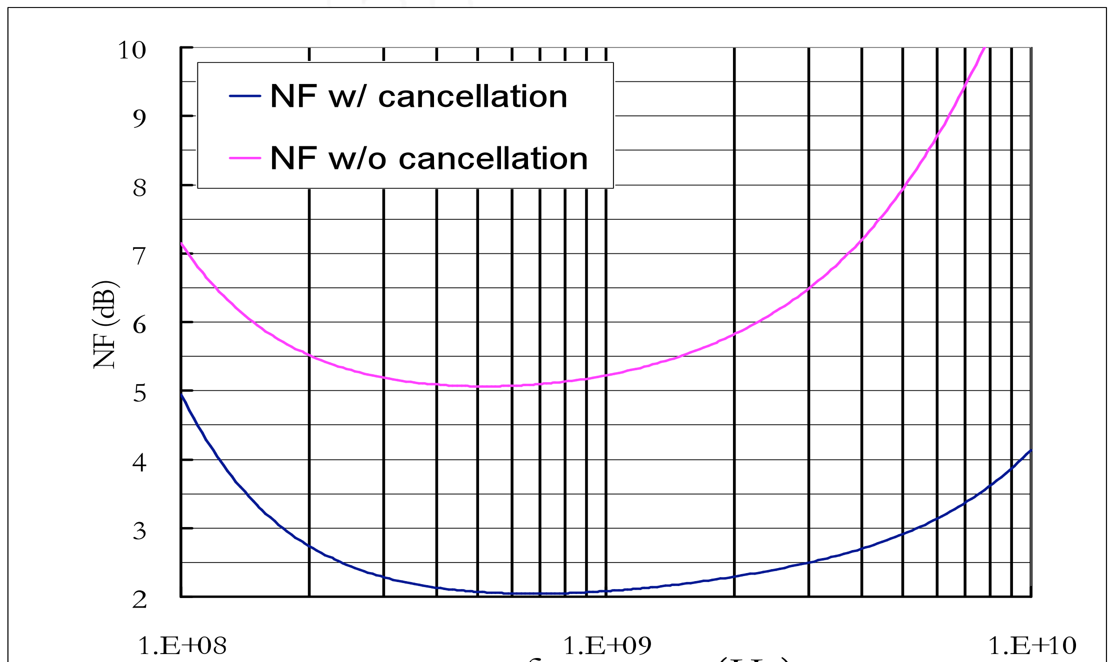


$$V_{out} = g_{m2} * V_{sig} + g_{m3} * \frac{R_1}{R_n} V_{sig} + g_{m3} * V_{noise} - g_{m2} * \frac{R_1}{R_s} V_{noise}$$

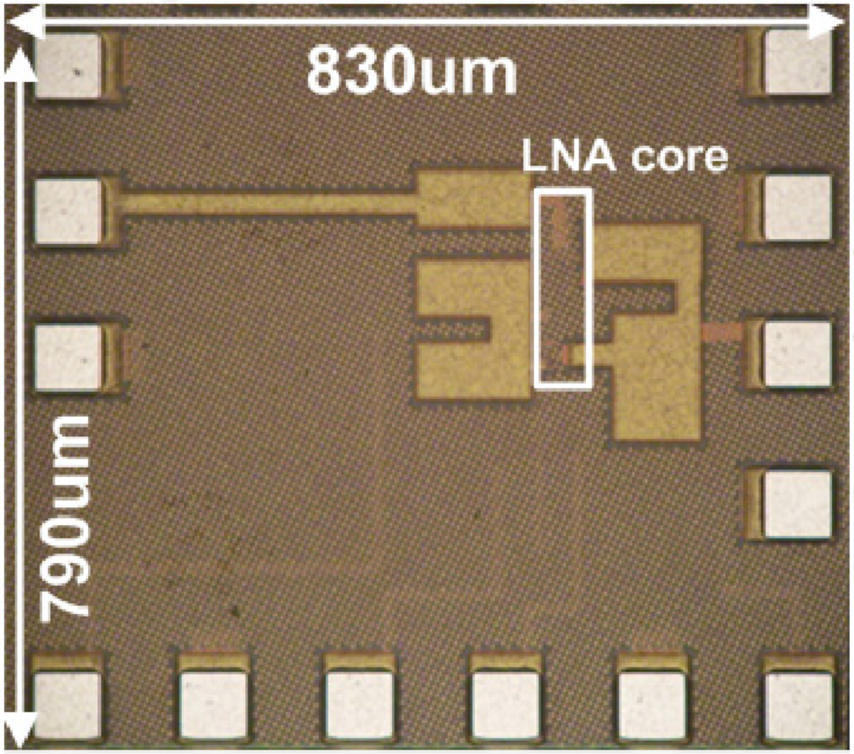

Rn
RL
Cx
Vout
M1
M5
Cxy
Vin
M4
M3
M2
Cz
Rp


Output Noise Breakdown
M4
35%
Rn
16%
M3
8%
M2
7%
M1
6%
Rp
28%
M1
M2
M3
M4
Rn
Rp


Bias Current Breakdown
M4, 80%
M3, 15%
M1,2, 5%
M3
M4
M1,2


NF w/ cancellation
NF w/o cancellation
NF (dB)
10
9
8
7
6
5
4
3
2
1.E+08
1.E+09
1.E+10
frequency (Hz)

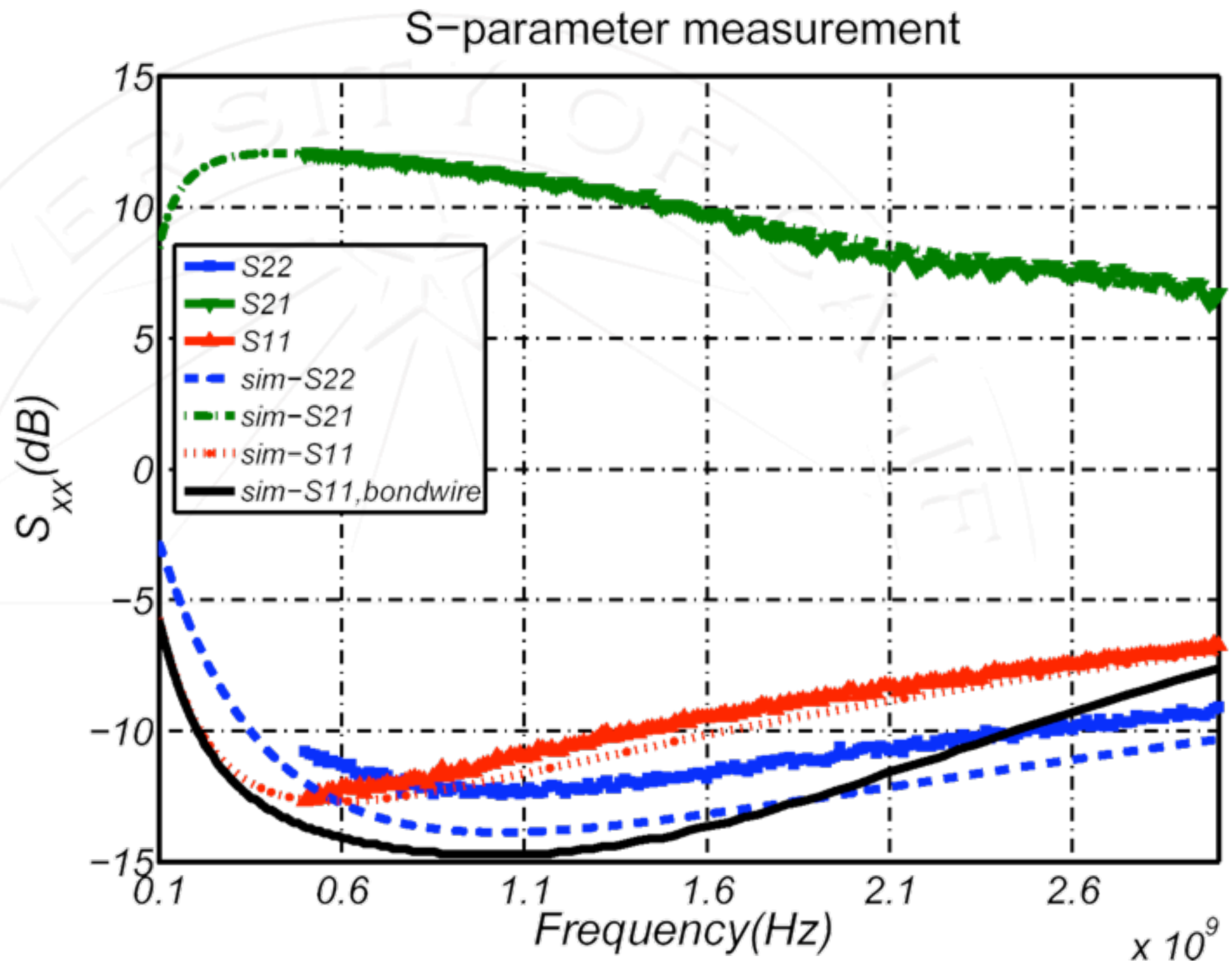
130nm LNA Prototype



- 130nm CMOS
- 1.5V, 12mA
- Employ only thin oxide transistors

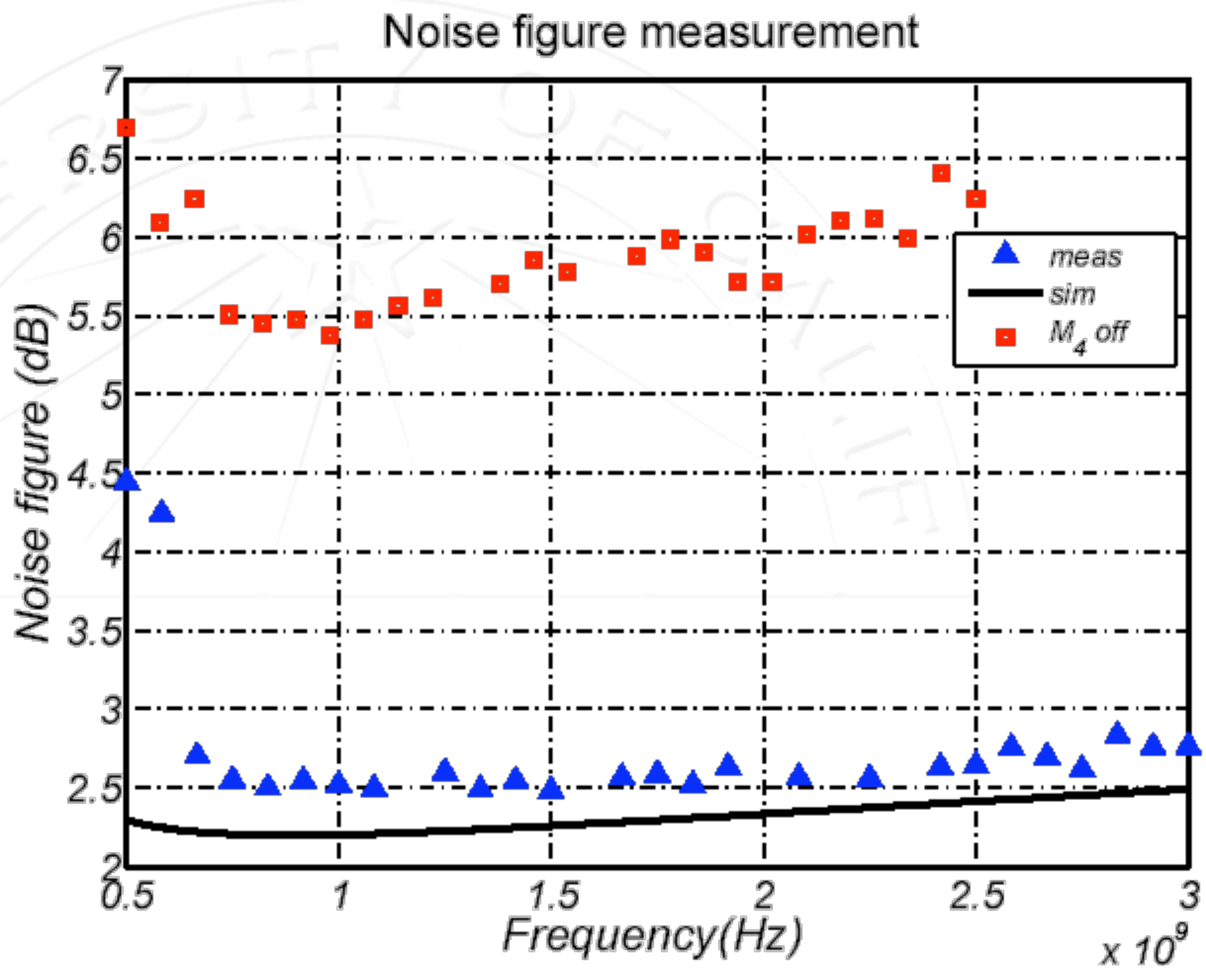
W.-H. Chen, G. Liu, Z. Boos, A. M. Niknejad, "A Highly Linear Broadband CMOS LNA Employing Noise and Distortion Cancellation," *IEEE Journal of Solid-State Circuits*, vol. 43, pp. 1164-1176, May 2008.

Measured S-Parameters



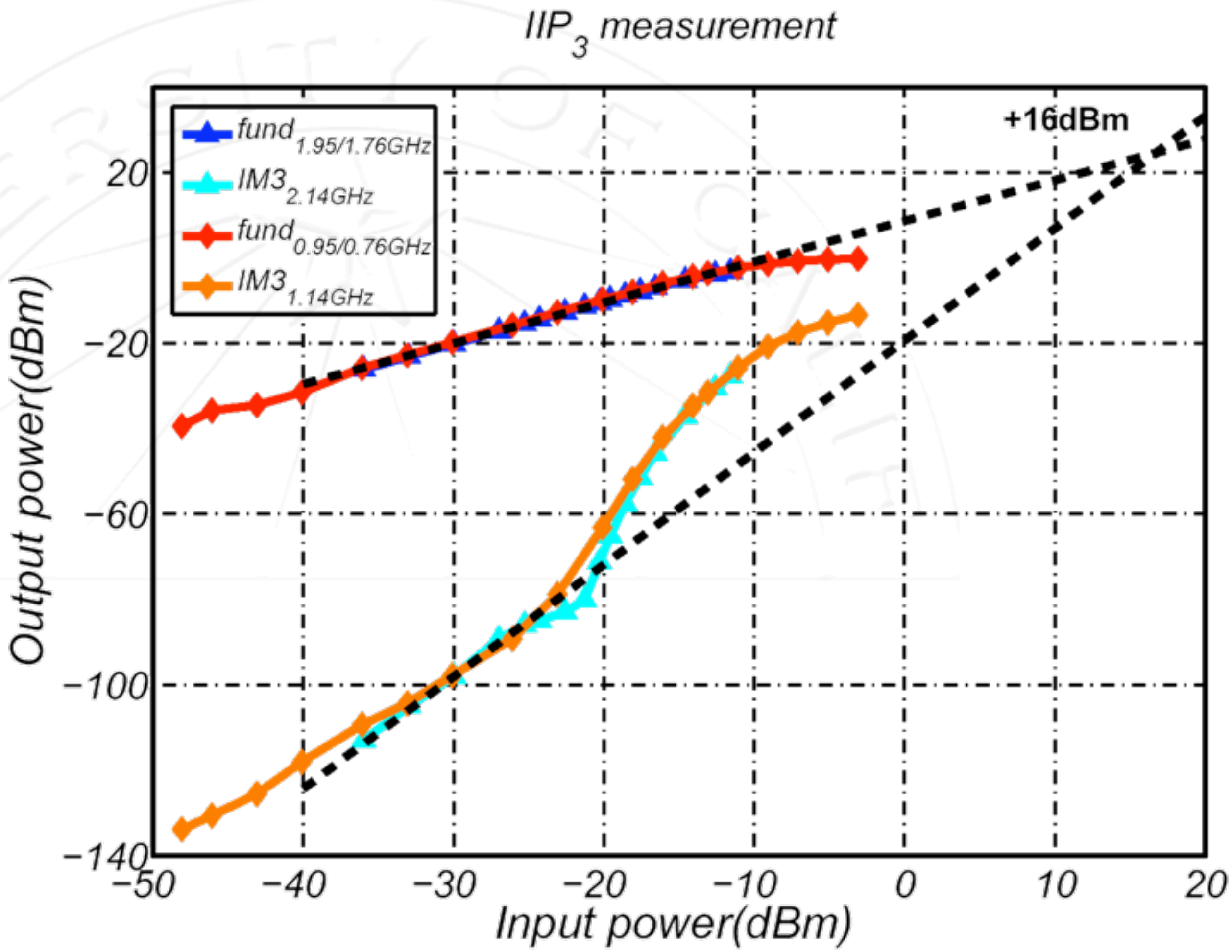
- Matches simulations well. Very broadband performance.

Measured Noise Performance



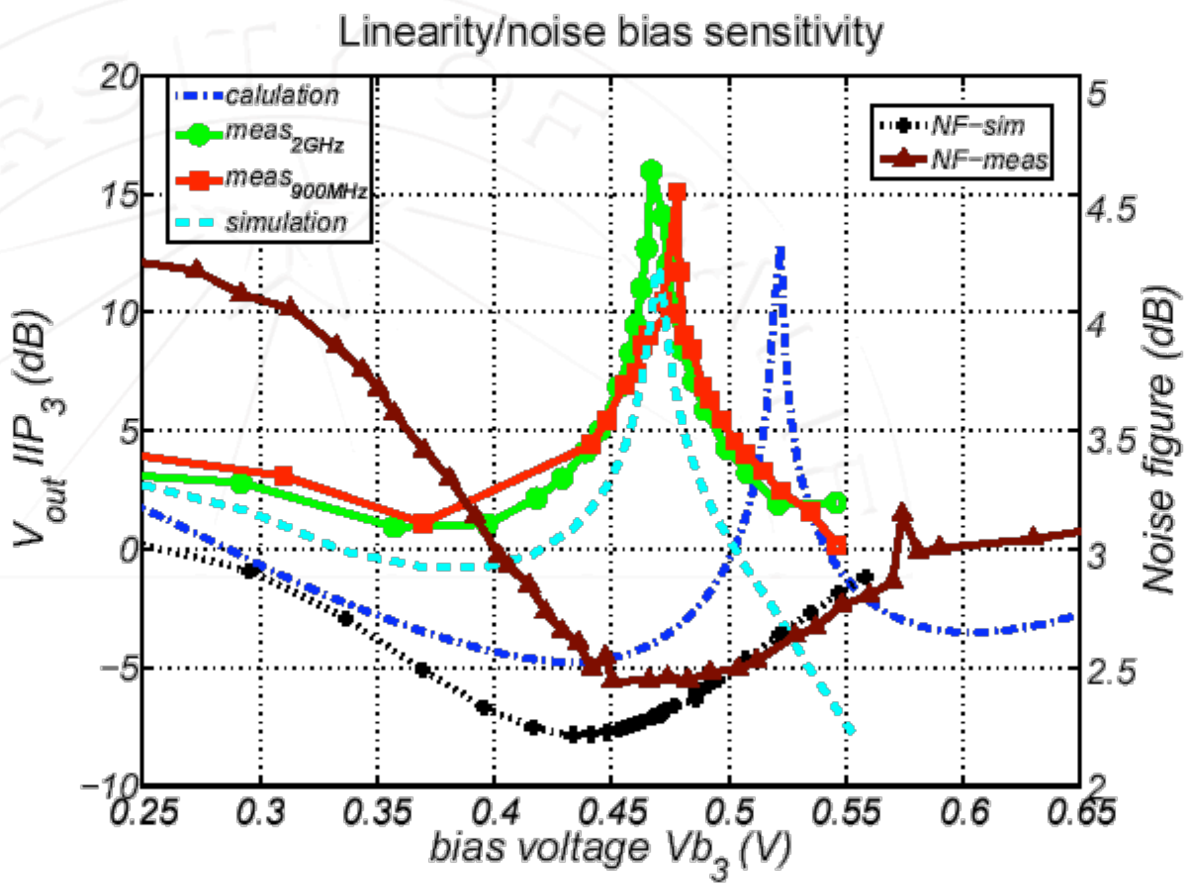
- Noise cancellation is clearly visible. This is also a "knob" for dynamic operation to save current.

Measured Linearity



- Record linearity of +16 dBm for out of band blockers.
- Linearity works over entire LNA band.

Linearity Bias Dependence



- As we vary the bias of key transistor, we simulate the effects of process/temp variation. There is a 50 mV window where the performance is still acceptable.

LNA Distortion Analysis

- Let's assume that the drain current is a power-series of the Vgs voltage:

$$i_{ds} = g_m \times v_{gs} + \frac{g_m'}{2!} \times v_{gs}^2 + \frac{g_m''}{3!} \times v_{gs}^3 + \ldots$$

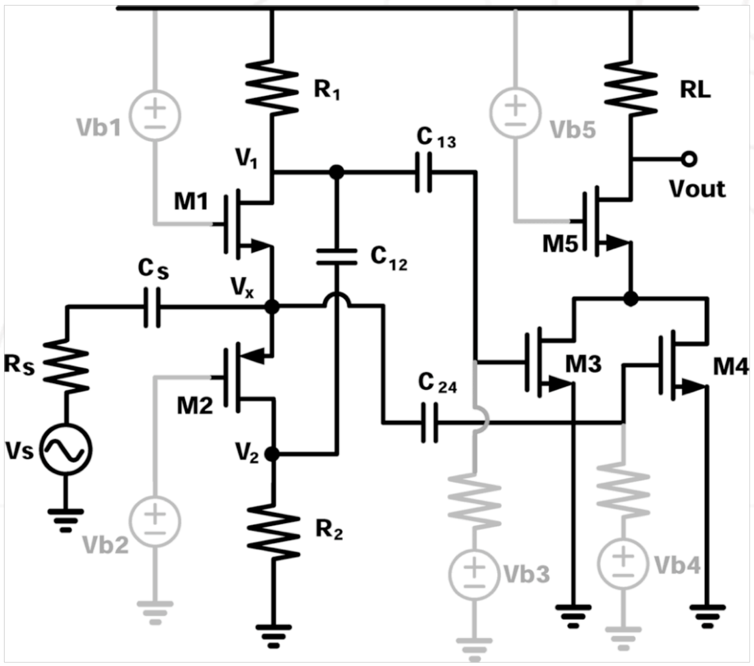
- The purpose of the "differential" input is to cancel the 2nd order distortion of the first stage (to minimize 2nd order interaction):

$$i_{out} = i_{ds,n} + i_{ds,p}$$

$$= \left(g_{m,N} \times v_{in} + \frac{g_{m,N}'}{2} \times v_{in}^2 + \frac{g_{m,N}''}{6} \times v_{in}^3 \right) - \left(g_{m,P} \times (-v_{in}) + \frac{g_{m,P}'}{2} \times (-v_{in})^2 + \frac{g_{m,P}''}{6} \times (-v_{in})^3 \right)$$

$$= (g_{m,N} + g_{m,P}) \times v_{in} + \frac{(g_{m,N}' - g_{m,P}')}{2} \times v_{in}^2 + \frac{(g_{m,N}'' + g_{m,P}'')}{6} \times v_{in}^3$$

Distortion Equivalent Circuit



- Assume R1/R2 and RL are small so that r_o non-linearity is ignored.

$$V_x = A_1(s_1) \circ V_s + A_2(s_1, s_2) \circ V_s^2 + A_3(s_1, s_2, s_3) \circ V_s^3$$

$$V_1 = B_1(s_1) \circ V_s + B_2(s_1, s_2) \circ V_s^2 + B_3(s_1, s_2, s_3) \circ V_s^3$$

$$V_2 = C_1(s_1) \circ V_s + C_2(s_1, s_2) \circ V_s^2 + C_3(s_1, s_2, s_3) \circ V_s^3$$

$$i_{m1} + \frac{V_x - V_1}{r_{o1}} + i_{m2} + \frac{V_x - V_2}{r_{o2}} + \frac{V_x}{Z_x(s)} = \frac{V_s - V_x}{Z_s(s)}$$

$$i_{m1} + \frac{V_x - V_1}{r_{o1}} = \frac{V_1}{Z_1(s)} + \frac{V_1 - V_2}{Z_{12}(s)}$$

$$i_{m2} + \frac{V_x - V_2}{r_{o2}} = \frac{V_2}{Z_2(s)} + \frac{V_2 - V_1}{Z_{12}(s)}$$

$$Z_1(s) = R_1 \parallel \frac{1}{sC_1}$$

$$Z_s(s) = R_s + \frac{1}{sC_s}$$

$$Z_x(s) = \frac{1}{sC_x}$$

Drain Current Non-Linearity

- Assuming that the gates of the input transistors are grounded at RF:

$$
\begin{aligned}
i_{m1} &= -\left(g_{m1}(-V_x) + \frac{g'_{m1}}{2}(-V_x)^2 + \frac{g''_{m1}}{6}(-V_x)^3 \right) \\
&= g_{m1}V_x - \frac{g'_{m1}}{2}V_x^2 + \frac{g''_{m1}}{6}V_x^3 \\
i_{m2} &= g_{m2}V_x + \frac{g'_{m2}}{2}V_x^2 + \frac{g''_{m2}}{6}V_x^3
\end{aligned}
$$

- First-order Kernels are found from:

$$
g_{m1}A_1(s) + \frac{A_1(s) - B_1(s)}{r_{o1}} + g_{m2}A_1(s) + \frac{A_1(s) - C_1(s)}{r_{o2}} + \frac{A_1(s)}{Z_x(s)} = \frac{1 - A_1(s)}{Z_s(s)}
$$

$$
g_{m1}A_1(s) + \frac{A_1(s) - B_1(s)}{r_{o1}} = \frac{B_1(s)}{Z_1(s)} + \frac{B_1(s) - C_1(s)}{Z_{12}(s)}
$$

$$
g_{m2}A_1(s) + \frac{A_1(s) - C_1(s)}{r_{o2}} = \frac{C_1(s)}{Z_2(s)} + \frac{C_1(s) - B_1(s)}{Z_{12}(s)}
$$

Simplified First-Order

- At the frequency of interest, Z12 ~ 0 and B1 ~ C1

$$A_1(s) = \frac{(Z_1(s) \parallel Z_2(s)) + (r_{o1} \parallel r_{o2})}{H(s)}$$

$$B_1(s) = \frac{Z_1(s) \parallel Z_2(s)}{\left(\frac{Z_1(s) \| Z_2(s) + (r_{o1} \| r_{o2})}{1 + (g_{m1} + g_{m2})(r_{o1} \| r_{o2})}\right)} A_1(s)$$

$$C_1(s) = B_1(s)$$

$$H(s) = Z_s(s)\Big(1 + (g_{m1} + g_{m2})(r_{o1} \parallel r_{o2})\Big) + \Big((Z_1(s) \parallel Z_2(s)) + (r_{o1} \parallel r_{o2})\Big)\left(1 + \frac{Z_s(s)}{Z_x(s)}\right)$$

Second-Order Terms

- Retaining only 2nd order terms in the KCL equations:

$$g_{m1}A_2(s_1,s_2) - \frac{g'_{m1}}{2}A_1(s_1)A_1(s_2) + \frac{A_2(s_1,s_2)-B_2(s_1,s_2)}{r_{o1}} +$$

$$g_{m2}A_2(s_1,s_2) + \frac{g'_{m2}}{2}A_1(s_1)A_1(s_2) + \frac{A_2(s_1,s_2)-C_2(s_1,s_2)}{r_{o2}} + \frac{A_2(s_1,s_2)}{Z_x(s_1+s_2)} = \frac{-A_2(s_1,s_2)}{Z_s(s_1+s_2)}$$

$$g_{m1}A_2(s_1,s_2) - \frac{g'_{m1}}{2}A_1(s_1)A_1(s_2) + \frac{A_2(s_1,s_2)-B_2(s_1,s_2)}{r_{o1}} = \frac{B_2(s_1,s_2)}{Z_1(s_1+s_2)} + \frac{B_2(s_1,s_2)-C_2(s_1,s_2)}{Z_{12}(s_1+s_2)}$$

$$A_2(s_1,s_2) = \frac{\frac{1}{2}(-g'_{m1}+g'_{m2})(r_{o1}\parallel r_{o1})Z_s(s_1+s_2)A_1(s_1)A_1(s_2) + \triangle A_2(s_1,s_2)}{H(s_1+s_2)+\triangle H(s_1,s_2)}$$

Second-Order Kernels

- Solving above equations we arrive at:

$$A_2(s_1,s_2) = \frac{\frac{1}{2}(-g'_{m1}+g'_{m2})(r_{o1} \parallel r_{o1})Z_s(s_1+s_2)A_1(s_1)A_1(s_2) + \triangle A_2(s_1,s_2)}{H(s_1+s_2)+\triangle H(s_1,s_2)}$$

$$B_2(s_1,s_2) = \frac{-\frac{Z_1(s_1+s_2)\|Z_2(s_1+s_2)}{Z_x(s_1+s_2)\|Z_s(s_1+s_2)}\Big(\frac{1}{2}(-g'_{m1}+g'_{m2})(r_{o1} \parallel r_{o1})Z_s(s_1+s_2)A_1(s_1)A_1(s_2)\Big) + \triangle B_2(s_1,s_2)}{H(s_1+s_2)+\triangle H(s_1,s_2)}$$

≈ 0

$$\triangle A_2(s_1,s_2) = \frac{1}{2}Z_{12}(s_1+s_2)A_1(s_1)A_1(s_2)\frac{Z_s(s_1+s_2)}{Z_1(s_1+s_2)+Z_2(s_1+s_2)}\times$$
$$\Big((g'_{m1}-g'_{m2})(r_{o1} \parallel r_{o2}) + \frac{g'_{m1}r_{o1}Z_2(s_1+s_2) - g'_{m2}r_{o2}Z_1(s_1+s_2)}{r_{o1}+r_{o2}}\Big)$$

≈ 0

$$\triangle B_2(s_1,s_2) = -\frac{1}{2}Z_{12}(s_1+s_2)A_1(s_1)A_1(s_2)\frac{Z_1(s_1+s_2)}{Z_1(s_1+s_2)+Z_2(s_1+s_2)}\frac{1}{r_{o1}+r_{o2}}\times$$
$$\Big(g'_{m1}r_{o1}\Big(Z_2(s_1+s_2)+r_{o2}\Big)\Big(1+\frac{Z_s(s_1+s_2)}{Z_x(s_1+s_2)}\Big) + Z_s(s_1+s_2)\Big(g'_{m2}r_{o2}(1+g_{m1}r_{o1}) + g'_{m1}r_{o1}(1+g_{m2}r_{o2})\Big)\Big)$$

≈ 0

$$\triangle H(s_1,s_2) = Z_{12}(s_1+s_2)\frac{Z_s(s_1,s_2)}{Z_1(s_1,s_2)+Z_2(s_1,s_2)}\frac{1}{(r_{o1}+r_{o2})}\times$$
$$\Big(\frac{(r_{o1}+Z_1(s_1+s_2))(r_{o2}+Z_2(s_1+s_2))}{Z_x(s_1+s_2)\parallel Z_s(s_1+s_2)} + \Big((1+g_{m1}r_{o1})\Big(r_{o2}+Z_2(s_1+s_2)\Big) + (1+g_{m2}r_{o2})\Big(r_{o1}+Z_1(s_1+s_2)\Big)\Big)\Big)$$

Third-Order Terms

$$g_{m1}A_3(s_1,s_2,s_3)+\frac{g''_{m1}}{6}A_1(s_1)A_1(s_2)A_1(s_3)-g'_{m1}\overline{A_1(s_1)A_2(s_2,s_3)}+\frac{A_3(s_1,s_2,s_3)-B_3(s_1,s_2,s_3)}{r_{o1}}$$

$$+g_{m2}A_3(s_1,s_2,s_3)+\frac{g''_{m2}}{6}A_1(s_1)A_1(s_2)A_1(s_3)+g'_{m2}\overline{A_1(s_1)A_2(s_2,s_3)}+\frac{A_3(s_1,s_2,s_3)-C_3(s_1,s_2,s_3)}{r_{o2}}$$

$$=-\frac{A_3(s_1,s_2,s_3)}{Z_s(s_1+s_2+s_3)}$$

$$g_{m1}A_3(s_1,s_2,s_3)+\frac{g''_{m1}}{6}A_1(s_1)A_1(s_2)A_1(s_3)-g'_{m1}\overline{A_1(s_1)A_2(s_2,s_3)}+\frac{A_3(s_1,s_2,s_3)-B_3(s_1,s_2,s_3)}{r_{o1}}$$

$$=\frac{B_3(s_1,s_2,s_3)}{Z_1(s_1+s_2+s_3)}+\frac{B_3(s_1,s_2,s_3)-C_3(s_1,s_2,s_3)}{Z_{12}(s_1+s_2+s_3)}$$

$$g_{m2}A_3(s_1,s_2,s_3)+\frac{g''_{m2}}{6}A_1(s_1)A_1(s_2)A_1(s_3)+g'_{m2}\overline{A_1(s_1)A_2(s_2,s_3)}+\frac{A_3(s_1,s_2,s_3)-C_3(s_1,s_2,s_3)}{r_{o1}}$$

$$=\frac{C_3(s_1,s_2,s_3)}{Z_2(s_1+s_2+s_3)}+\frac{C_3(s_1,s_2,s_3)-B_3(s_1,s_2,s_3)}{Z_{12}(s_1+s_2+s_3)}$$

Third-Order Kernel

- Assuming $Z_{12} \sim 0$ (at $s_1+s_2+s_3$):

$$A_3(s_1,s_2,s_3) = \frac{-Z_s(r_{o1} \parallel r_{o2})\Big(-(g'_{m1}+g'_{m2})\overline{A_1(s_1)A_2(s_2,s_3)} + \frac{1}{6}(g''_{m1}+g''_{m2})A_1(s_1)A_1(s_2)A_1(s_3)\Big)}{H(s_1+s_2+s_3)}$$

$$B_3(s_1,s_2,s_3) = \frac{-Z_1(s_1+s_2+s_3)}{Z_x(s_1+s_2+s_3) \parallel Z_s(s_1+s_2+s_3)} A_3(s_1,s_2,s_3)$$

Simplified Results Z_{12}~0

- The simplified equations are summarized here:

$$A_1(s) = \frac{Z_1(s) + r_{o1}}{H(s)}$$

$$A_2(s_1, s_2) = \frac{\frac{1}{2} g'_{m1} r_{o1} Z_s(s_1 + s_2) A_1(s_1) A_1(s_2)}{H(s_1 + s_2)}$$

$$A_3(s_1, s_2, s_3) = \frac{-Z_s(s1 + s2 + s3) r_{o1} \left(-g'_{m1} \overline{A_1(s_1) A_2(s_2, s_3)} + \frac{1}{6} g''_{m1} A_1(s_1) A_1(s_2) A_1(s_3) \right)}{H(s_1 + s_2 + s_3)}$$

$$B_1(s) = \frac{Z_1(s) \times (1 + g_{m1} r_{o1})}{Z_1(s) + r_{o1}} A_1(s)$$

$$B_2(s_1, s_2) = \frac{-Z_1(s_1 + s_2)}{Z_x(s_1 + s_2) \parallel Z_s(s_1 + s_2)} A_2(s_1, s_2)$$

$$B_3(s_1, s_2, s_3) = \frac{-Z_1(s_1 + s_2 + s_3)}{Z_x(s_1 + s_2 + s_3) \parallel Z_s(s_1 + s_2 + s_3)} A_3(s_1, s_2, s_3)$$

Output Voltage

- The output voltage is given by a new Volterra series. Assume for simplicity the following:

$$V_{out} = \left((g_{m3} \times V_1 + \frac{g'_{m3}}{2!} \times V_1^2 + \frac{g''_{m3}}{3!} \times V_1^3) + (g_{m4} \times V_x + \frac{g'_{m4}}{2!} \times V_x^2 + \frac{g''_{m4}}{3!} \times V_x^3) \right) \times Z_L(s)$$

- The fundamental and third-order output are therefore:

$$V_{out,fund} = \Big((A_1(s) \circ V_s) \times g_{m4} + (B_1(s) \circ V_s) \times g_{m3} \Big) \times Z_L(s)$$

$$\begin{aligned} V_{out,3^{rd}} = \Bigg(& \Big((A_3(s_1,s_2,s_3) \circ V_s^3) \times g_{m4} + (B_3(s_1,s_2,s_3) \circ V_s^3) \times g_{m3} \Big) \\ & + \Big((A_1(s) \circ V_s)^3 \times \frac{g''_{m4}}{6} + (B_1(s) \circ V_s)^3 \times \frac{g''_{m3}}{6} \Big) \\ & + \Big((\overline{A_1(s_1)A_2(s_2,s_3)} \circ V_s^3) \times g'_{m4} + (\overline{B_1(s_1)B_2(s_2,s_3)} \circ V_s^3) \times g'_{m3} \Big) \Bigg) \times Z_L(s) \end{aligned}$$

Focus on Third-Order Output

- At low frequencies:
  - $A_1/B_1 \sim R_{in}/R_1$
  - $A_2/B_2 \sim -R_s/R_1$
  - $A_3/B_3 \sim -R_s/R_1$

$$\frac{g_{m4}}{g_{m3}} = \frac{R_1}{R_s}, \quad and \quad \frac{g''_{m4}}{g''_{m3}} = \frac{R_1}{R_{in}} = \frac{R_1}{R_s} \times \frac{1}{1+\epsilon_{rr}}$$

Cancels like thermal noise

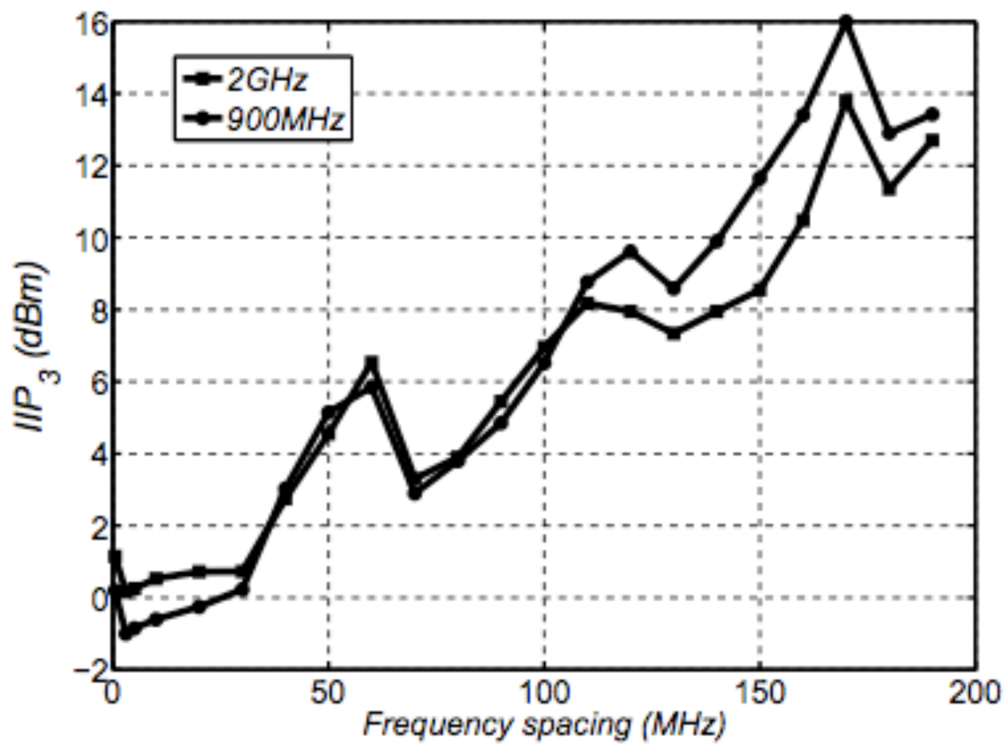
New distortion gen at output. Cancel with MGTR.

$$V_{out,3^{rd}} = \Bigg(\Big(\left(A_3(s_1,s_2,s_3)\circ V_s^3\right)\times g_{m4} + \left(B_3(s_1,s_2,s_3)\circ V_s^3\right)\times g_{m3}\Big)$$
$$+\left((A_1(s)\circ V_s)^3 \times \frac{g''_{m4}}{6} + (B_1(s)\circ V_s)^3 \times \frac{g''_{m3}}{6}\right)$$
$$+\left((\overline{A_1(s_1)A_2(s_2,s_3)}\circ V_s^3)\times g'_{m4} + (\overline{B_1(s_1)B_2(s_2,s_3)}\circ V_s^3)\times g'_{m3}\right)\Bigg)\times Z_L(s)$$

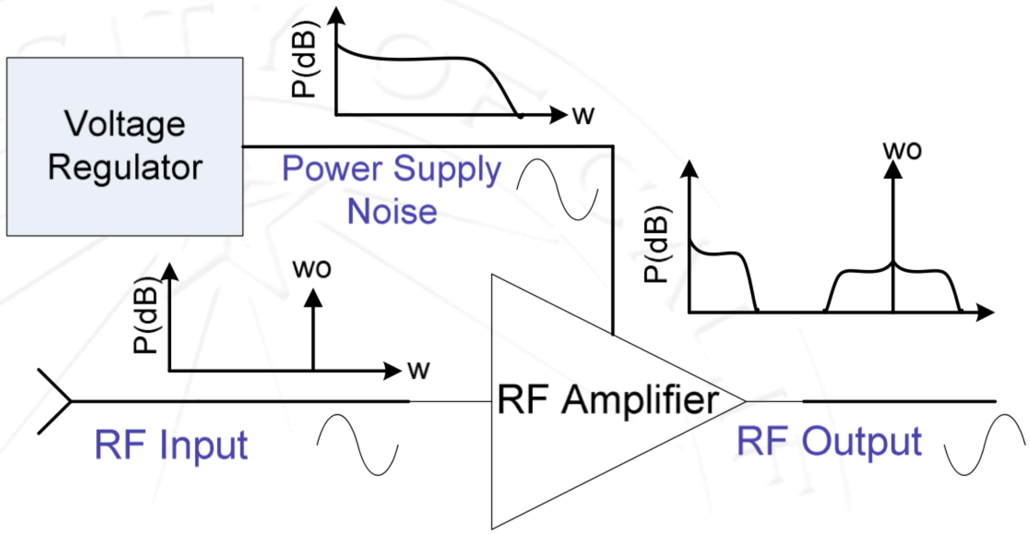
Second-order interaction: Must use g' =0

Two-Tone Spacing Dependence

- Because 2nd order interaction is minimized by using a PMOS and NMOS in parallel, the capacitor C_{12} plays an important role.
- When second order distortion is generated at low frequencies, f_1-f_2, the capacitor C_{12} has a high reactance and distortion cancellation does not take place.
- There is therefore a dependency to the two-tone spacing.

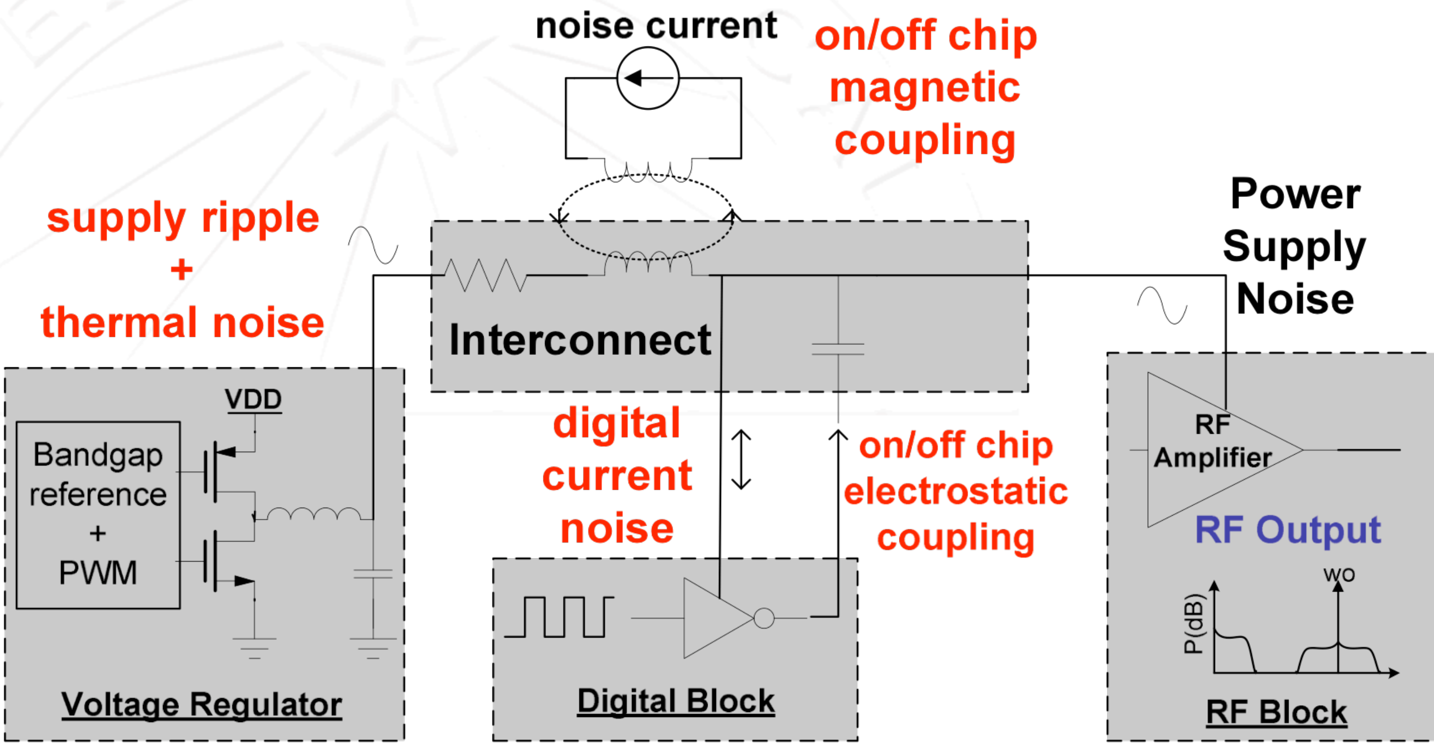


Power Supply Ripple



- In RF systems, the supply ripple can non-linearity transfer noise modulation on the supply to the output.
- This problem was recently analyzed by Jason Stauth: "Energy Efficient Wireless Transmitters: Polar and Direct-Digital Modulation Architectures," Ph.D. Dissertation, U.C. Berkeley

Supply Noise Sources



Multi-Port Memoryless Non-linearity

- The output voltage is a non-linear function of both the supply voltage and the input voltage. A two-variable Taylor series expansion can be used if the system is memory-less:

$$\begin{aligned} S_{out}(S_{in}, S_{vdd}) &= a_{10}S_{in} + a_{20}S_{in}^2 + a_{30}S_{in}^3 \cdots \\ &+a_{11}S_{in}S_{vdd} + a_{21}S_{in}^2S_{vdd} + \cdots \\ &+a_{01}S_{vdd} + a_{02}S_{vdd}^2 + a_{03}S_{vdd}^3 \cdots \end{aligned}$$

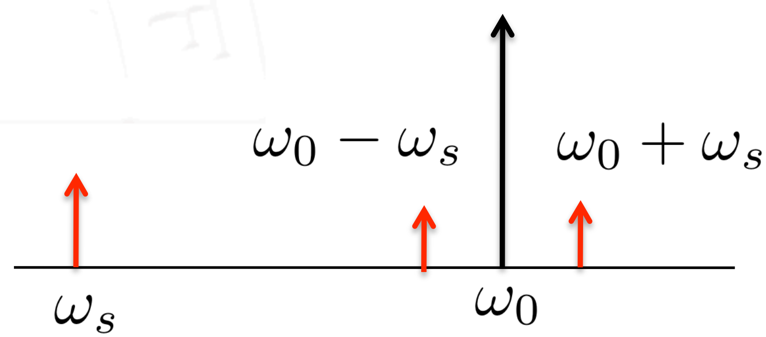
Supply Noise Sideband

- Assume the input is at RF and the supply noise is a tone. Then the output signal will contain a noise sideband given by:

$$S_{in} = v_i \cos(\omega_0 t)$$

$$S_{vdd} = v_s \cos(\omega_S t)$$

$$v_{out}(\omega_0 \pm \omega_S) = \frac{1}{2} a_{11} v_i v_S$$



$$Sideband(dBc) = dB\left(\frac{2a_{10}}{a_{11}} \cdot \frac{1}{v_s}\right)$$

$$PSRR(dBV) = dB\left(\frac{2a_{10}}{a_{11}}\right)$$

Multi-Port Volterra Series

- Extending the concept of a Volterra Series to a two input-port system, we have

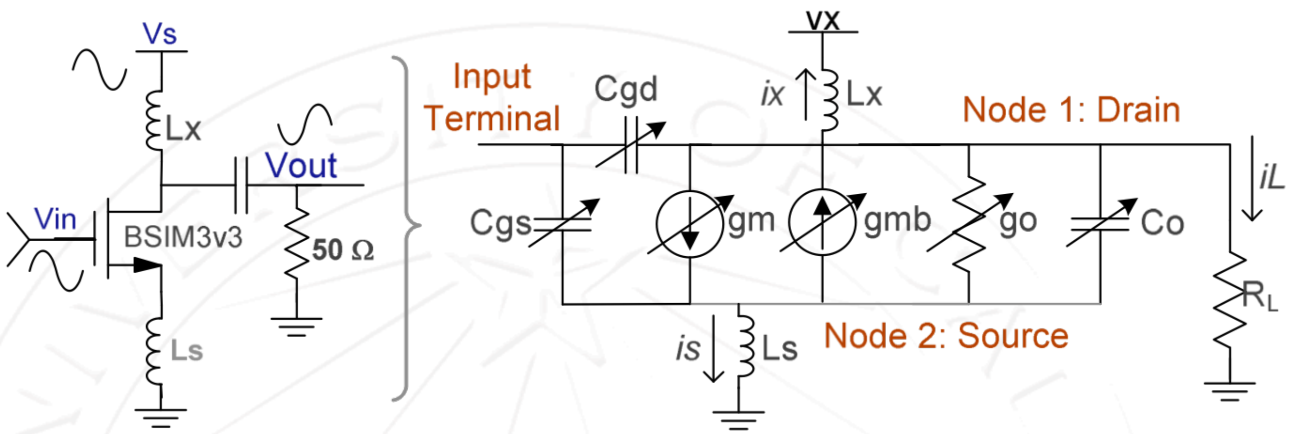
$$v_{out}(t) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} F_{mn}(v_1(t), v_2(t))$$

$$F_{mn}(v_1(t), v_2(t)) = \int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} h_{mn}(\tau_1,\ldots,\tau_{m+n}) v_1(t-\tau_1)\ldots v_1(t-\tau_m) v_2(t-\tau_{m+1})\ldots v_2(t-\tau_{m+n}) d\tau_1 \ldots d\tau_{m+n}$$

$$\begin{aligned} S_{out} &= A_{10}(j\omega_a)\circ S_1 + A_{20}(j\omega_a, j\omega_b)\circ S_1^2 + A_{30}(j\omega_a, j\omega_b, j\omega_c)\circ S_1^3 + \cdots \\ &+ A_{01}(j\omega_a)\circ S_2 + A_{02}(j\omega_a, j\omega_b)\circ S_2^2 + A_{03}(j\omega_a, j\omega_b, j\omega_c)\circ S_2^3 + \cdots \\ &+ A_{11}(j\omega_a, j\omega_b)\circ S_1 S_2 + A_{21}(j\omega_a, j\omega_b, j\omega_c)\circ S_1^2 S_2 + A_{12}(j\omega_a, j\omega_b, j\omega_c)\circ S_1 S_2^2 + \cdots \end{aligned}$$

$$PSRR = dB\left|\frac{2A_{10}(j\omega_0)}{A_{11}(j\omega_0, j\omega_S)}\right|$$

Example



$$
\begin{aligned}
i_d = \quad & g_{m1} v_{gs} + g_{m2} v_{gs}^2 + g_{m3} v_{gs}^3 + \cdots \\
& - g_{mb1} v_{sb} - g_{mb2} v_{sb}^2 - g_{mb3} v_{sb}^3 + \cdots \\
& + g_{mo11} v_{ds} \cdot v_{gs} + g_{mo12} v_{ds} \cdot v_{gs}^2 + g_{mo21} v_{ds}^2 \cdot v_{gs} + \cdots \\
& + C_1 \frac{d}{dt}(v_{db}) + \frac{C_2}{2} \frac{d}{dt}(v_{db}^2) + \frac{C_3}{3} \frac{d}{dt}(v_{db}^3) + \cdots
\end{aligned}
$$

- Several important terms:
  - g_m,g_o non-linearity is usual transconductance and output resistance terms
  - g_{mo} is the interaction between the input/output
  - C_j is the output voltage non-linear capacitance

First Order Terms

- First-order transfer function:

$$A_{10}^{1}(j\omega_a) = -y_S(j\omega_a)\frac{gm_1}{K_0(j\omega_a)}$$, where (RF Node Transfer)

$$K_0(j\omega_a) = (gm_1 + gmb_1 + y_1)\cdot(y_X + y_L) + y_S\cdot(y_X + y_1 + y_L).$$

$$A_{01}^{1}(j\omega_a) = \frac{gm_1(y_X + y_L)}{K_0},$$ (Supply Node Transfer) $y_L = (R_L)^{-1}$

Node

$$A_{10}^{2}(j\omega_a) = \frac{y_X(gm_1 + y_1 + gmb_1 + y_S)}{K_0}$$, and

$$y_1(j\omega_a) = go_1 + j\omega_a C_1$$

$$y_x(j\omega_a) = (j\omega_a L_C)^{-1}$$

$$A_{01}^{2}(j\omega_a) = \frac{y_X y_L}{K_0}.$$

$$y_S(j\omega_a) = (j\omega_a L_S)^{-1}$$

Mixing Product

- The most important term for now is the supply-noise mixing term:

$$v_{out}(\omega_o \pm \omega_S) = A_{11}^1(j\omega_0, j\omega_S) \circ \left[Vi(\omega_0), Vs(\omega_S)\right],$$

$$A_{11}^1(j\omega_a, j\omega_b) = y_S \frac{gmo_{11}K_1 + 2y_2K_2 + 2gm_2K_3 - 2gmb_2K_4}{K_0},$$

$$K_1(j\omega_a, j\omega_b) = A_{01}^2(j\omega_b)\left[1 + A_{10}^1(j\omega_a) - 2A_{10}^2(j\omega_a)\right] - A_{01}^1(j\omega_b)\left[1 - A_{10}^2(j\omega_a)\right],$$

$$K_2(j\omega_a, j\omega_b) = A_{01}^2(j\omega_b)\left[A_{10}^1(j\omega_a) - A_{10}^2(j\omega_a)\right] + A_{01}^1(j\omega_b)\left[A_{10}^2(j\omega_a) - A_{10}^1(j\omega_a)\right],$$

$$K_3(j\omega_a, j\omega_b) = A_{01}^2(j\omega_b)\left[1 - A_{10}^1(j\omega_a)\right], \text{ and}$$

$$K_4(j\omega_a, j\omega_b) = A_{10}^2(j\omega_a)A_{01}^2(j\omega_b).$$

PSSR Reduction

$$PSRR = dB\left|\frac{gm_1}{gmo_{11}K_1 + 2y_2K_2 + 2gm_2K_3 - 2gmb_2K_4}\right|$$

- Increase g_{m1}
- Reduce second order conductive non-linearity at drain (g_{o2})
- Reduce the non-linear junction capacitance at drain
- Reduce cross-coupling term by shielding the device drain from supply noise (cascode)

Output Conductance Non-Linearity

- For short-channel devices, due to DIBL, the output has a strong influence on the drain current. A complete description of the drain current is therefore a function of $f(v_{ds}, v_{gs})$.
- This is especially true if the device is run close to triode region (large swing or equivalently high output impedance):

$$i_{ds}(v_{gs}, v_{ds}) = g_{m1}v_{gs} + g_{ds1}v_{ds} + g_{m2}v_{gs}^2 + g_{ds2}v_{ds}^2 + x_{11}v_{gs}v_{ds} + g_{m3}v_{gs}^3 + g_{ds3}v_{ds}^3 + x_{12}v_{gs}v_{ds}^2 + x_{21}v_{gs}^2v_{ds} + \ldots$$

$$g_{mk} = \frac{1}{k!}\frac{\partial^k I_{DS}}{\partial V_{GS}^k}; \quad g_{dsk} = \frac{1}{k!}\frac{\partial^k I_{DS}}{\partial V_{DS}^k}; \quad x_{pq} = \frac{1}{p!q!}\frac{\partial^{p+q} I_{DS}}{\partial V_{GS}^p \partial V_{DS}^q}.$$

Total Distortion

- Including the output conductance non-linearity modifies the distortion as follows

$$v_{ds} = c_1 v_{gs} + c_2 v_{gs}^2 + c_3 v_{gs}^3 + \ldots$$

$$\begin{aligned}
c_1 &= -\, g_{m1} \cdot (R_{CS} // (1/g_{ds1})) \\
c_2 &= -\left(g_{m2} + g_{ds2} c_1^2 + x_{11} c_1\right) \cdot (R_{CS} // (1/g_{ds1})) \\
c_3 &= -\left(g_{m3} + g_{ds3} c_1^3 + 2 g_{ds2} c_1 c_2 + x_{11} c_2 + x_{12} c_1^2 + x_{21} c_1\right) \\
&\quad \cdot (R_{CS} // (1/g_{ds1})).
\end{aligned}$$

Source: S. C. Blaakmeer, *E. A. M. Klumperink, D. M. W. Leenaerts,, B. Nauta,* “Wideband Balun-LNA With Simultaneous Output Balancing, Noise-Canceling and Distortion-Canceling,” *JSSC*, vol. 43, June 2008.

Example IIP Simulation

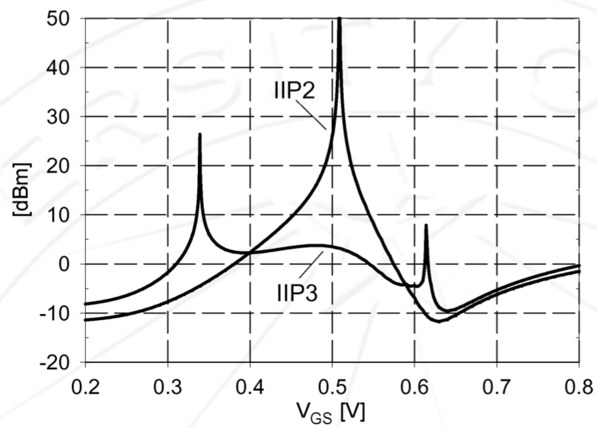


Fig. 6. Simulated IIP2 and IIP3 of a resistively loaded CS-stage.

$$IIP2_{dBm} = 20 \cdot \log_{10}\left(\left|\frac{c_1}{c_2}\right|\right) + 10\ dB$$

$$IIP3_{dBm} = 20 \cdot \log_{10}\left(\sqrt{\left|\frac{4}{3}\frac{c_1}{c_3}\right|}\right) + 10\ dB$$

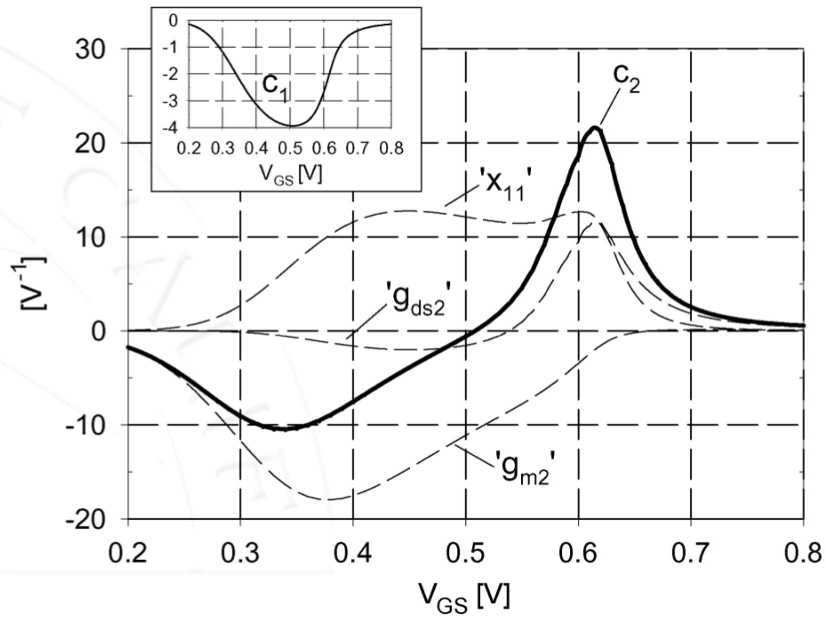
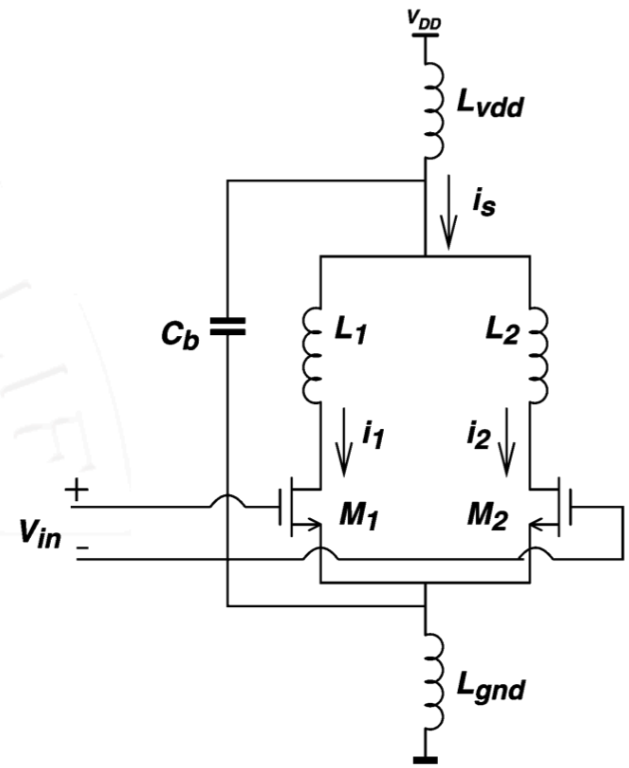


Fig. 5. Simulated second-order nonlinearity coefficient (c_2) and individual contributions due to the transistor coefficients $(g_{m2}, g_{ds2}$ and $x_{11})$. Inset: linear gain coefficient (c_1) of the CS-stage.

- Contributions to c_2 are shown above.
- For low bias, g_{ds2} contributes very little but x_{11} and g_{m2} are significant. They also have opposite sign.

PA Power Supply Modulation

- When we apply a 1-tone to a class AB PA, the current drawn from the supply is constant.
- For when we apply 2-tones, there is a low-frequency component to the input:



$$V_{\rm in} = A\sin(\omega_1 t) + A\sin(\omega_2 t)$$
$$= 2A\cos\left(\frac{\omega_1 - \omega_2}{2}\right)t\sin\left(\frac{\omega_1 + \omega_2}{2}\right)t$$
$$= 2A\cos(\omega_m t)\sin(\omega_c t)$$

- This causes a low frequency current to be drawn from the supply as well, even for a differential circuit.

P. Haldi, D. Chowdhury, P. Reynaert, G. Liu, A. M Niknejad, "A 5.8 GHz 1 V Linear Power Amplifier Using a Novel On-Chip Transformer Power Combiner in Standard 90 nm CMOS," *IEEE Journal of Solid-State Circuits*, vol. 43, pp.1054-1063, May 2008.

Supply Current

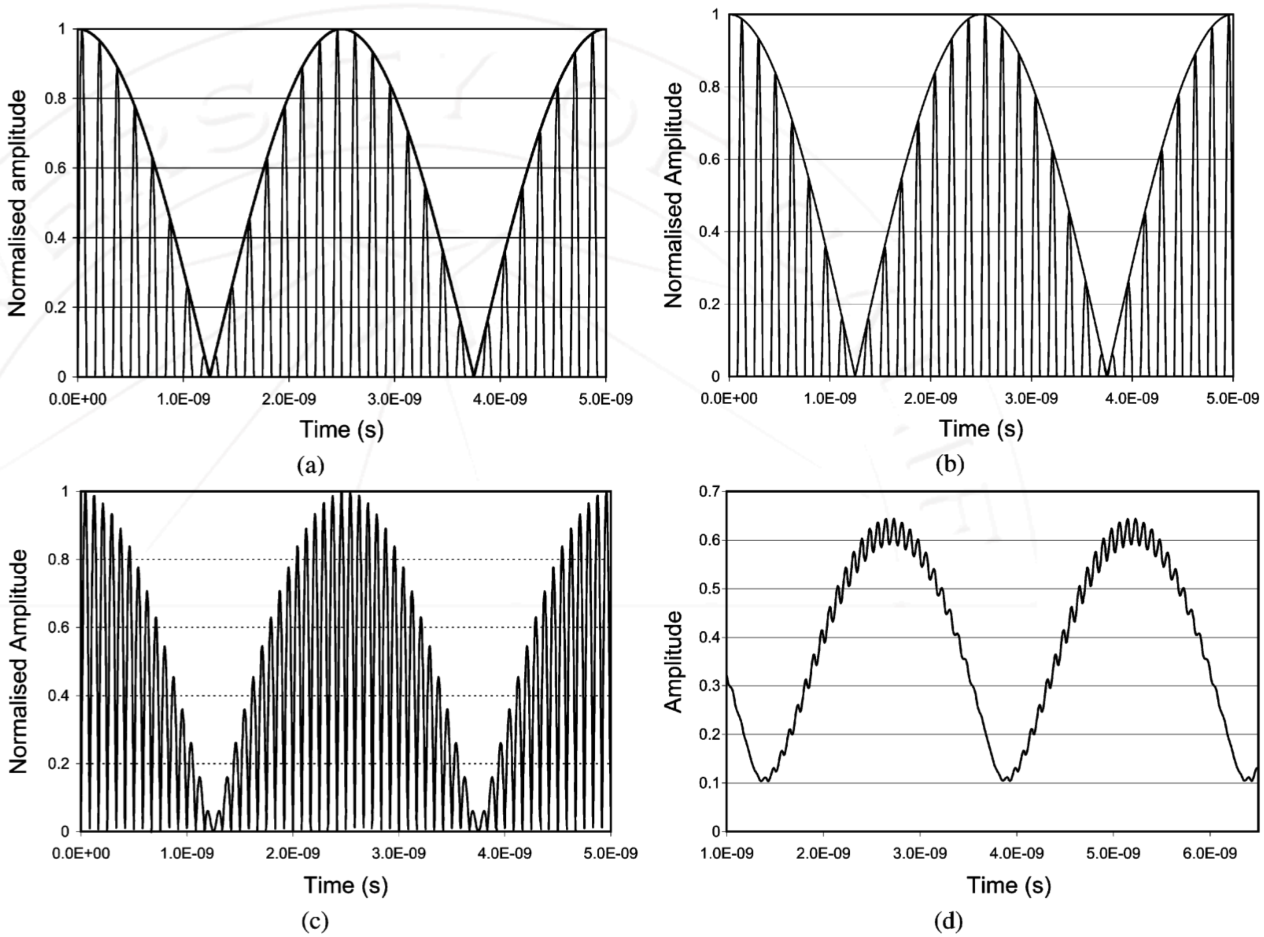


Fig. 9. (a) Drain current waveform of M1, operating in Class B, under two-tone excitation. (b) Drain current waveform of M2. (c) Sum of drain currents of M1 and M2. (d) Supply current after on-chip bypassing.

- The supply current is a full-wave rectified sine.

Fourier Components of i_s

- Substitute the Fourier series for the "rectified" sine and cosine.
- Note that an on-chip bypass can usually absorb the higher frequencies ($2f_c$) but not the low frequency beat (f_s and harmonics)

$$
\begin{aligned}
i_s &= k\left(\frac{2}{\pi} + \frac{4}{\pi}\frac{\cos(2\omega_m t)}{3} - \dots\right) \\
&\quad \times \left(\frac{2}{\pi} - \frac{4}{\pi}\frac{\cos(2\omega_c t)}{3} - \dots\right) \\
&= k\left(\dots - \frac{8}{\pi^2}\frac{\cos(2\omega_c t)}{3}\right. \\
&\quad \left. + \frac{8}{\pi^2}\frac{\cos(2\omega_m t)}{3} - \dots\right) \\
&= k\left(\dots - \frac{8}{\pi^2}\frac{\cos(2\omega_c t)}{3}\right. \\
&\quad \left. + \frac{8}{\pi^2}\frac{\cos(\omega_s t)}{3} - \dots\right)
\end{aligned}
$$

Supply Ripple Voltage

$$V_{dd} = V_{\mathrm{DD}} + A_2 \cdot \cos(\omega_s t) + \text{higher harmonics of } \omega_s.$$

- The finite impedance of the supply means that the supply ripple has the following form. Assuming a multi-port Volterra description for the transistor results in:

$$\begin{aligned} S_o = {} & F_1(\omega_a) \circ S_1 + F_2(\omega_a, \omega_b) \circ S_1^2 \\ & + F_3(\omega_a, \omega_b, \omega_c) \circ S_1^3 + \ldots \\ & + G_1(\omega_a) \circ S_2 + G_2(\omega_a, \omega_b) \circ S_2^2 \\ & + G_3(\omega_a, \omega_b, \omega_c) \circ S_2^3 + \ldots \\ & + H_{11}(\omega_a, \omega_b) \circ (S_1 \cdot S_2) \\ & + H_{12}(\omega_a, \omega_b, \omega_c) \circ S_1 S_2^2 \\ & + H_{21}(\omega_a, \omega_b, \omega_c) \circ S_1^2 S_2 + \ldots \end{aligned}$$

$$S(\omega_1 \pm \omega_s) = H_{11} \circ S_1 \cdot S_2$$

Experimental Results

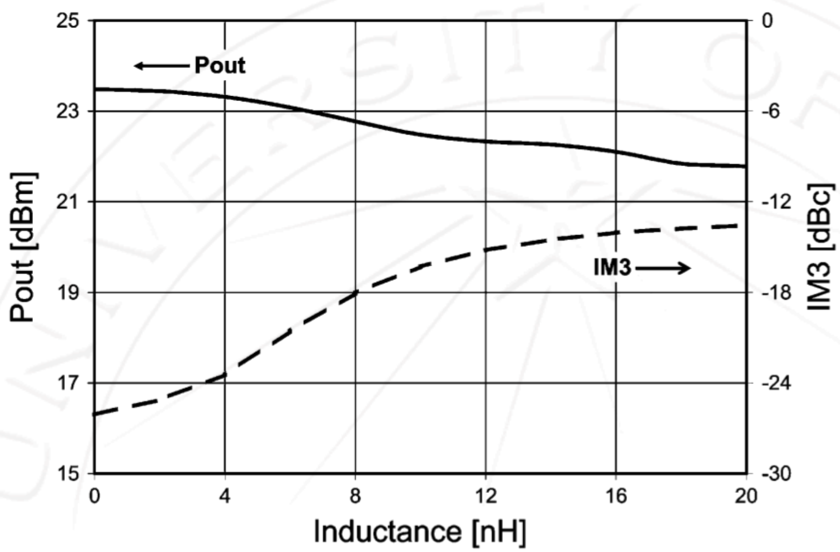


Fig. 14. Degradation in IM3 with increased supply inductance.

- Even though the PA is fully balanced, the supply inductance impacts the linearity.
- Measurements confirm the source of the IM3 at low offsets arising from supply modulation.

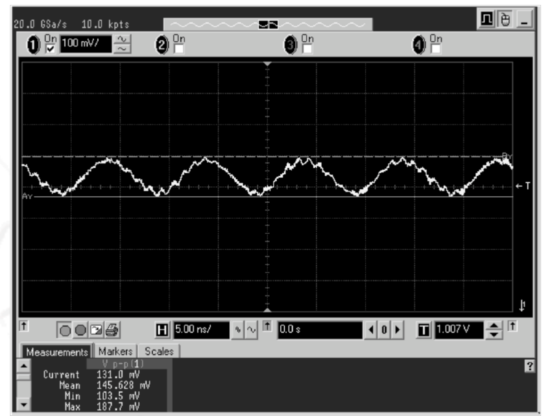


Fig. 15. Measured supply voltage ripple in a two-tone test with 100 MHz tone spacing.

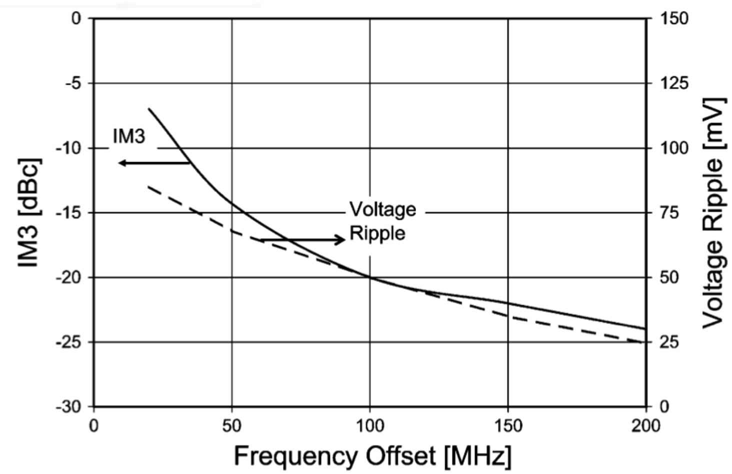
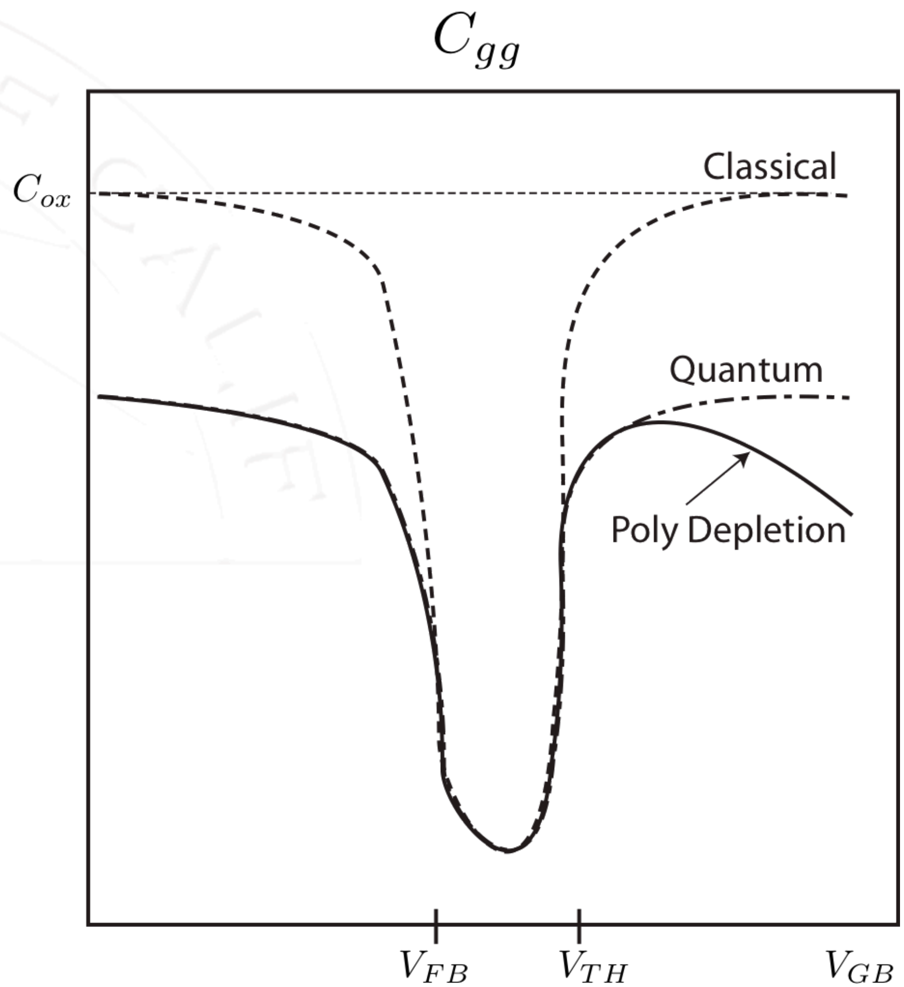


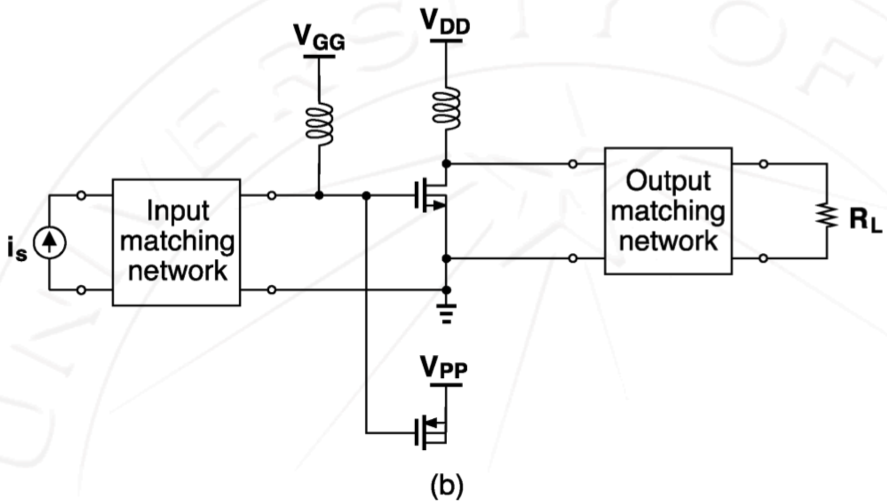
Fig. 16. Measured IM3 degradation and supply voltage ripple in a two-tone test with different tone spacings.

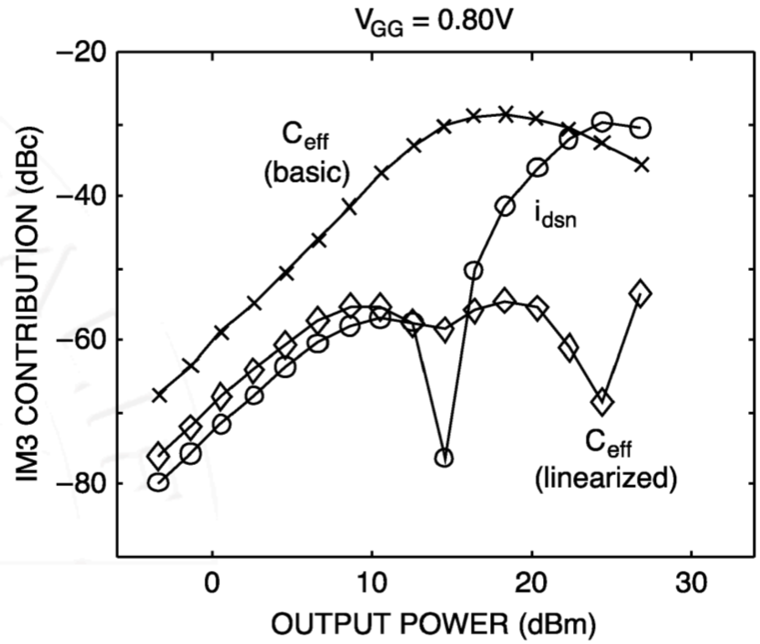
MOS CV Non-Linearity

- C_{gs}, C_u, and C_{db} all contribute to then non-linearity.
- As expected, the contribution is frequency dependent and very much a strong function of the swing (drain, gate).
- Gate cap is particular non-linear.



PMOS Compensation Technique





- Make an overall flat CV curve by adding an appropriately sized PMOS device.

Source: C. Wang, M. Vaidyanathan, L. Larson, "A Capacitance-Compensation Technique for Improved Linearity in CMOS Class-AB Power Amplifiers," *JSSC,* vol. 39, Nov. 2004.

General References

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- **UCB EECS 242 Class Notes,** Ali M Niknejad, Spring 2002, Robert G. Meyer, Spring 1995.