EE242 Homework 1

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February 9, 2009

1 Solution:

In a Weaver receiver shown below, the RF signal is

$$RF = m_r cos(\omega_{LO_1} + \omega_{IF_1})t + m_i cos(\omega_{LO_1} - \omega_{IF_1})t$$

We can calculate the signal at A,B,C,D assuming the gain mismatch is α, β and the phase mismatch is θ, ϕ respectively.

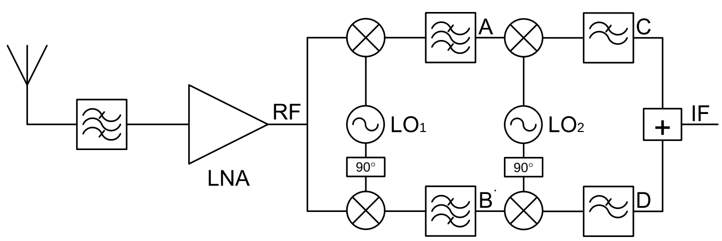

Figure 1: Weaver Architecture

$$
\begin{aligned}
A_{LPF} &= (1+\alpha)cos(\omega_{LO_1}t+\frac{\theta}{2})\times RF \\
&= \frac{m_r}{2}(1+\alpha)cos(\omega_{IF_1}t-\frac{\theta}{2})+\frac{m_i}{2}(1+\alpha)cos(\omega_{IF_1}t+\frac{\theta}{2}) \\
B_{LPF} &= (1-\alpha)sin(\omega_{LO_1}t-\frac{\theta}{2})\times RF \\
&= -\frac{m_r}{2}(1-\alpha)sin(\omega_{IF_1}t+\frac{\theta}{2})+\frac{m_i}{2}(1-\alpha)sin(\omega_{IF_1}t-\frac{\theta}{2}) \\
C_{LPF} &= (1+\beta)cos(\omega_{LO_2}t+\frac{\phi}{2})\times A_{LPF} \\
&= (1+\alpha)(1+\beta)[\frac{m_r}{4}cos(\omega_{IF}t+\frac{\theta+\phi}{2})+\frac{m_i}{4}cos(\omega_{IF}t+\frac{\phi-\theta}{2})] \\
D_{LPF} &= (1-\beta)cos(\omega_{LO_2}t-\frac{\phi}{2})\times B_{LPF} \\
&= (1-\alpha)(1-\beta)[-\frac{m_r}{4}cos(\omega_{IF}t-\frac{\theta+\phi}{2})+\frac{m_i}{4}cos(\omega_{IF}t-\frac{\phi-\theta}{2})]
\end{aligned} \tag{1}
$$

Then we can combine the C and D signal at the output node as

$$
\begin{aligned}
&IF = C - D = \\
&\frac{m_r}{4}[(1+\alpha)(1+\beta)cos(\omega_{IF}t+\frac{\phi+\theta}{2})+(1-\alpha)(1-\beta)cos(\omega_{IF}t-\frac{\phi+\theta}{2})] \\
&+\frac{m_i}{4}[(1+\alpha)(1+\beta)cos(\omega_{IF}t+\frac{\phi-\theta}{2})-(1-\alpha)(1-\beta)cos(\omega_{IF}t-\frac{\phi-\theta}{2})] \\
&=\frac{m_r}{2}[(1+\alpha\beta)cos(\frac{\phi+\theta}{2})cos(\omega_{IF}t)-(\alpha+\beta)sin(\frac{\phi+\theta}{2})sin(\omega_{IF}t)] \\
&+\frac{m_i}{2}[(\alpha+\beta)cos(\frac{\phi-\theta}{2})cos(\omega_{IF}t)-(1+\alpha\beta)sin(\frac{\phi+\theta}{2})sin(\omega_{IF}t)]
\end{aligned} \tag{2}
$$

The image reject ratio is as following:

$$
\begin{aligned}
IR &= 10log[\frac{(1+\alpha\beta)^2cos^2(\frac{\phi+\theta}{2})+(\alpha+\beta)^2sin^2(\frac{\phi+\theta}{2})}{(\alpha+\beta)^2cos^2(\frac{\phi-\theta}{2})+(1+\alpha\beta)^2sin^2(\frac{\phi-\theta}{2})}] \\
&= 10log\frac{(1+\alpha\beta)^2[1+cos(\phi+\theta)]+(\alpha+\beta)^2[1-cos(\phi+\theta)]}{(\alpha+\beta)^2[1+cos(\phi-\theta])+(1+\alpha\beta)^2[1-cos(\phi-\theta)]}
\end{aligned} \tag{3}
$$

2 Solution:

For a 1MHz communication system, the wavelength is 300m which is 100X bigger than antenna size. So we can take the antenna as a small antenna and the radiation power factor can be calculated as[1]

$$PF_{rad} = \frac{1}{6\pi}\frac{kV}{l^3} \tag{4}$$

where k is the shape factor and $l = \frac{\lambda}{2\pi}$. And the efficiency of the antenna can be written as

$$\eta = \frac{PF_{rad}}{PF_{rad} + PF_{loss}} \tag{5}$$

where PF_{loss} is the loss power factor. Now we assume the shape factor $k = 2$ and then the radiation power factor is

$$PF_{rad} = \frac{1}{6\pi}\frac{2 \times 3}{(c/2\pi f)^3} = 2.92 \times 10^{-6} \tag{6}$$

Assuming the loss power factor is 0.0001, we can get the efficiency of the antenna is

$$\eta = 2.92 \times 10^{-2} = -15.3dB \tag{7}$$

So totally we will have -30dB loss from antenna.
Then we calculate the link budget according to following assumptions

Type of parameters	value of parameters
Data rate	10kHz
Symbol rate	10kHz
Noise figure	10dB
Io/No	-3dB
Required Eb/No	30dB
Antenna gain (TX)	1.76 dB
Antenna gain (RX)	1.76dB
TX antenna hight	260m
RX antenna hight	2m
Carrier frequency	1MHz
Distance	100 miles
Transmission loss	2dB
Reception loss	10dB
Link margin	10dB

Table 1: Assumptions for link budget calculation

According to these assupmtions, we can calculate the required output power should be +26dBm. Plug in the efficiency of the antenna, the total output

power at the output of PA will be +56dBm which is about 400W.
Comparing AM with FM, since FM requires more bandwidth, the noise power in FM system is higher than AM which leads to more power consumption.

3 Solution:

Because it is not matched after the signal is down convertered, we express the gain and NF of the mixer, IF filter and VGA in term of voltage instead of power.
The noise figure of mixer + IF filter + VGA is

$$NF_{3,4,5} = NF_3 + \frac{NF_4 - 1}{G_3} + \frac{NF_5 - 1}{G_3 G_4} = \frac{\overline{v_n^2}}{\overline{v_{n0}^2}} + 1 \tag{8}$$

Given the input impedance of the mixer is 300Ω, the input referred noise power should be

$$P_{n3,4,5} = \frac{(NF_{3,4,5} - 1) \times \overline{v_{n0}^2}}{R_{300}} = \frac{(NF_{3,4,5} - 1) \times P_{n0} \times R_0}{R_{300}} \tag{9}$$

So the noise figure of mixer + IF filter + VGA in term of power should be

$$\frac{(NF_{3,4,5} - 1) \times R_0}{R_{300}} + 1 \tag{10}$$

So the noise figure of the entire front-end should be

$$NF = NF_1 + \frac{NF_2 - 1}{G_1} + \frac{R_0}{G_1 G_2 R_{300}} \times (NF_3 - 1 + \frac{NF_4 - 1}{G_3} + \frac{NF_5 - 1}{G_3 G_4}) \tag{11}$$

Here, G_1, G_2, NF_1, NF_2 are expressed in power and $G_3, G_4, G_5, NF_3, NF_4, NF_5$ are in term of voltage
The voltage gain of the system is

$$G = \frac{R_{300}}{R_0} G_1 \times G_2 \times G_3 \times G_4 \times G_5 \tag{12}$$

The IIP3 can be calculated as

$$\frac{1}{IIP3} = \frac{1}{IIP3_1} + \frac{G_1}{IIP3_2} + \frac{R_{300}}{R_0}(\frac{G_1 G_2}{IIP3_3} + \frac{G_1 G_2 G_3}{IIP3_4} + \frac{G_1 G_2 G_3 G_4}{IIP3_5}) \tag{13}$$

If we specify the Gain,NF and IIP3 for each block as table2, we can get the overall performance as table3, which meets the requirement.

Block	Gain	NF	IIP3
RF Filter	-1dB	1dB	+100dBm
LNA	15dB	1.5dB	+0dBm
mixer	10dB	8dB	+16dBm
IF filter	-5dB	5dB	+100dBm(referred to 50Ω)
VGA	+65dB	15dB	+20dBm(referred to 50Ω)

Table 2: Spec for each block

Voltage Gain	91.78dB
NF	2.74dB
IIP3	-9.71dBm

Table 3: Overall performance