EECS 240
Analog Integrated Circuits

Topic 12: Settling Time

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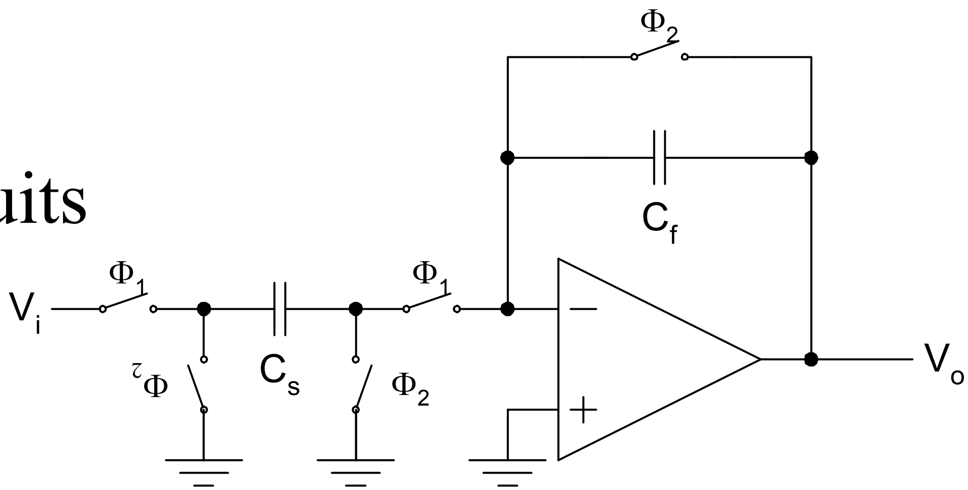
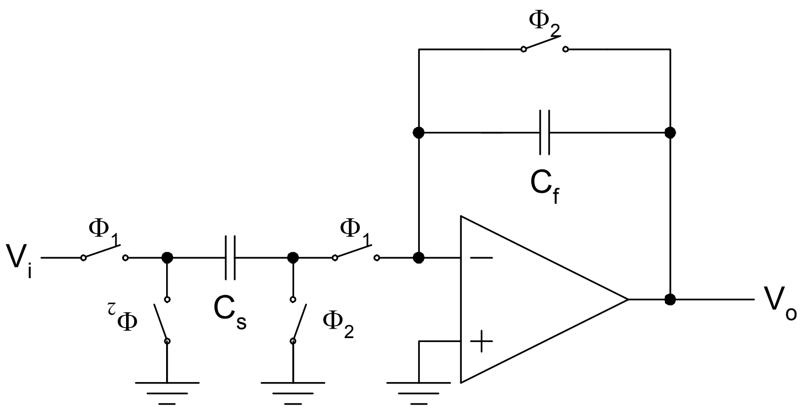
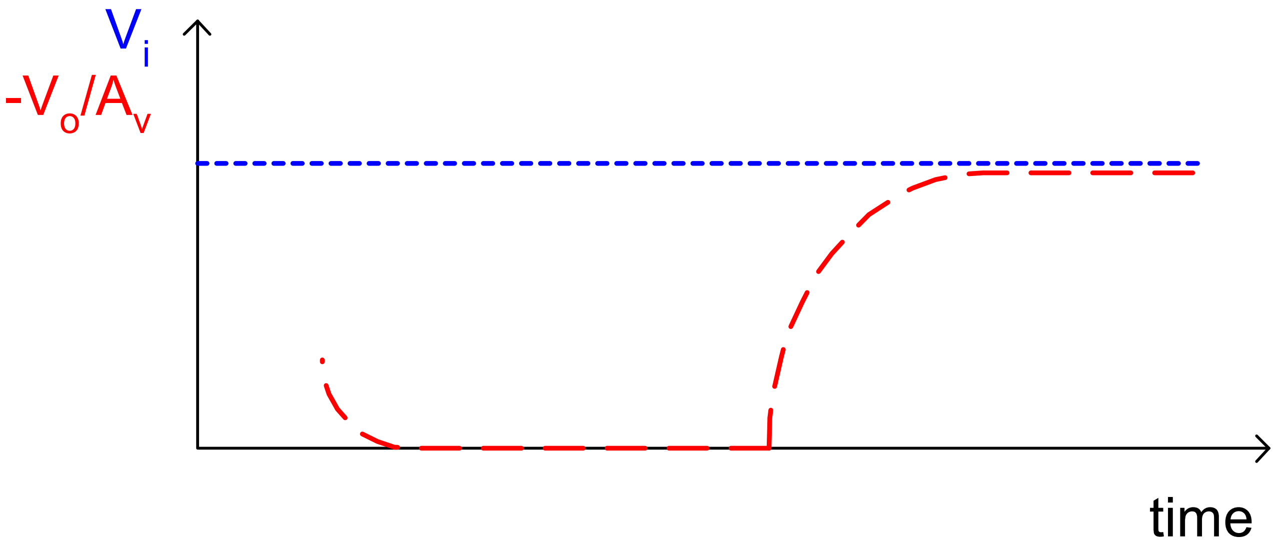
Settling

- Speed/accuracy metrics
- Continuous time circuits
  - Bandwidth
  - Loop-gain, slew rate → distortion
- Switched capacitor circuits
  - Step response
  - Loop-gain → settling accuracy
  - Loop-bandwidth → settling time

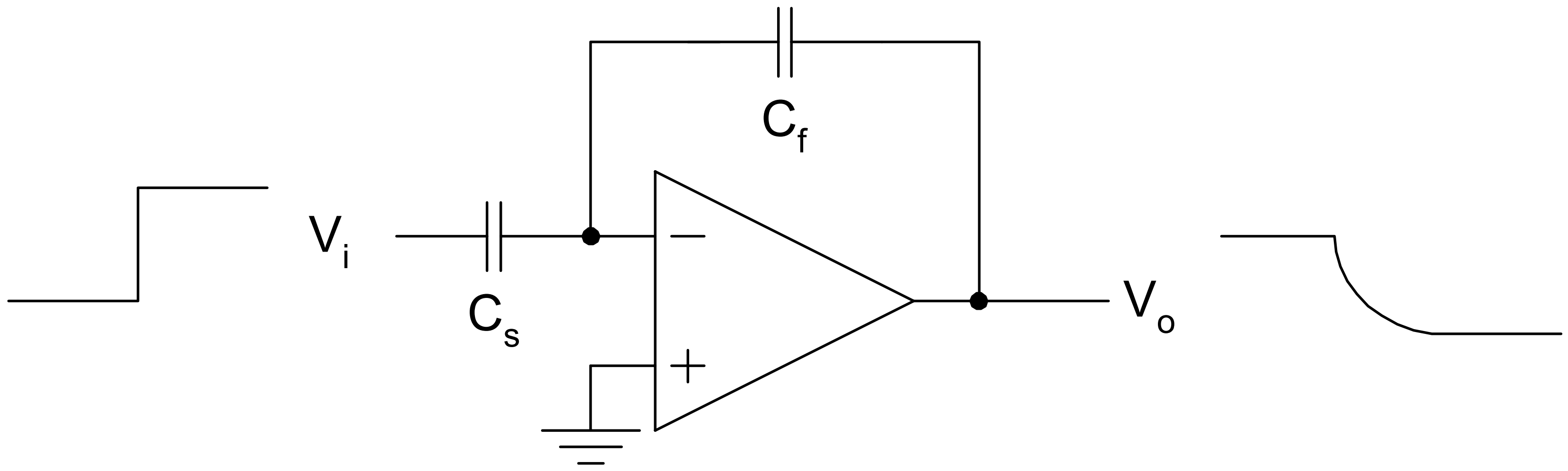


Step Response



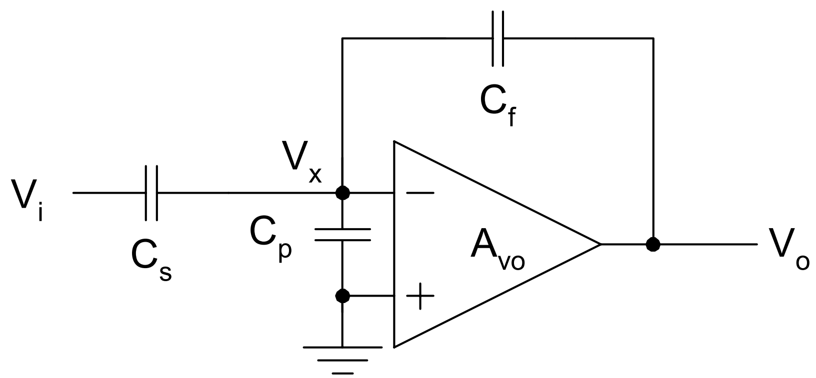


Step Response Analysis



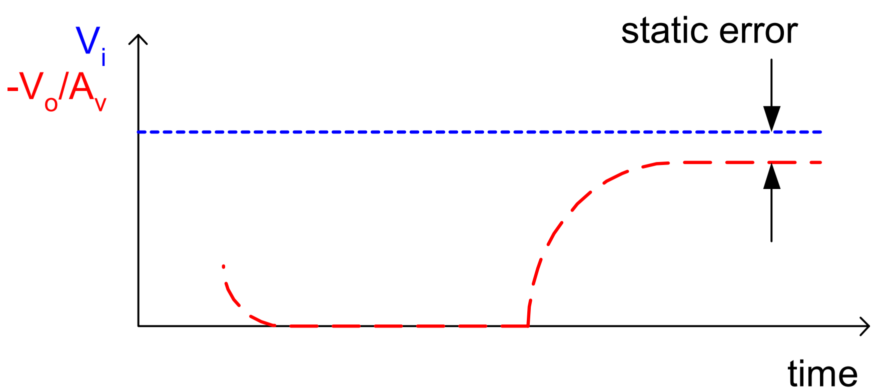
- Static error
- Dynamic error

Static Error



KCL →

$$\frac{V_o}{V_i} = -\frac{c}{1+\frac{1}{\underbrace{FA_{vo}}_{T_o}}}$$



with

$$F = \frac{C_f}{C_f + C_s + C_p}$$

$$\frac{V_o}{V_x} = -A_{vo}$$

$$c = \frac{C_s}{C_f}$$

Static Error (cont.)

Example:

$$\frac{V_o}{V_i} = -\frac{c}{1+\frac{1}{FA_{vo}}}$$

$$\cong -c\left(1 - \underbrace{\frac{1}{FA_{vo}}}_{\text{relative error}}\right)$$

Closed loop gain: c = -4

C_f = 1pF, C_s = 4pF, C_p = 1pF
→ F = 1/6 Note: C_p hurts!

Error specification: <0.1%

→ FA_{vo} > 1000
→ A_{vo} > 6000 over output range

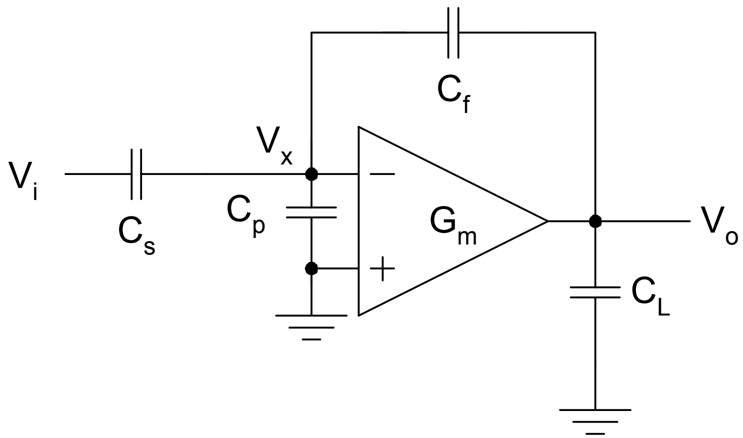
Beware: other (dynamic errors) add!

Dynamic Errors

- Error sources
  - Finite bandwidth
  - Feedforward zero
  - Non-dominant poles
  - Doublets
  - Nonlinear effects: slewing

- Analysis approach
  - One error at a time!
  - In particular: treat static and dynamic errors separately
  - Final result: superposition of all individual errors

Linear Settling
Single Time Constant



- Note: R_o irrelevant at frequencies of interest (and for $T_o >> 1$)
- Solve KCL

$$v_x s C_T - v_o s C_f - v_i s C_s = 0$$

$$v_o s\left(C_f + C_L\right) - v_x s C_f + G_m v_x = 0$$

$$\frac{V_o}{V_i} = -c\,\frac{1 - s\dfrac{C_f}{G_m}}{1 + s\dfrac{C_L + (1-F)C_f}{FG_m}}$$

Linear Settling (cont.)

$$\frac{V_o}{V_i} = -c\frac{1 - s\dfrac{C_f}{G_m}}{1 + s\dfrac{C_L + (1-F)C_f}{FG_m}}$$

$$= -c\frac{1+\dfrac{s}{z}}{1+\dfrac{s}{p}}$$

with

$$z = +\frac{G_m}{C_f}$$

$$p = -\frac{FG_m}{C_L + (1-F)C_f}$$

- Loop bandwidth sets pole
- Feedforward through C_f contributes zero → sets initial response

Step Response

Frequency domain:

input step:

$$V_{i,step} = \frac{V_{step}}{s}$$

output step:

$$V_{o,step} = -c\frac{1+\frac{s}{z}}{1+\frac{s}{p}}V_{i,step}$$

$$= -c\frac{1+\frac{s}{z}}{1+\frac{s}{p}}\frac{V_{step}}{s}$$

Time domain:
(inverse Laplace transform)

$$v_{o,step}(t) = \underbrace{-V_{step}c}_{\text{ideal response}}\left[1 - \underbrace{\left(\underbrace{1-\frac{p}{z}}_{\substack{\text{initial error} \\ \text{(feedforward)}}}\right)e^{pt}}_{\text{exponentially decaying error}}\right]$$

Note: For p=z the error is zero and the circuit has infinite bandwidth. Applications?

Step Response (cont.)

$$v_{o,step}(t) = \underbrace{-V_{step}c}_{\text{ideal response}}\left[1-\left(1-\frac{p}{z}\right)e^{-t/\tau}\right]$$

with :

$$\tau = \frac{C_{Leff}}{FG_m}$$

$$\frac{p}{z} = -F\frac{C_f}{C_{Leff}}$$

$$C_{Leff} = C_L + (1-F)C_f$$

Case 1: -p/z << 1

$$v_{o,step}(t) \cong \underbrace{-V_{step}c}_{\text{ideal response}}\left[1-e^{-t/\tau}\right]$$

Relative settling error:

$$\varepsilon = \frac{v_o(t\to\infty)-v_o(t=t_s)}{v_o(t\to\infty)} = e^{-t/\tau}$$

$$\frac{t_s}{\tau} = -\ln\varepsilon$$

ε	t_s/τ
0.01	4.6
0.001	6.9
10^{-4}	9.2
10^{-5}	11.5
10^{-6}	13.8

Example:

Specification:

$$c = 4$$

$$C_f = C_p = 1\text{pF},\quad C_s = cC_f = 4\text{pF},\quad C_L = 5\text{pF}$$

$$t_s = 10\text{ns} \qquad \varepsilon = 10^{-3}$$

Solution:

$$F = 0.17$$

$$C_{Leff} = 5.83\text{pF}$$

$$\left|\frac{p}{z}\right| = F\frac{C_f}{C_{Leff}} = 0.03 << 1$$

$$\tau = \frac{10\text{ns}}{6.9} = 1.45\text{ns}$$

$$G_m = \frac{C_{Leff}}{F\tau} = \frac{5.83\text{pF}}{0.17\times 1.45\text{ns}} = \underline{\underline{24\text{mS}}}$$

Case 2: -p/z not negligible

$$v_{o,step}(t) \cong \underbrace{-V_{step}c}_{\text{ideal response}}\left[1-\left(1-\frac{p}{z}\right)e^{-t/\tau}\right]$$

Relative settling error:

$$\varepsilon = \frac{v_o(t\to\infty) - v_o(t=t_s)}{v_o(t\to\infty)} = \left(1-\frac{p}{z}\right)e^{-t/\tau}$$

$$\frac{t_s}{\tau} = -\ln\left(\frac{\varepsilon}{1+F\dfrac{C_f}{C_{Leff}}}\right)$$

Example:

Specification:

$$c = 0.25$$

$$C_f = 1\text{pF},\quad C_s = cC_f = 250\text{fF},\quad C_p = 250\text{fF},\quad C_L = 1\text{pF}$$

$$t_s = 10\text{ns} \qquad \varepsilon = 10^{-3}$$

Solution:

$$F = 0.67$$

$$C_{Leff} = 1.3\text{pF}$$

$$\frac{t_s}{\tau} = -\ln\left(\frac{10^{-3}}{1+0.67\times 0.77}\right) = 7.3$$

$$\frac{v_o(t=0^+)}{V_{step}c} = -\frac{p}{z}$$

$$= F\frac{C_f}{C_{Leff}} = 0.52$$

Non-Dominant Pole

- Ignore feed-forward zero for simplicity (homework?)

$$H(s) = \frac{V_o}{V_{in}} = -c\frac{1}{1 + s\frac{C_{Leff}}{FG_m}}$$

- Model for non-dominant pole

$$G_m(s) = \frac{G_{mo}}{1 - \frac{s}{p_2}}$$

$$p_2 = -K\omega_u$$

ω_u is unity gain bandwidth of $T(s)$

- Step response

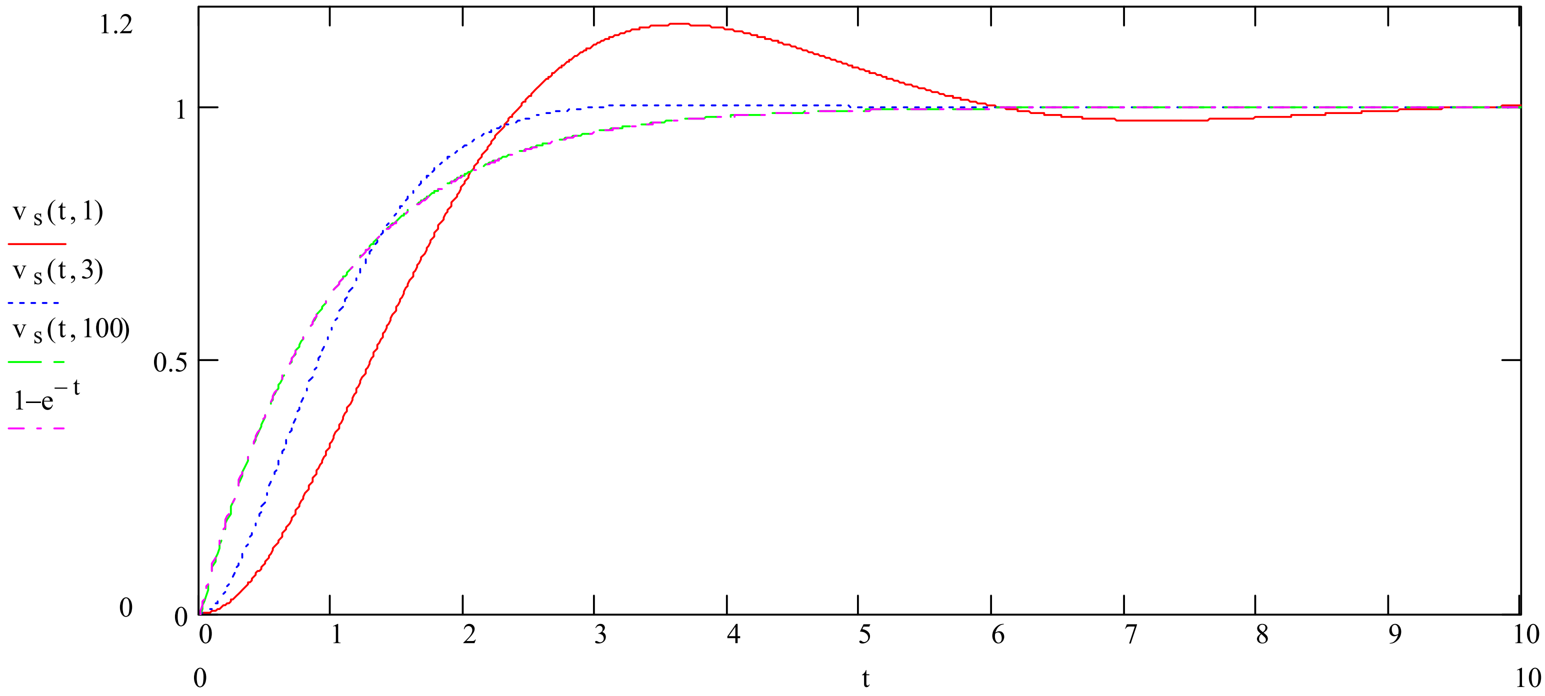
$$V_{o,step}(s) = H(s)\frac{V_{in,step}}{s}$$

$$v_{o,step}(t) = L^{-1}\{V_{o,step}(s)\}$$

$$= \ldots$$

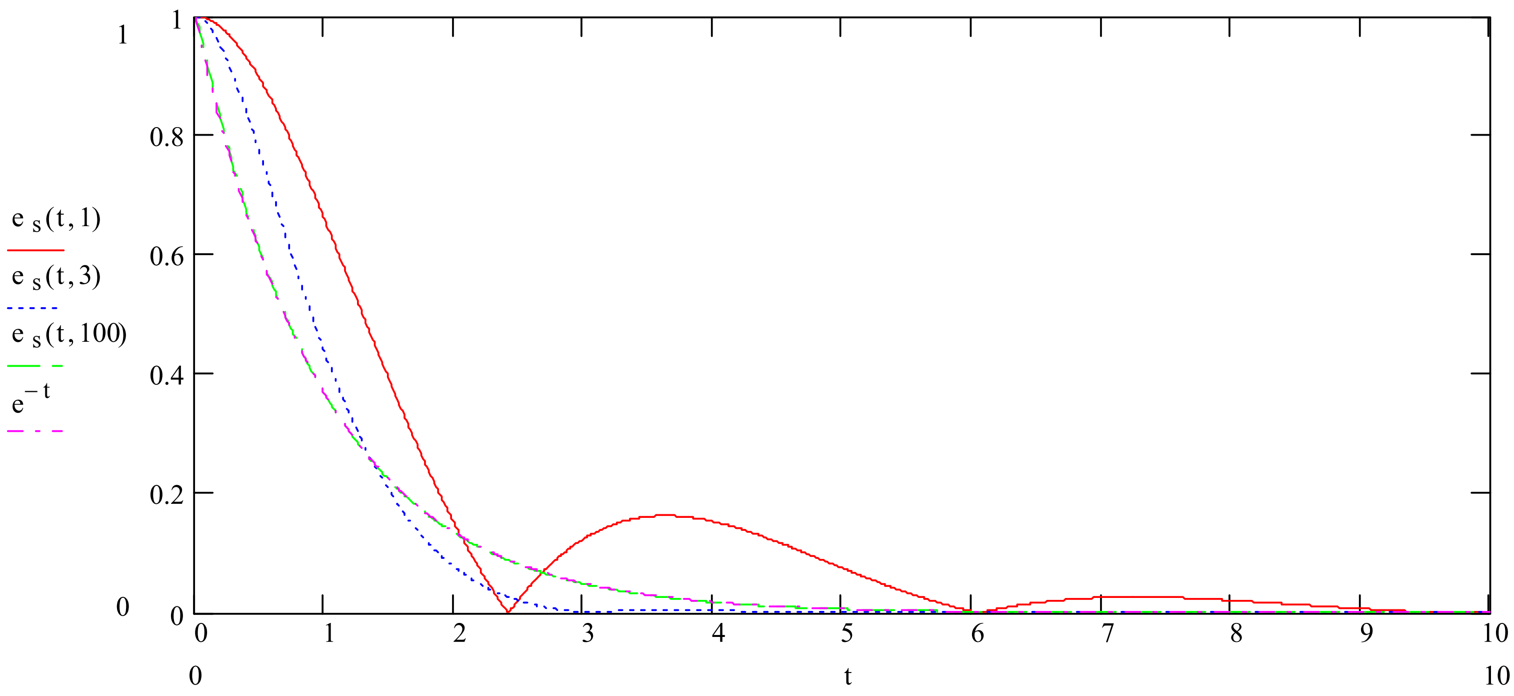
Non-Dominant Pole (cont.)

$$\text{Step response}: \qquad v_s(t,K) = -\frac{v_{o,step}(t)\big|_{\tau=1}}{cV_{in,step}}$$



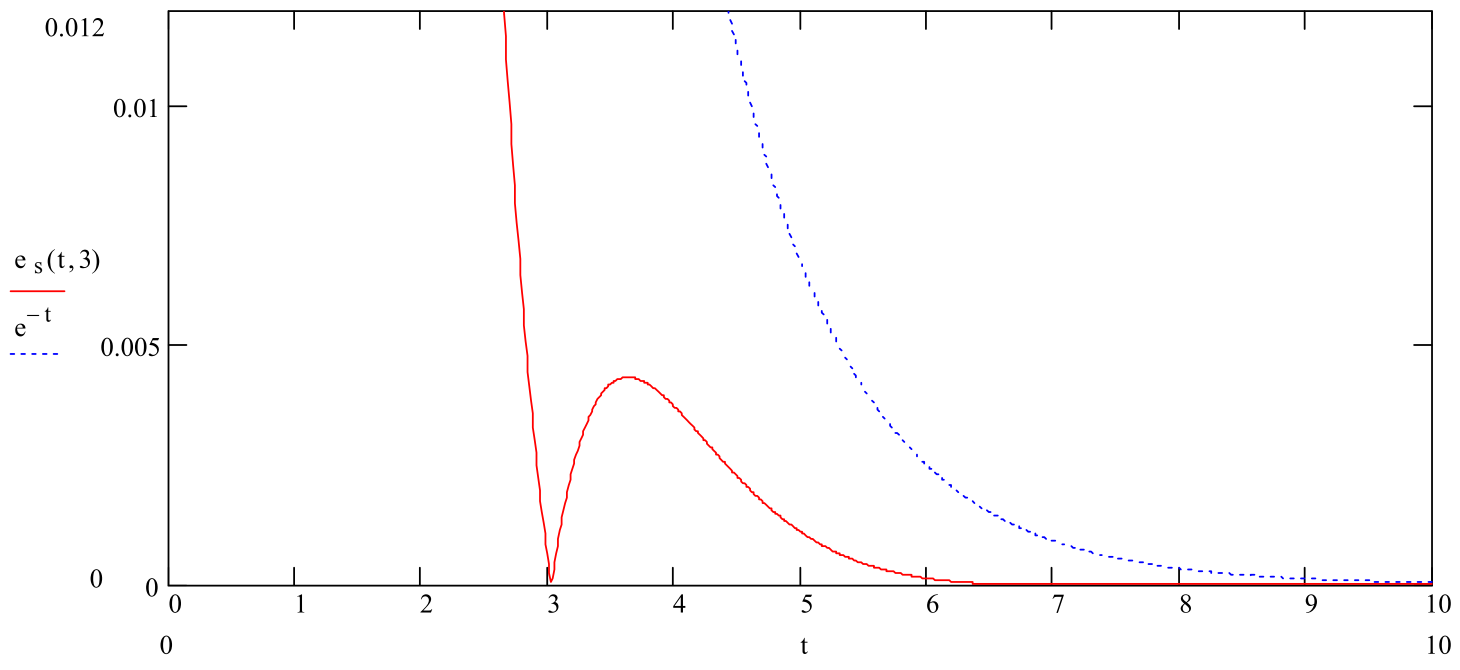
Non-Dominant Pole (cont.)

Relative error : $\varepsilon = \left|1 - v_s(t, K)\right|$



Non-Dominant Pole (cont.)

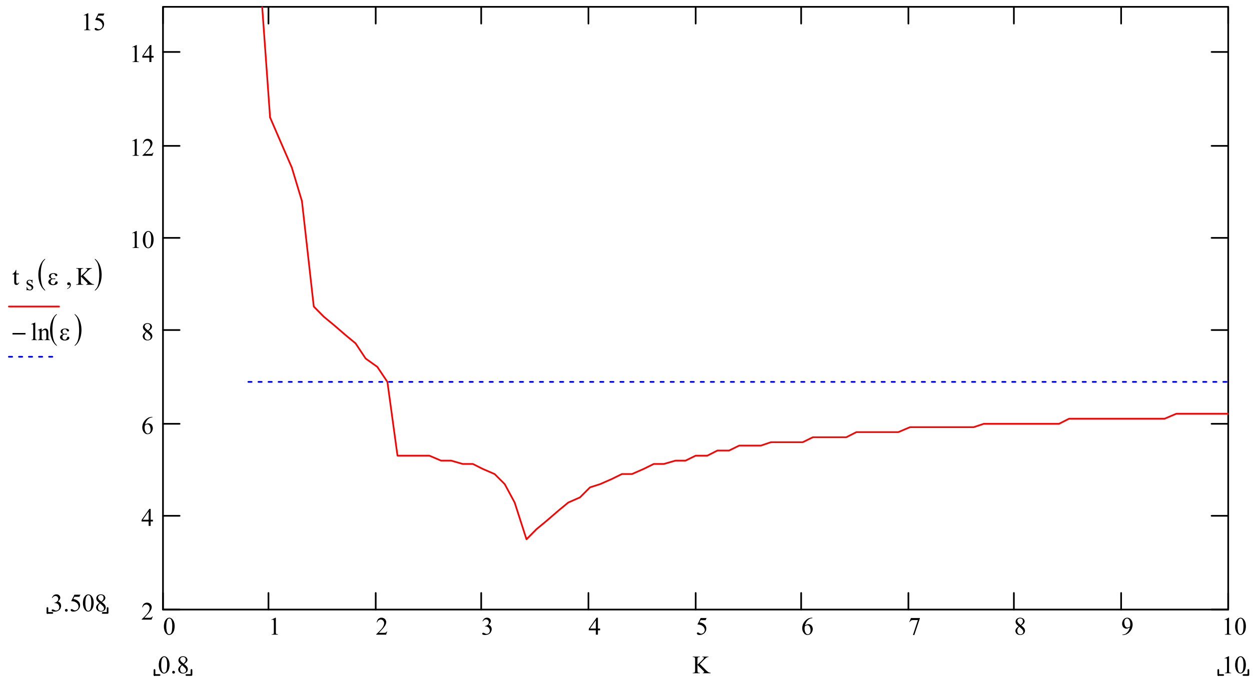
Relative error : detail for K = 3



Non-dominant pole <u>can</u> speed up settling!

Non-Dominant Pole (cont.)

Settling time: $t_s(\mathrm{K})$ for $\varepsilon = 10^{-3}$, $\tau = 1$



- Optimum for K=3.3
- Avoid K < 2

Doublets

- Doublet = "closely spaced pole/zero pair"
- Origins:
  - Feedforward path (e.g. Miller capacitor)
  - Frequency dependent degeneration (cascode, gain boosting)
  - etc.
- Concerns:
  - Ringing (if high-Q)
  - Slow settling if doublet frequency < ω_{-3dB} of T(s)
  - Hard to see from SPICE output (esp. ac analysis)

Doublet Analysis

- Amplifier model:

$$G_m(s) = G_{mo}\frac{1+\dfrac{s}{\omega_z}}{1+\dfrac{s}{\omega_p}}$$

with

$$\omega_p = \beta\omega_{-3dB}, \qquad \omega_{-3dB} \text{ is bandwidth of } T(s)$$

$$\omega_z = \frac{\omega_p}{\alpha}$$

$$\alpha = 1+\varepsilon \qquad \text{with} \qquad |\varepsilon| << 1$$

- Closed-loop gain (ignore feedforward zero):

$$\frac{V_o}{V_{in}} = -c\frac{1}{1+s\dfrac{C_{Leff}}{FG_m(s)}} \cong -\frac{c}{1+\dfrac{s}{\omega_{-3dB}}}\left(\frac{1+\dfrac{s}{\omega_z}}{1+\dfrac{s}{\omega_{pp}}}\right)$$

with

$$\omega_{-3dB} = \frac{FG_{mo}}{C_{Leff}}$$

$$\omega_{pp} \cong \omega_p$$

Doublet Analysis (cont.)

- Step response

$$v_{o,step}(t) = -cV_{step}\left(1 + Ae^{-t\omega_{-3dB}} + Be^{-t\omega_{pp}}\right)$$

with

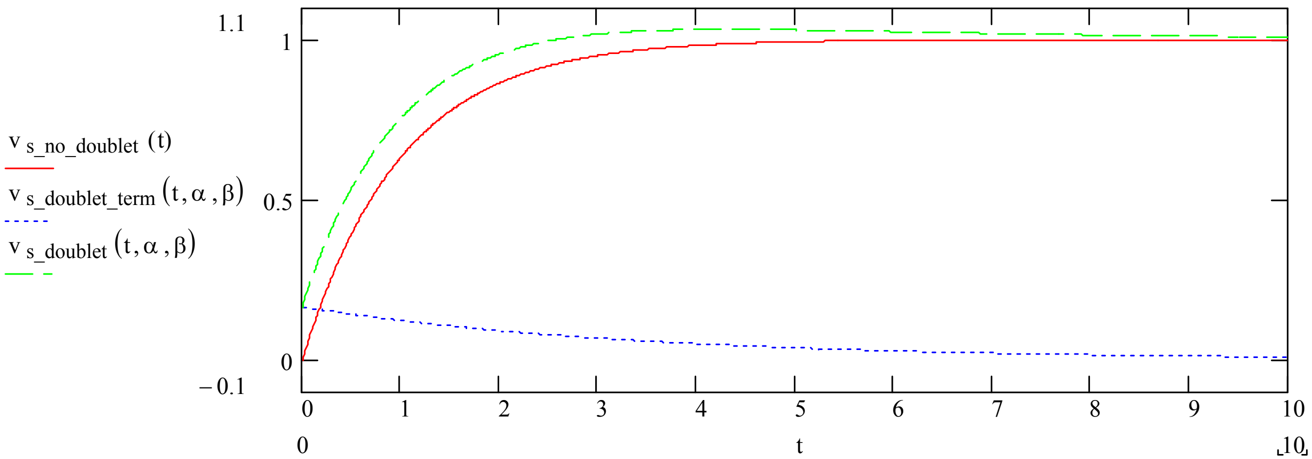
$$A \cong -1$$

$$B \cong \varepsilon \frac{\beta}{1-\beta^2}$$

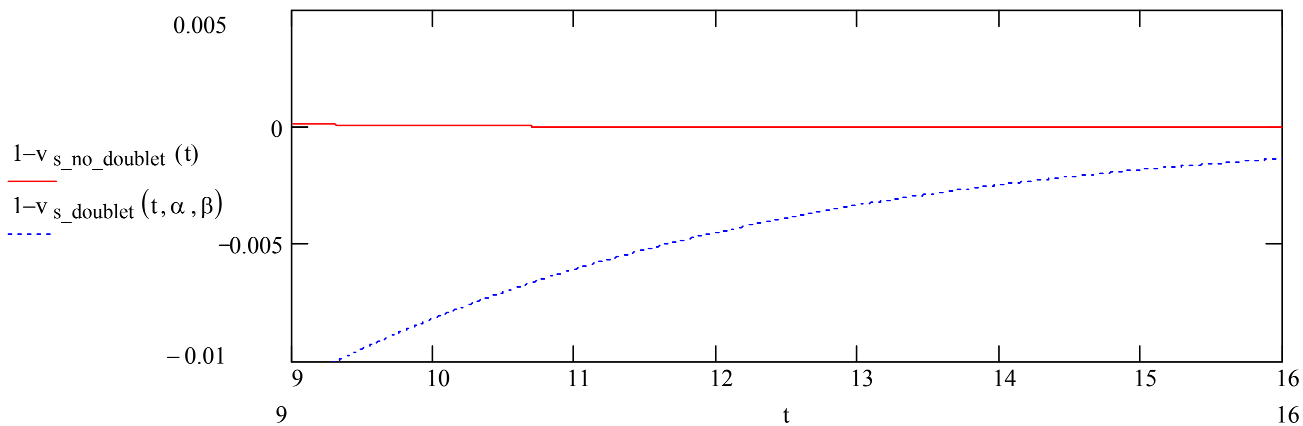
- Two exponentially decaying errors with time constants

$$\tau_1 = 1/\omega_{-3dB} \quad \text{and} \quad \tau_2 = 1/\omega_{pp}$$

Doublet Example



$$\alpha = 1.5$$
$$\beta = 0.3$$



Doublet Conclusions

- Case A: $\tau_2 \leq \tau_1$ i.e. $\beta \geq 1$
  – Doublet settles much faster than amplifier
  – Has no impact on overall settling time

- Case B: $\tau_2 > \tau_1$
  – Doublet settles more slowly than amplifier
  – Determines overall settling time (unless ε within settling accuracy requirements … only met in "low accuracy" situations, cf. scope probes)

- → Avoid "slow" doublets!

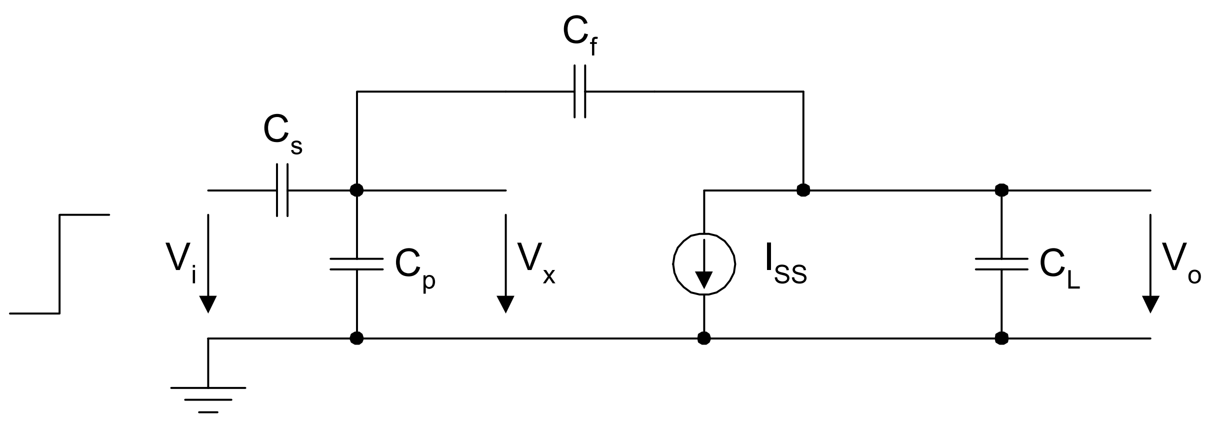
Slewing

- Transconductor
  - Differential pair
  - Class (A)B input stage

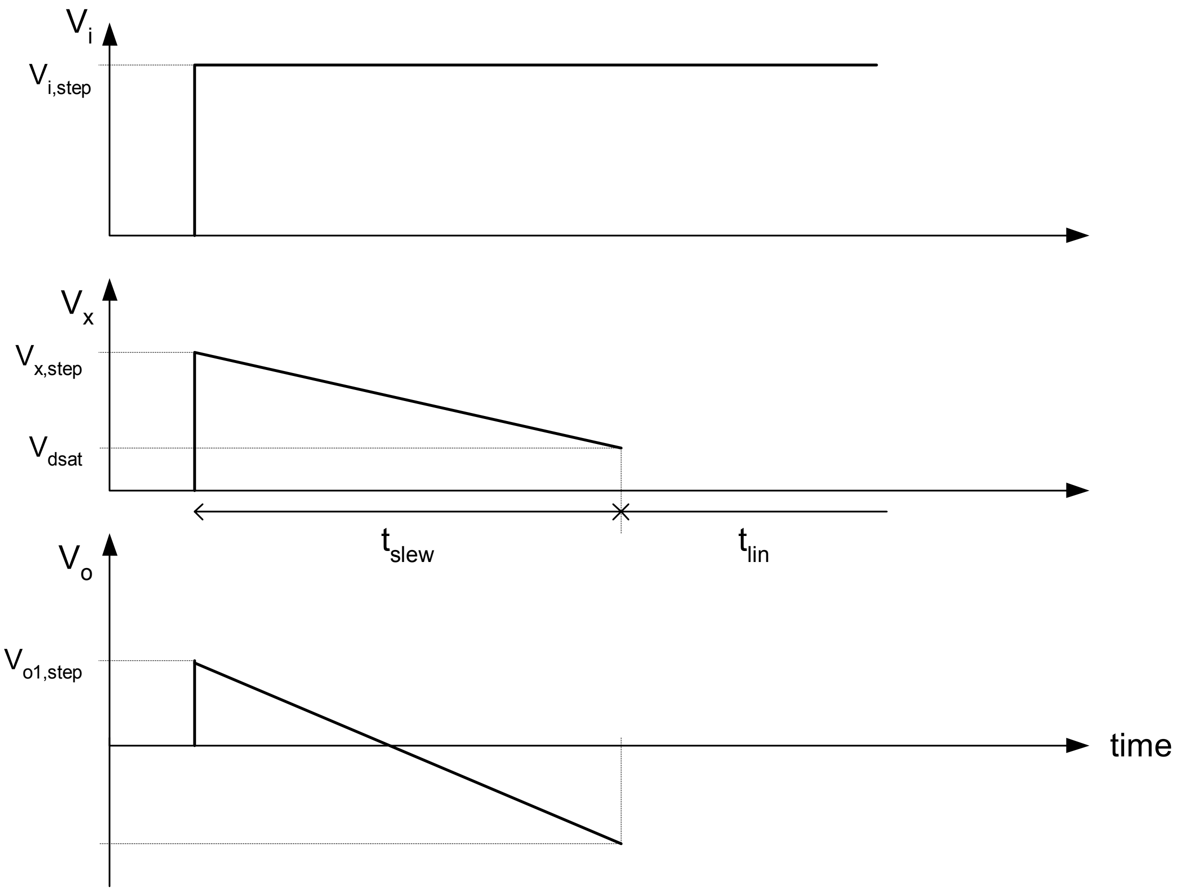
- Model for (nonlinear) slewing amplifier
  - Piecewise linear approximation:
    - Slewing with constant current, followed by
    - Linear settling exponential
    - $t_s = t_{slew} + t_{lin}$

Slewing Analysis

- Circuit model during slewing:



Slewing Analysis (cont.)



Slewing Analysis (cont.)

- Slewing period:

$$V_{x,step} = V_{i,step} \frac{C_s}{C_s + C_2} \qquad \text{with} \qquad C_2 = C_p + \frac{C_f C_L}{C_f + C_L}$$

$$\Delta V_x = V_{x,step} - V_d^{sat}$$

$$\Delta V_o = \frac{\Delta V_x}{F}$$

$$t_{slew} = \frac{\Delta V_o}{SR} \qquad \text{with} \qquad SR = \frac{I_{SS}}{C_{Leff}}$$

- Linear settling: reduced step size!
  - Complete step at output: $cV_{i,step}$
  - Step during linear settling: V_d^{sat} / F
  - Scaled accuracy: $\varepsilon_{lin} = \varepsilon \frac{cV_{i,step} F}{V_d^{sat}} > \varepsilon$