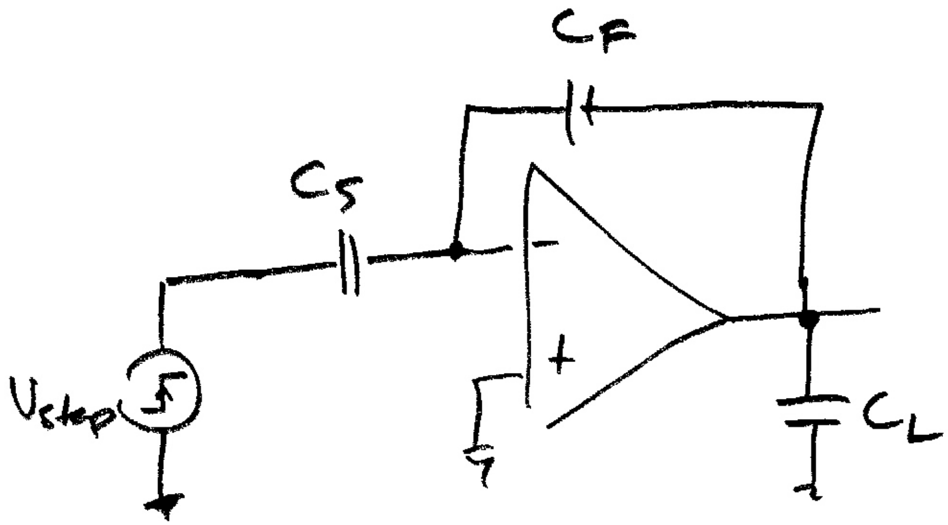
LECT 16 SETTLING TIME (CONT) PART II

POLE & ZERO

$$v_{o,step}(t) = -V_{step} \cdot C \left\{ 1 - \underbrace{\left(1 + \frac{p}{z}\right) e^{-t/\tau}}_{\text{DYNAMIC ERROR TERM}} \right\}$$



$$p = -\frac{F\,G_m}{C_{Leff}}$$

$$z = +\frac{G_m}{C_F}$$

$$\frac{p}{z} = 1 + \varepsilon \qquad v_{o,step}(t) = -V_{step} \cdot C \left\{ 1 - \varepsilon\, e^{-t/\tau} \right\}$$

TWO CASES : I. $\left|\frac{P}{Z}\right| \ll 1$

$\rightarrow$ EFFECTIVELY ZERO NOT THERE

II. $\left|\frac{P}{Z}\right| \sim 1$ $\rightarrow$ CANNOT NEGLECT ZERO

AMP SETTLING

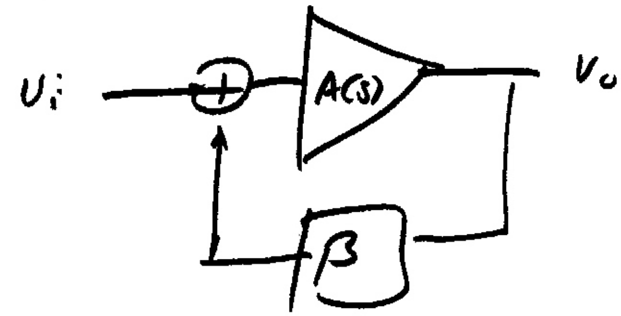
OPEN LOOP : $A(s) = \dfrac{A_0}{(1 + s/\omega_{p1})}$

$$|A(j\omega_{ua})| = 1 \simeq \left.\frac{A_0}{s/\omega_{p1}}\right|_{s=\omega_{ua}}$$

$$\boxed{\omega_{ua} \cong A_0 \cdot \omega_{p1}}$$

MIDBAND GAIN : $\boxed{A(s) = \dfrac{A_0\,\omega_{p1}}{s} = \dfrac{\omega_{ua}}{s}}$

CLOSED LOOP



$$A_{CL} = \frac{A(s)}{1 + \beta A(s)}$$

MIDBAND

$$A_{CL} = \frac{\omega_{ua}/s}{1 + \beta \frac{\omega_{ua}}{s}} = \frac{1}{\beta} \frac{1}{1 + s/\beta\omega_{ua}}$$

$$\omega_{-3dB} = \beta\, \omega_{ua}$$

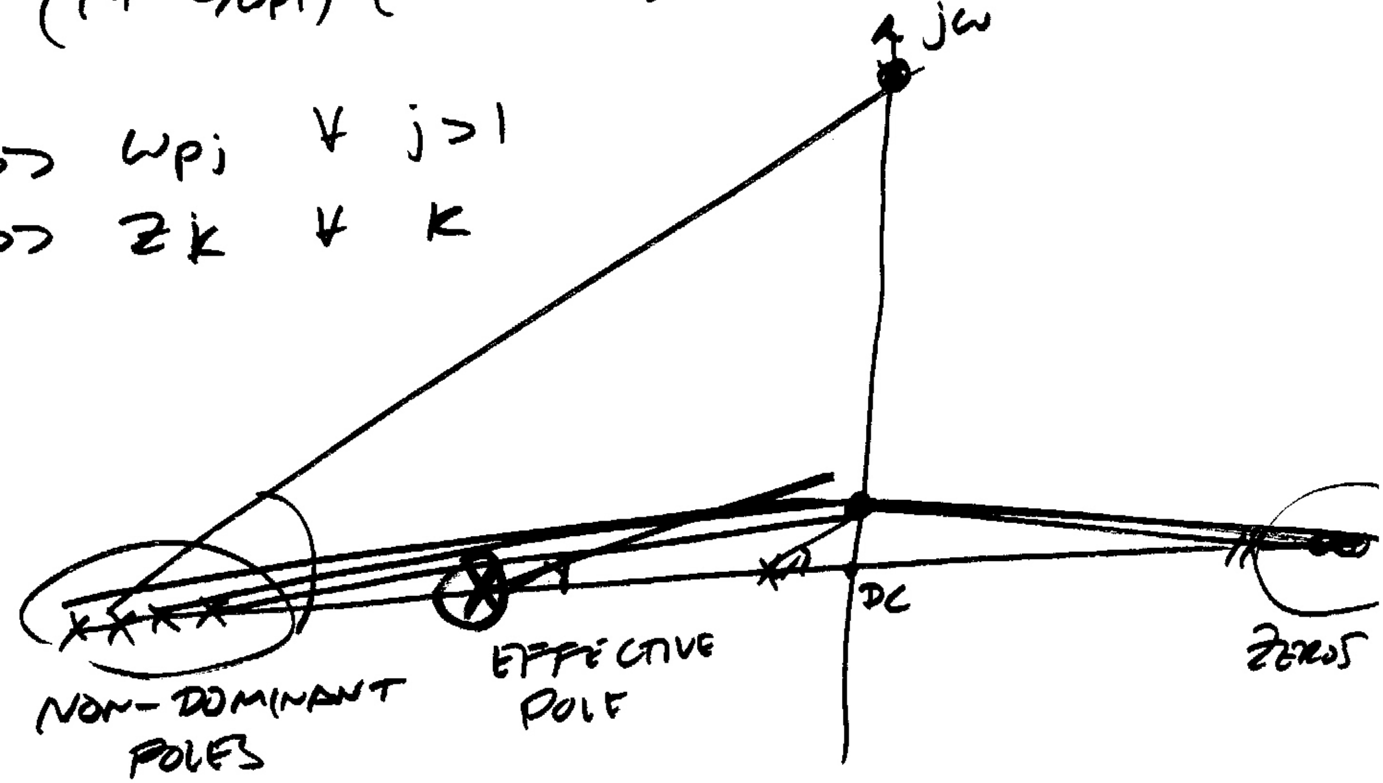
BASIC GAIN/BW TRADEOFF

MULTI-POLE/ZERO SYSTEM

$$A(s) = \frac{A_0 \left(1 + s/z_1\right) \cdots \left(1 + s/z_m\right)}{\left(1 + s/\omega_{p1}\right)\left(1 + s/\omega_{p2}\right) \cdots \left(1 + s/\omega_{pn}\right)}$$

$$\omega_{p1} \gg \omega_{pj} \quad \forall \; j > 1$$

$$\gg z_k \quad \forall \; k$$



$$\angle A(s) = \sum \angle z_i \big|_{j\omega} - \sum \angle p_i \big|_{j\omega}$$

POLES

$$\frac{1}{\left(1+s/\omega_{p2}\right)\left(1+s/\omega_{B3}\right)\cdots} \overset{s/\omega_{pk} \ll 1}{\simeq} \frac{1}{1+s/\omega_{p2}+s/\omega_{p3}+s/\omega_{p4}+\cdots}$$

$$= \frac{1}{1+s\underbrace{\left(\frac{1}{\omega_{p2}}+\frac{1}{\omega_{p3}}+\cdots\right)}_{\frac{1}{\omega_{eff,1}}}}$$

ZEROS

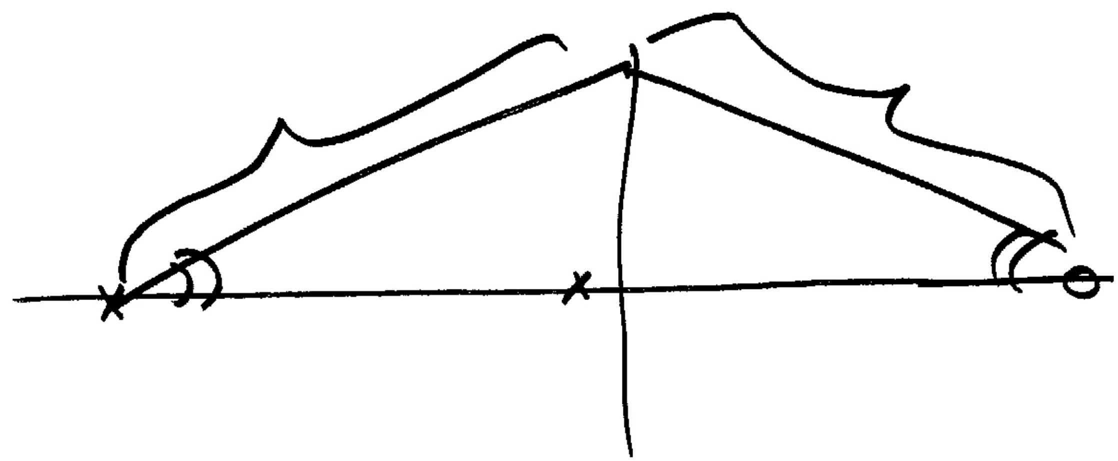
$$\left(1+s/z_1\right)\left(1+s/z_2\right)\cdots\left(1+s/z_m\right) =$$

$$= 1+s\left(\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}+\cdots\right) = 1+s\sum_{k=1}^{m}\frac{1}{z_k}$$

$$= 1+\delta = \frac{1}{1-\delta} = \frac{1}{1-s\left(\sum \frac{1}{z_k}\right)}$$ — $\omega_{eff\,2}$

$$A(s) \simeq \frac{1}{\left(1+s/\omega_{p1}\right)\left(1+s/\omega_{eff1}\right)\left(1-s/\omega_{eff2}\right)}$$

$$\simeq \frac{1}{\left(1+s/\omega_{p1}\right)\left(1+s\left(\frac{1}{\omega_{eff1}}-\frac{1}{\omega_{eff2}}\right)\right)}$$



$\underline{\omega \gg \omega_{p1}}$ FREQ HIGHER THAN DOMINANT POLE

$$A(s) = \frac{A_o}{\left(\frac{s}{\omega_{p1}}\right)\left(1 + s/\omega_{eq}\right)} = \frac{A_o \omega_{p1}}{s\left(1 + s/\omega_{eq}\right)}$$

$$= \frac{\omega_{ua}}{s\left(1 + s/\omega_{eq}\right)}$$

$$|T(s)| = |\beta A(s)| = \left| \frac{\beta \omega_{ua}}{s\left(1 + s/\omega_{eq}\right)} \right|_{s = j\omega_t} = 1$$

$$\frac{\beta \omega_{ua}}{\omega_t \sqrt{1 + \left(\frac{\omega_t}{\omega_{eq}}\right)^2}} = 1$$

$$\boxed{\beta \omega_{ua} = \omega_t \sqrt{1 + \left(\frac{\omega_t}{\omega_{eq}}\right)^2}}$$

CHECK $\omega_{eq} \gg \omega_t$

$$\underline{\beta\,\omega_{ua} = \omega_t}$$ ✓

$$\angle T(\omega_t) = -90^\circ - \tan^{-1}\left(\frac{\omega_t}{\omega_{eq}}\right)$$

$$PM = \angle T - (-180^\circ) = 90^\circ - \tan^{-1}\frac{\omega_t}{\omega_{eq}}$$

$$\tan^{-1}\frac{\omega_t}{\omega_{eq}} = 90^\circ - PM$$

$$\frac{\omega_t}{\omega_{eq}} = \tan(90^\circ - PM)$$

$$\omega_t = \omega_{eq}\tan(90^\circ - PM)$$

$PM = 45^\circ \Rightarrow \tan 45 = 1$

$$\boxed{\omega_t = \omega_{eq}}$$

POLE/ZERO DOUBLET

$$\frac{z_x}{p_x} = 1 + \varepsilon \qquad \varepsilon \ll 1$$

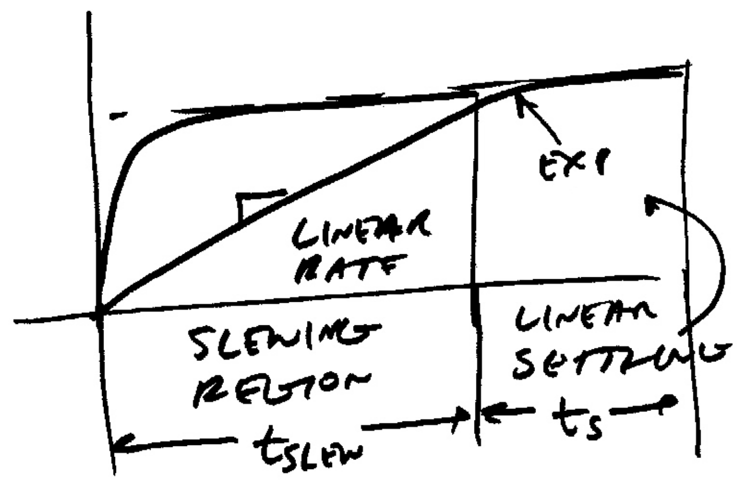
$$H(s) = \frac{(1 + s/z_x)}{(1 + s/p_x)(1 + s/p_1)}$$

$$\mathcal{L}^{-1}\left\{ \frac{1}{s} H(s) \right\} = \frac{A}{s} + \frac{B}{1 + s/p_x} + \frac{C}{1 + s/p_1}$$

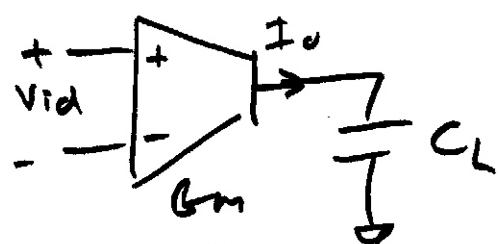
$$A = 1 \qquad B = -\frac{p_1}{p_x}\frac{\varepsilon}{p_x} \qquad C = \frac{1 - \frac{p_1}{z_x}}{1 - \frac{p_1}{p_x}}$$

$$y(t) = u(t)\Big(1 - \underbrace{e^{-tp_1}}_{\text{DOMINANT POLE}}\Big) - \underbrace{\left(\frac{p_1}{p_x}\right)\varepsilon}_{\text{ERROR TERM}}\ \underbrace{e^{-tp_x}}_{\text{SETTLING TIME OF ERROR TERM}}$$

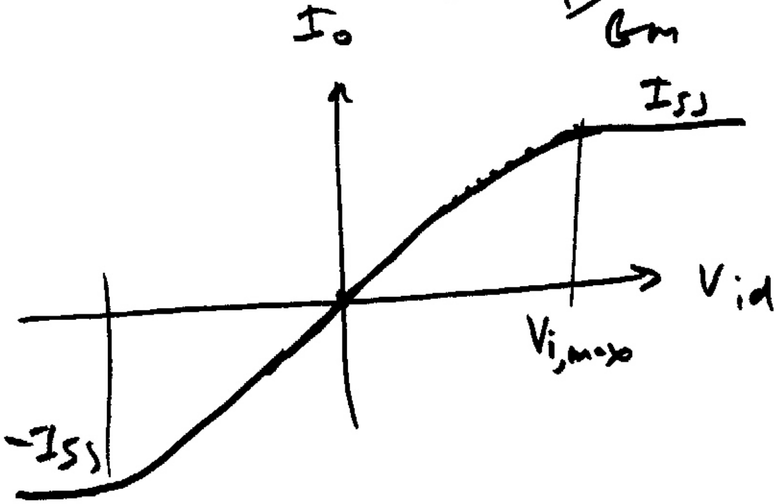
SLEWING



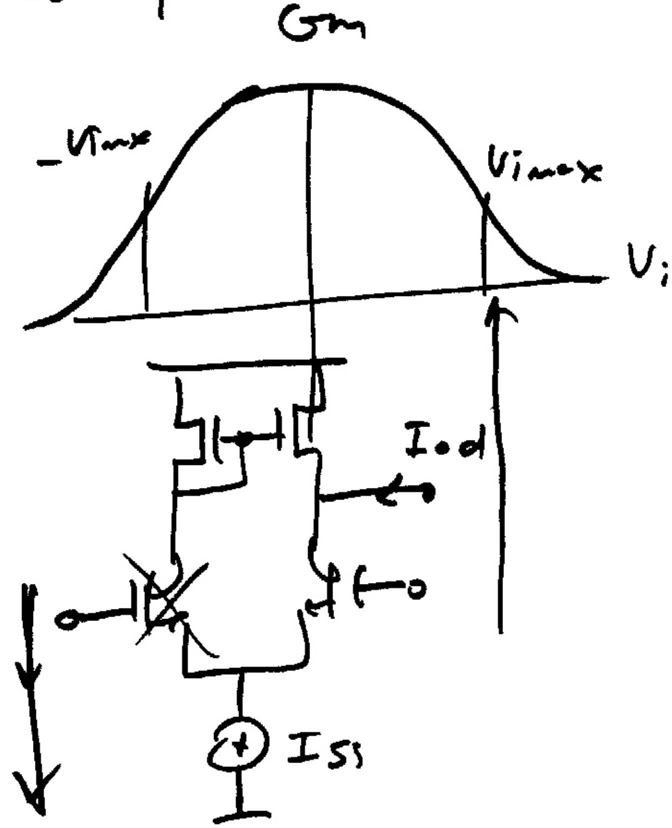
EXP
LINEAR RATE
SLEWING REGION
LINEAR SETTLING
t_{SLEW}
t_S



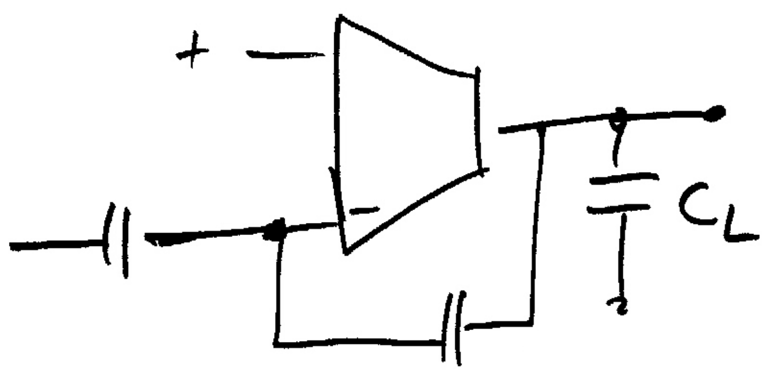
+
V_{id}
-
+
-
I_o
C_L
G_m



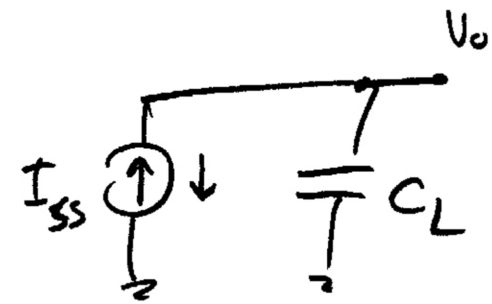
I_o
I_{SS}
V_{id}
$V_{i,max}$
$-I_{SS}$



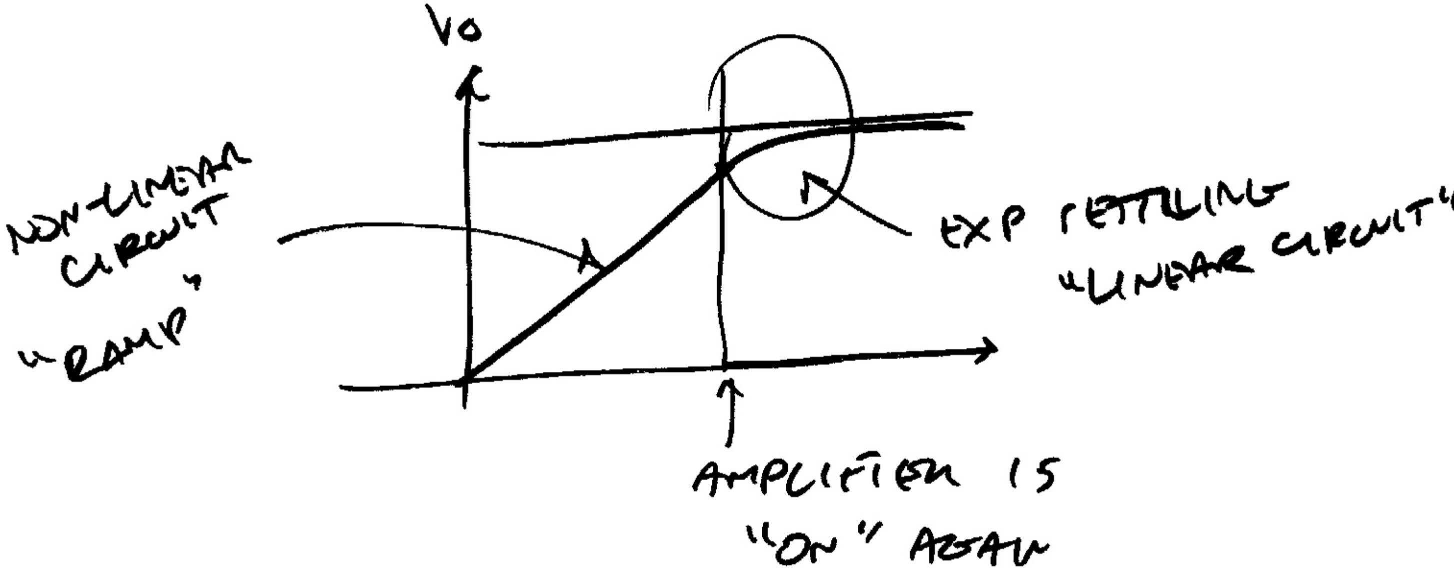
G_m
$-V_{imax}$
V_{imax}
V_i
I_{od}
I_{SS}

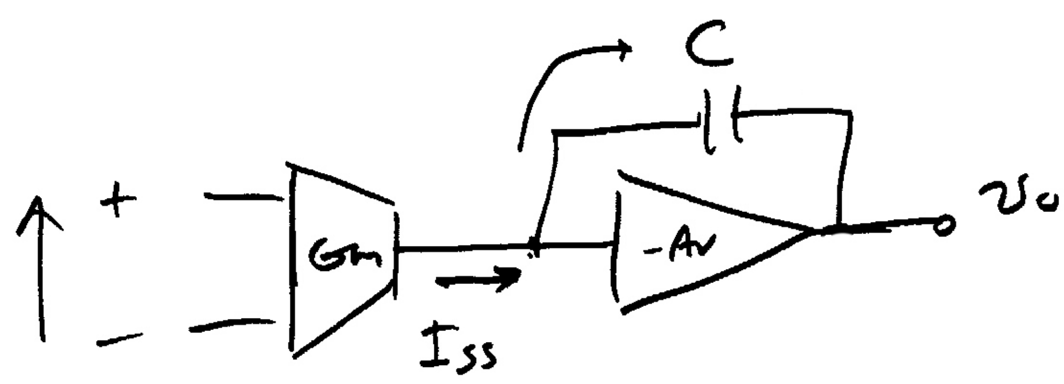


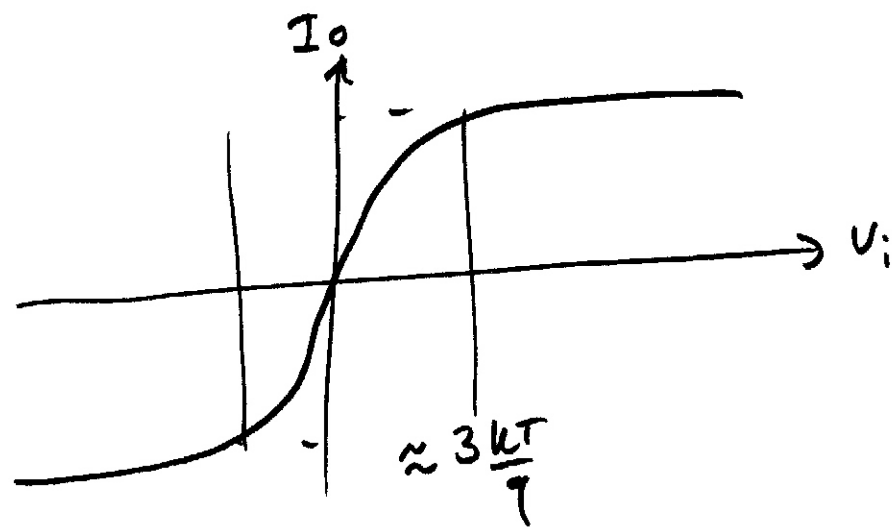
$\Rightarrow$



$$C_L \frac{dV_o}{dt} = I_{SS} \qquad \frac{dV_o}{dt} = \frac{I_{SS}}{C_L}$$

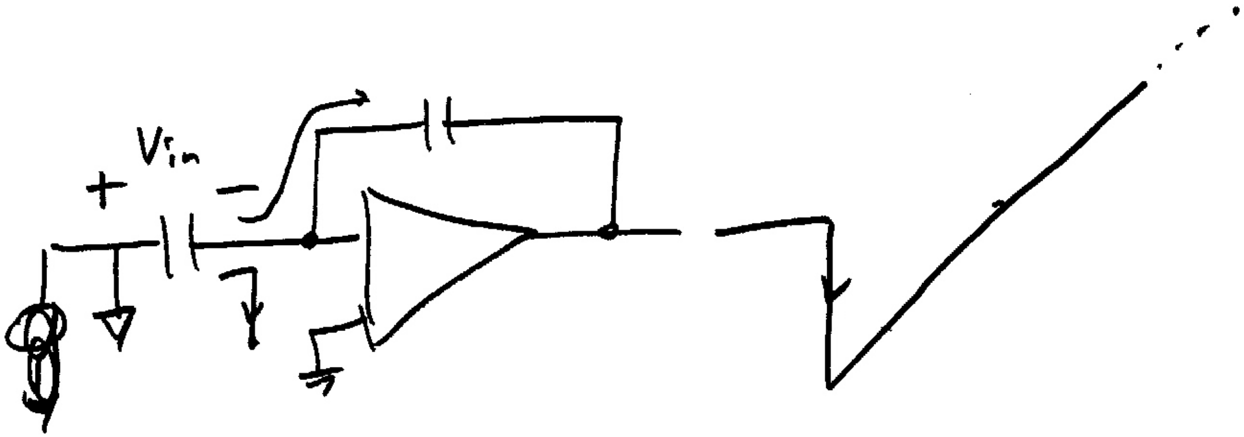






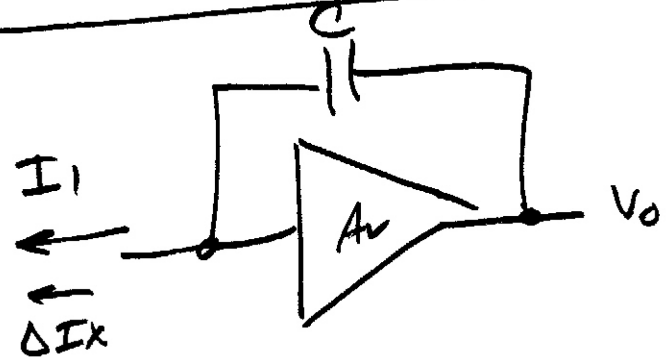
MOSFET $\simeq \sqrt{2}\ V_{od} = \sqrt{2}\ V^*$

SQUARE LAW



STEP INPUT IS REAL!

G&M



$$\frac{dV_o}{dt} = \frac{I_1}{C}$$

$$V_o(t) = \frac{I_1}{C} t$$

$$g_m \triangleq \frac{\Delta I_x}{\Delta V_x}$$

$$\frac{\Delta V_o}{\Delta I_x} = \frac{1}{sC}$$

SECOND STAGE IS ACTING LIKE INTEGRATOR

$$\frac{\Delta V_o}{\Delta I_x} = \frac{1}{j\omega C}$$

$$\boxed{\frac{\Delta V_o}{\Delta V_i} = \frac{g_m}{j\omega C}}$$

$$\frac{g_m}{\omega_u C} = 1$$

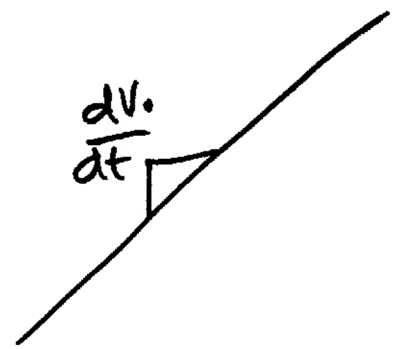
$$\omega_2 = K\,\omega_u \qquad K = 1 \rightarrow 45^\circ \text{ PM}$$

$$\frac{1}{C} = \frac{\omega_u}{g_m} = \frac{\omega_2}{g_m}$$

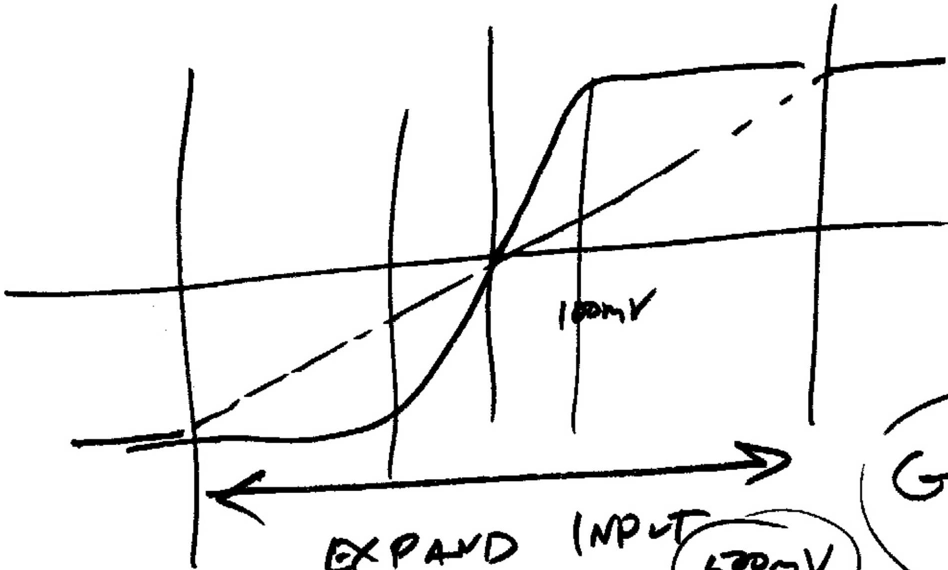
LARGE SIG

$$\boxed{\frac{dV_o}{dt} = \frac{I_1}{C} = \frac{I}{g_m}\,\omega_2}$$

"FIXED"



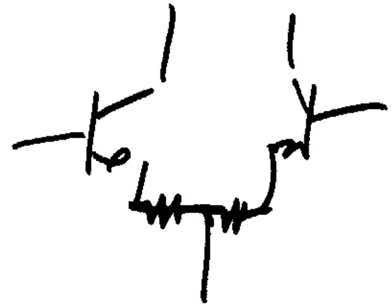
DECREASE $\frac{g_m}{I}$???



EXPAND INPUT RANGE

500mV

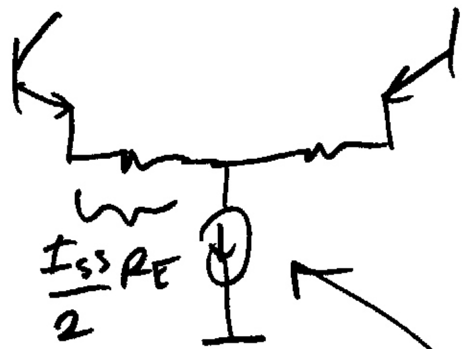
$$G_m = \frac{g_m}{1 + g_m R_E}$$



$$SR_{NO,FB} = \frac{I_{SS}}{g_m}\omega_2 = 2\frac{kT}{q}\omega_2$$

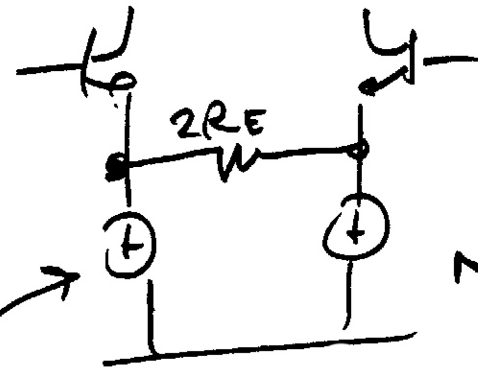
$$SR_{R_E} = \frac{I_{SS}}{G_m}\omega_2 = 2\frac{kT}{q}\omega_2\left(1 + g_m R_E\right)$$

INCREASE SR BY LOOP GAIN!



HEADROOM
AN ISSUE

CORRELATED
NOISE ⇒ ZERO
FOR DIFF O/P



NO HEADROOM
SHRINKAGE!

HIGHER NOISE!!!

$$SR = \frac{I_{SS}}{g_{m1}} \omega_2 = \frac{2 I_1}{\sqrt{2 \mu C_{ox} \left(\frac{W}{L}\right) I_1}} \omega_2$$

$$= \frac{2 \omega_2}{\sqrt{2 \mu C_{ox} \left(\frac{W}{L}\right)}} \sqrt{I_1}$$

DECREASE (W/L)

g_m DROPS $\longrightarrow$ MUST DROP C_L ...

AMP DESIGN : SPEED, RESOLUTION

$$t_s \,,\ \varepsilon$$

(1) STATIC SETTLING $\Rightarrow$ A_{v0} $\qquad \varepsilon' = \frac{1}{2}\varepsilon$

(2) DYNAMIC SETTLING

$$t_s = t_{s,lin} + t_{s,slew}$$

$$\begin{tikzcd}
t_{s,lin} \arrow[d] & t_{s,slew} \arrow[d] \\
I_{SS} \arrow[dr] & G_m \arrow[d, "V^*"] \\
& I_{SS} \arrow[d] \\
& \text{TAKE LARGER!}
\end{tikzcd}$$