EECS 217

Lecture 7: Properties of Scattering Parameters

Prof. Niknejad

University of California, Berkeley

Scattering Parameters of a Lossless Network

- Consider the total power dissipated by a network (must sum to zero)

$$P_{av} = \frac{1}{2}\Re\left(v^t i^*\right) = 0$$

- Expanding in terms of the wave amplitudes

$$= \frac{1}{2}\Re\left((v^+ + v^-)^t Z_0^{-1}(v^+ - v^-)^*\right)$$

- Where we assume that Z_0 are real numbers and equal. The notation is about to get ugly

$$= \frac{1}{2Z_0}\Re\left(v^{+t}v^{+*} - v^{+t}v^{-*} + v^{-t}v^{+*} - v^{-t}v^{-*}\right)$$

- Notice that the middle terms sum to a purely imaginary number. Let $x = v^+$ and $y = v^-$

$$y^t x^* - x^t y^* = y_1 x_1^* + y_2 x_2^* + \cdots - x_1 y_1^* + x_2 y_2^* + \cdots = a - a^*$$

Lossless (cont)

- We have shown that

$$P_{av} = \frac{1}{2Z_0}\left(\underbrace{{v^+}^t v^+}_{\text{total incident power}} - \underbrace{{v^-}^t {v^-}^*}_{\text{total reflected power}}\right) = 0$$

- This is a rather obvious result. It simply says that the incident power is equal to the reflected power (because the N port is lossless). Since $v^- = Sv^+$

$${v^+}^t v^+ = (Sv^+)^t (Sv^+)^* = {v^+}^t S^t S^* {v^+}^*$$

- This can only be true if S is a unitary matrix

$$S^t S^* = I$$

$$S^* = (S^t)^{-1}$$

Orthogonal Properties of S

- Expanding out the matrix product

$$\delta_{ij} = \sum_k (S^T)_{ik} S^*_{kj} = \sum_k S_{ki} S^*_{kj}$$

- For $i = j$ we have

$$\sum_k S_{ki} S^*_{ki} = 1$$

- For $i \neq j$ we have

$$\sum_k S_{ki} S^*_{kj} = 0$$

- The dot product of any column of S with the conjugate of that column is unity while the dot product of any column with the conjugate of a different column is zero. If the network is reciprocal, then $S^t = S$ and the same applies to the rows of S.
- Note also that $|S_{ij}| \leq 1$.

Shift in Reference Planes

- Note that if we move the reference planes, we can easily recalculate the S parameters.
- We'll derive a new matrix S' related to S. Let's call the waves at the new reference ν

$$v^- = Sv^+$$

$$\nu^- = S'\nu^+$$

- Since the waves on the lossless transmission lines only experience a phase shift, we have a phase shift of $\theta_i = \beta_i \ell_i$

$$\nu_i^- = v^- e^{-j\theta_i}$$

$$\nu_i^+ = v^+ e^{j\theta_i}$$

Reference Plane (cont)

- Or we have

$$\begin{bmatrix} e^{j\theta_1} & 0 & \cdots & \\ 0 & e^{j\theta_2} & \cdots & \\ 0 & 0 & e^{j\theta_3} & \cdots \\ \vdots & & & \end{bmatrix} \nu^- = S \begin{bmatrix} e^{-j\theta_1} & 0 & \cdots & \\ 0 & e^{-j\theta_2} & \cdots & \\ 0 & 0 & e^{-j\theta_3} & \cdots \\ \vdots & & & \end{bmatrix} \nu^+$$

- So we see that the new S matrix is simply

$$S' = \begin{bmatrix} e^{-j\theta_1} & 0 & \cdots & \\ 0 & e^{-j\theta_2} & \cdots & \\ 0 & 0 & e^{-j\theta_3} & \cdots \\ \vdots & & & \end{bmatrix} S \begin{bmatrix} e^{-j\theta_1} & 0 & \cdots & \\ 0 & e^{-j\theta_2} & \cdots & \\ 0 & 0 & e^{-j\theta_3} & \cdots \\ \vdots & & & \end{bmatrix}$$