EECS 217

Lecture 10: Transmission Line Transformers

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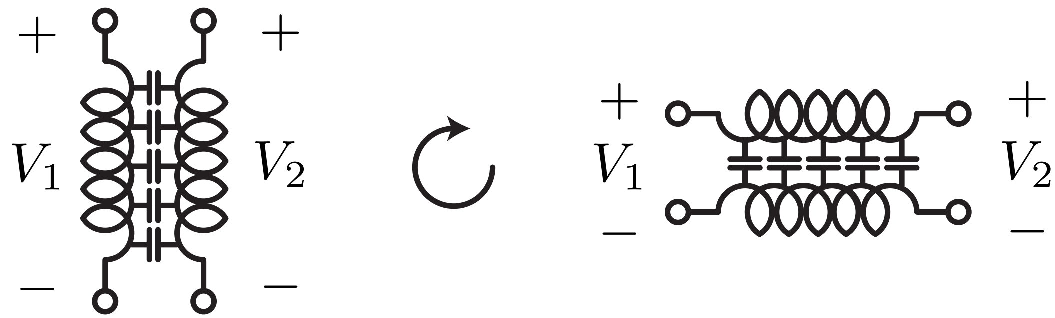
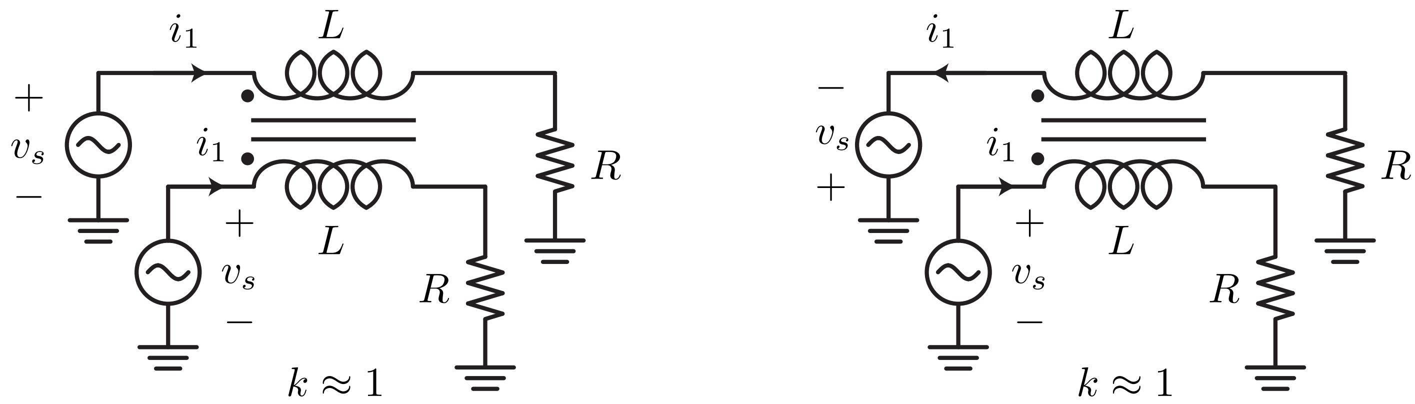
Transformers at High Frequency



- Transformers pose several problems at high frequencies. As we have seen, the distributed winding capacitance limits the high frequency application. The windings cannot be isolated too much due to the lack of a lossless magnetic core, and overall a coupling factor $k \sim 0.8 - 0.9$ can be acheived as a compromise.
- But if we view a transformer as a transmission line, then we see that the winding capacitance is in fact a critical part of the circuit. This is a very convenient building block, especially at lower frequencies when ordinary transmission lines are too bulky.

Low Frequencies: A Common Mode Choke



- We begin by noting that the operation of a simple $1:1$ transmission line transformer at low frequency. The input impedance for a *common-mode* signal (even mode) is simply given by

$$Z_{in} = \frac{v_s}{i_1} = j\omega L + j\omega M + R = j\omega(L+M) + R = j\omega L(1+k) + R \approx j\omega L(1+k)$$

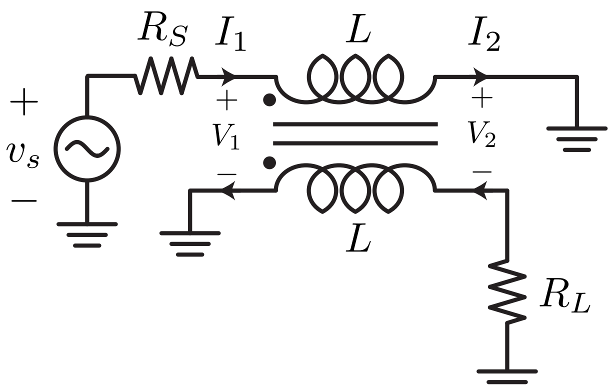
- Assuming $\omega L \gg R$. Thus the common mode input impedance is very high, especially at low frequency when a core can boost the inductance to a large value.
- For a differential mode input (odd mode), though, the input impedance is much lower

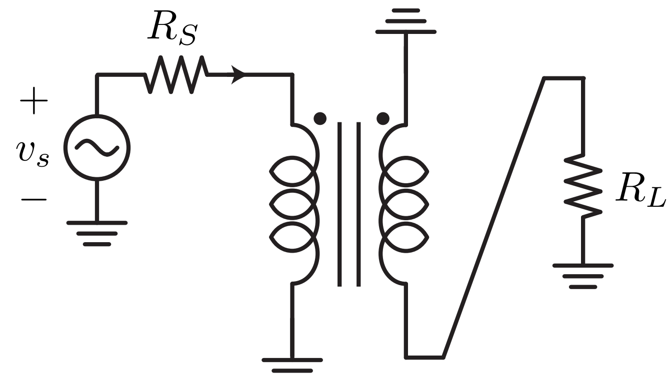
$$Z_{in} = \frac{v_s}{i_1} = j\omega L - j\omega M + R = j\omega(L-M) + R = j\omega L(1-k) + R \approx R$$

Common Mode Rejection

- Assuming $k \approx 1$. We see that such a structure by its very nature prevents common mode AC currents from flowing. Furthermore, since a common mode signal energizes the core, whereas a differential signal does not, the loss will be much higher for a common mode signal at high frequencies.
- A given excitation can be written as a superposition of the “even” and “odd” modes. Thus the even component will attenuate more as it travels down the T-line. Thus at the end of the T-line, we expect the odd mode to survive whereas the even mode will decay away.

Broadband Inverter





- We now excite the transformer as a transmission line with a differential (balanced) signal, as shown above. Note that at low frequency the circuit simply inverts the input signal and applies it to the load R_L. At high frequency, though, we employ the transmission line equations. From the $ABCD$ matrix

$$V_1 = \cosh\gamma\ell V_2 + Z_0 \sinh\gamma\ell I_2$$

$$V_2 = I_2 R_L$$

Broadband Inverter (cont)

- Assume that the differential characteristic impedance $Z_0 = \sqrt{\frac{L-M}{C}} = R_L$ so that no reflections occur at the load. Then we have

$$V_1 = (\cosh \gamma\ell + \sinh \gamma\ell)\, V_2 = e^{\gamma\ell} V_2$$

- We see that the output signal is given by

$$v_L = -V_2 = -e^{-\gamma\ell} V_1$$

- For the lossless case, $\gamma\ell = jk\ell$, and thus the circuit behaves like

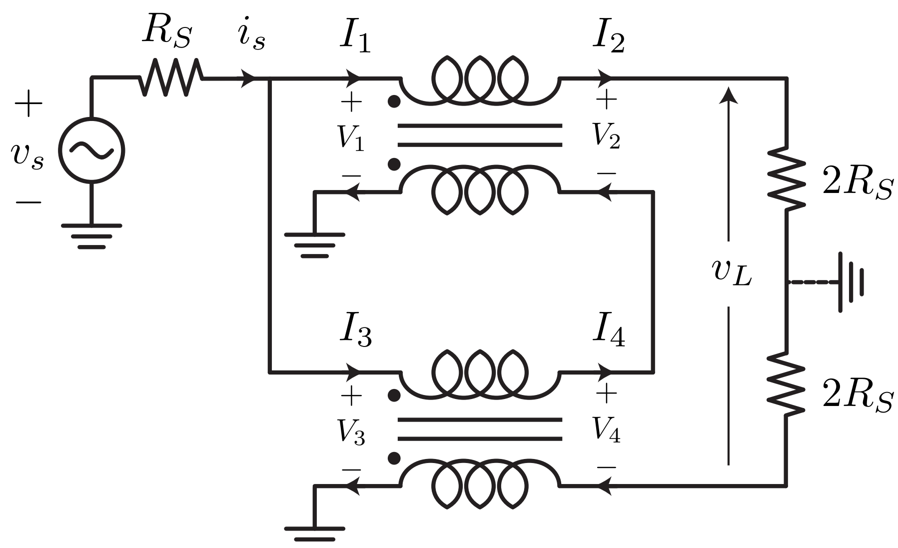
$$v_L = -e^{-jk\ell} V_1$$

- Thus the output signal is equal to the input over a broad frequency range

$$|v_L| = |V_1|$$

- The phase of the transfer function is 180° as long as $k\ell \ll 1$. The input impedance $Z_{in} = Z_0 = R_L$ is likewise matched over a broad frequency range.

Transmission Line Transformer Balun



- Consider now the circuit shown above, where the input drives two transformers, the top configured as a non-inverting delay line and the botton acting like an inverting delay line, as derived in the last section.
- Using the notation form the figure, by observation we have

$$I_2 = I_4$$

$$V_L = V_2 + V_4$$

$$V_1 = V_3 = v_s$$

$$i_s = I_1 + I_3$$

$$I_2 = I_4 = \frac{v_L}{4R_S}$$

T-Line Balun Equations

- Now apply the transmission line equations (assuming a lossless line)

$$v_L = V_2 + V_4 = \cos k\ell V_s - jZ_0 \sin k\ell I_1 + \cos k\ell V_s - jZ_0 \sin k\ell I_3$$

$$= 2\cos k\ell V_s - jZ_0 \sin k\ell \underbrace{(I_1 + I_3)}_{i_s}$$

$$i_s = I_1 + I_3 = jY_0 \sin k\ell \underbrace{(V_2 + V_4)}_{v_L} + 2\cos k\ell \underbrace{I_2}_{=I_4}$$

- Combining the above equations we have

$$i_s = \left(jY_0 \sin k\ell + \frac{2\cos k\ell}{4R_S} \right) v_L$$

$$v_L + jZ_0 \sin k\ell i_s = 2\cos k\ell v_s$$

$$v_L + jZ_0 \sin k\ell \left(jY_0 \sin k\ell + \frac{\cos k\ell}{2R_S} \right) v_L = 2\cos k\ell v_s$$

T-Line Balun (cont)

- Finally, we can solve for the output voltage

$$v_L = v_s \frac{2\cos k\ell}{1 - \sin^2 k\ell + j\sin k\ell \cos k\ell \frac{Z_0}{2R_S}}$$

- The voltage gain is given by

$$G_v = \frac{v_L}{v_s} = \frac{2}{\cos k\ell + j\sin k\ell \frac{Z_0}{2R_S}} = \frac{2}{e^{-jk\ell}}$$

- where the last equality holds if we select $Z_0 = 2R_S$. We see that the output voltage is twice the input voltage plus a delay. This relation is a broadband for a low-loss circuit

$$|G_v| = 2$$

Input Impedance

- We can also derive the input impedance

$$2v_s e^{-jk\ell} + jZ_0 \sin k\ell i_s = 2\cos k\ell v_s$$

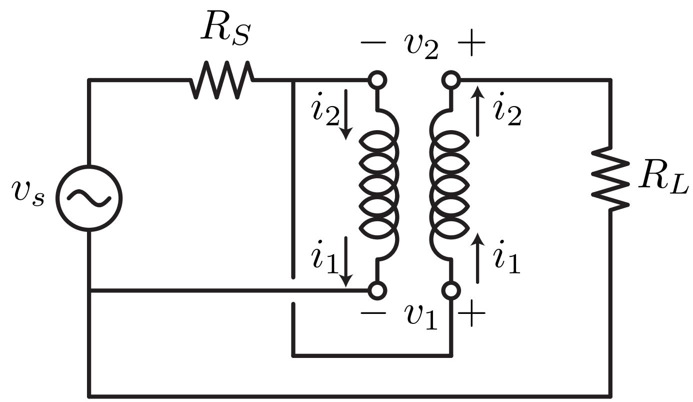
$$v_s \left(2\cos k\ell - 2e^{-jk\ell}\right) = j \underbrace{Z_0}_{2R_s} \sin k\ell i_s$$

- Simplifying we have

$$\frac{v_s}{i_s} = R_s$$

- Which shows that the circuit behaves like a $4:1$ impedance matching circuit.

A 4:1 Unbalanced Transformer



- Consider the above circuit. Intuitively, when the transmission line is electrically short, we can see that the source current is twice as large as the load current. We thus expect that at impedance match occurs for a load four times as large as the source.
- We can verify this at high frequency by applying KVL around the source and load loop

$$v_s = i_s R_S - v_2 + i_2 R_L = (i_1 + i_2) R_S - v_2 + i_2 R_L$$

- We can also take a KVL loop around the source

$$v_s = i_s R_S + v_1 = (i_1 + i_2) R_S + v_1$$

4:1 Transformer T-Line Analysis

- Assuming the transformer is acting like a differential transmission line, the current and voltage are related by

$$v_1 = v_2 \cos\beta\ell + j i_2 Z_0 \sin\beta\ell$$

$$i_1 = i_2 \cos\beta\ell + j v_2 Y_0 \sin\beta\ell$$

- The above four equations contain four unknowns v_1,v_2,i_1, and i_2. Solving for the load current i_2 we have the output power

$$P_o = \frac{1}{2}|i_2|^2 R_L$$

- To find the optimal T-line characteristic impedance Z_0 we differentiate the output power to find that

$$\frac{\partial P_o}{\partial Z_0} = 0$$

if

$$Z_0 = \sqrt{R_S R_L}$$

- The above result holds independent of the T-line length ℓ.

Optimal Load Impedance

- The optimal load impedance is given by

$$\frac{\partial P_o}{\partial R_L} = 0 \qquad \text{if} \qquad R_L = \frac{2R_S(1+\cos\beta\ell)}{\cos\beta\ell} \approx 4R_S$$

- The last equality holds if the line length $\beta\ell \ll 1$ is sufficiently small.
- The ratio of the load power to the available power is given by

$$P_a = \frac{v_s^2}{8R_S}$$

$$\frac{P_o}{P_a} = \frac{1+\cos\beta\ell}{\frac{5}{4}\left(1+\frac{6}{5}\cos\beta\ell + \cos^2\beta\ell\right)}$$

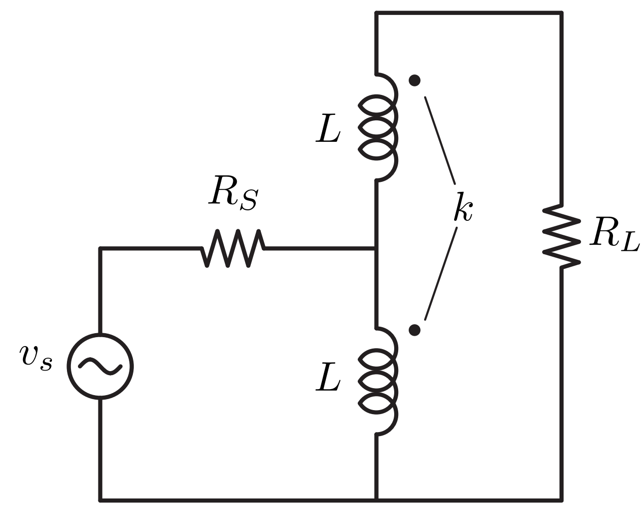
- The load power drops to zero if $\beta\ell = \pi$ or if the transmission line approaches a half-wavelength, $\ell = \lambda/2$. Since

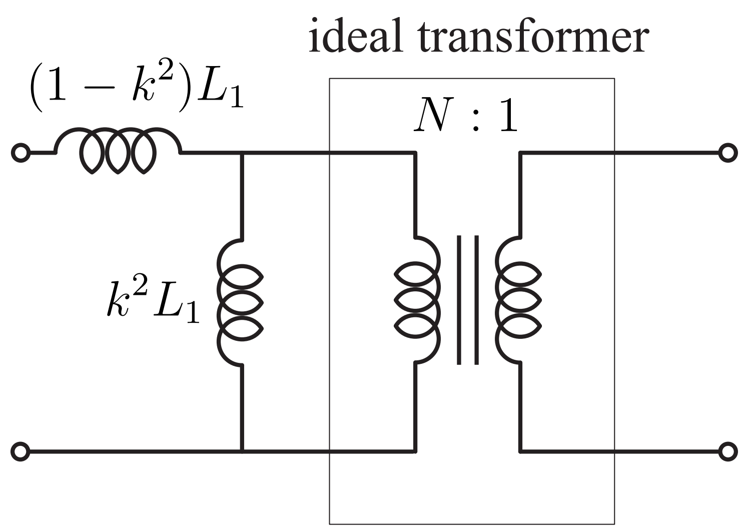
$$v_L = v_1 + v_2$$

- we can see that the load voltage is zero when the phase shift from input to output is 180°. So this transformer is only effective when $\ell \ll \lambda/2$. Typically we keep $\ell < \lambda/10$.

Low Frequency Limit

- At low frequencies the equivalent circuit of the transformer is shown. Note that the transmission line behavior is replaced by magnetic flux coupling of the auto-transformer.
- An equivalent circuit for the transformer, shown below, can be used to find the LF cutoff





- In general we see tha the low frequency signal is shunted to ground by the inductance k^2L.

Ideal (Easy) Analysis

- Since the transformer only works when the electrical T-line length is small, we can simplify the analysis by assuming that

$$i_1 = i_2 = i$$

$$v_1 = v_2 = v$$

- Thus we have

$$v_L = v_1 + v_2 = 2v \qquad i_L = i$$

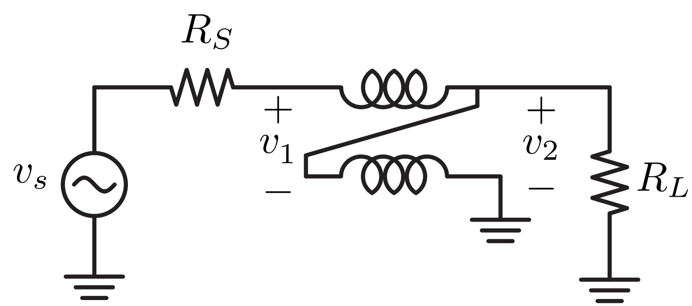
$$R_L = \frac{v_L}{i_L} = \frac{2v}{i}$$

$$Z_{in} = \frac{v}{2i}$$

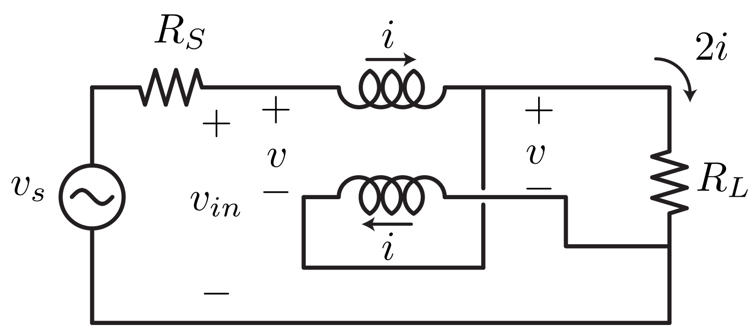
$$\frac{v}{i} = 2Z_{in}$$

$$R_L = 4Z_{in} \qquad Z_{in} = \frac{R_L}{4}$$

Step Up Transformer



- The above circuit doubles the current to the load and thus the input impedance is boosted to $Z_{in} = 4R_L$. Let's redraw the circuit and analyze it using the ideal equations



$$i_s = i$$

$$v_{in} = v + 2iR_L$$

$$v_L = v = 2iR_L$$

$$v_{in} = 2iR_L \times 2 = 4iR_L$$

$$Z_{in} = \frac{v_{in}}{i} = 4R_L$$

References

- Rotholz E., “Transmission Line Transformers,” *IEEE Trans. on Microwave Theory and Techniques*, vol. MTT-29, no. 4, Aprial 1981.
- Sevick J., “A Simplified Analysis of the Broadband Transmission Line Transformer,” *High Frequency Electronics*, Feb. 2004.
- Smith, *Modern Communication Circuits*