Integrated Circuits for Communication

Berkeley

Two-Ports and Power Gain

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Power Gain

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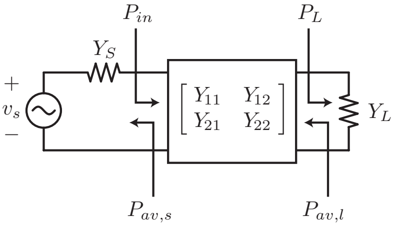


- We can define power gain in many different ways. The *power gain* G_p is defined as follows

$$G_p = \frac{P_L}{P_{in}} = f(Y_L, Y_{ij}) \neq f(Y_S)$$

- We note that this power gain is a function of the load admittance Y_L and the two-port parameters Y_{ij}.

Derivation of Power Gain

- The power gain is readily calculated from the input admittance and voltage gain

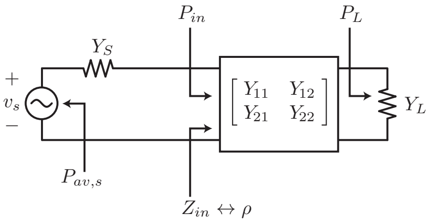
$$P_{in} = \frac{|V_1|^2}{2} \Re(Y_{in})$$

$$P_L = \frac{|V_2|^2}{2} \Re(Y_L)$$

$$G_p = \left|\frac{V_2}{V_1}\right|^2 \frac{\Re(Y_L)}{\Re(Y_{in})}$$

$$G_p = \frac{|Y_{21}|^2}{|Y_L + Y_{22}|^2} \frac{\Re(Y_L)}{\Re(Y_{in})} = f(Y_{i,j}, Y_L)$$

Is G_p a complete picture of power gain?



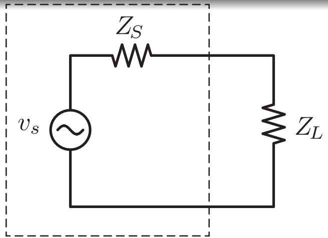
- Notice that G_p is the power gain normalized to the input power. That seems natural, but consider the following situation:

$$P_{in} = (1 - |\rho|^2)P_{avs}$$

- The input power is the available power from the source minus the "reflected" power. In other words, if we have a mismatched input, ($\rho \sim 1$) the output power might be *very* low:

$$P_L = P_{in}G_p = (1 - |\rho|^2)P_{avs} \approx 0$$

Review: Maximum Power Transfer Theorem



Fixed Source

- Suppose that a voltage source with a given (fixed) source impedance Z_S is connected to a load Z_L. What is the maximum power we can extract from the source given that we can choose any (passive) load ?
- The solution is easily derived and the optimum load is the conjugate matched load:

$$Z_{L,opt} = Z_S^*$$

- Under a matched condition, the power available from the source is the *maximum available power*:

$$P_{av,S} = P_L\big|_{Z_L = Z_{L,opt}} = \frac{|V_S|^2}{8\Re(Z_S)}$$

Power Gain (part 2)

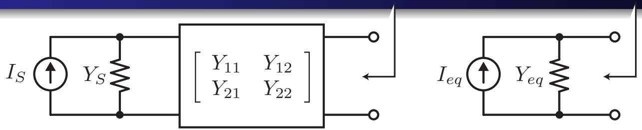
- The *available power gain* is defined as follows

$$G_a = \frac{P_{av,L}}{P_{av,S}} = f(Y_S, Y_{ij}) \neq f(Y_L)$$

- The available power from the two-port is denoted $P_{av,L}$ whereas the power available from the source is $P_{av,S}$.
- Finally, the *transducer gain* is defined by

$$G_T = \frac{P_L}{P_{av,S}} = f(Y_L, Y_S, Y_{ij})$$

- This is a measure of the efficacy of the two-port as it compares the power at the load to a simple conjugate match.



- To derive the available power gain, consider a Norton equivalent for the two-port where (short port 2)

$$I_{eq} = -I_2 = Y_{21} V_1 = \frac{-Y_{21}}{Y_{11} + Y_S} I_S$$

- The Norton equivalent admittance is simply the output admittance of the two-port

$$Y_{eq} = Y_{22} - \frac{Y_{21} Y_{12}}{Y_{11} + Y_S}$$

Available Gain (cont)

- The available power at the source and load are given by

$$P_{av,S} = \frac{|I_S|^2}{8\Re(Y_S)} \qquad P_{av,L} = \frac{|I_{eq}|^2}{8\Re(Y_{eq})}$$

$$G_a = \left|\frac{I_{eq}}{I_S}\right|^2 \frac{\Re(Y_S)}{\Re(Y_{eq})}$$

$$G_a = \left|\frac{Y_{21}}{Y_{11} + Y_S}\right|^2 \frac{\Re(Y_S)}{\Re(Y_{eq})} = f(Y_{i,j}, Y_S)$$

Transducer Gain Derivation

- The transducer gain is given by

$$G_T = \frac{P_L}{P_{av,S}} = \frac{\frac{1}{2}\Re(Y_L)|V_2|^2}{\frac{|I_S|^2}{8\Re(Y_S)}} = 4\Re(Y_L)\Re(Y_S)\left|\frac{V_2}{I_S}\right|^2$$

- We need to find the output voltage in terms of the source current. Using the voltage gain we have and input admittance we have

$$\left|\frac{V_2}{V_1}\right| = \left|\frac{Y_{21}}{Y_L + Y_{22}}\right|$$

$$I_S = V_1(Y_S + Y_{in})$$

$$\left|\frac{V_2}{I_S}\right| = \left|\frac{Y_{21}}{Y_L + Y_{22}}\right| \frac{1}{|Y_S + Y_{in}|}$$

Transducer Gain (cont)

$$|Y_S + Y_{in}| = \left| Y_S + Y_{11} - \frac{Y_{12} Y_{21}}{Y_L + Y_{22}} \right|$$

- We can now express the output voltage as a function of source current as

$$\left| \frac{V_2}{I_S} \right|^2 = \frac{|Y_{21}|^2}{|(Y_S + Y_{11})(Y_L + Y_{22}) - Y_{12} Y_{21}|^2}$$

- And thus the transducer gain

$$G_T = \frac{4\Re(Y_L)\Re(Y_S)|Y_{21}|^2}{|(Y_S + Y_{11})(Y_L + Y_{22}) - Y_{12} Y_{21}|^2} = f(Y_{i,j}, Y_S, Y_L)$$

Power Gain in Other Two-Port Parameters

$$G_T = \frac{4\Re(Y_L)\Re(Y_S)|Y_{21}|^2}{|(Y_S + Y_{11})(Y_L + Y_{22}) - Y_{12}Y_{21}|^2}$$

$$G_T = \frac{4\Re(Z_L)\Re(Z_S)|Z_{21}|^2}{|(Z_S + Z_{11})(Z_L + Z_{22}) - Z_{12}Z_{21}|^2} = f(Z_{i,j}, Z_S, Z_L)$$

- It's interesting to note that *all* of the gain expression we have derived are in the exact same form for the impedance, hybrid, and inverse hybrid matrices.
- This is true since power is a product of current and voltage, so by duality, if we replace voltage with current and current with voltage, we end up with the same expressions.

Which Power Gain is Best?

- Like most things in life, there's no "best" definition of power gain. It all depends on your perspective.
- If you're trying to design a low noise amplifier, it turns out that you are very much concerned about the source impedance, and so G_a is a convenient gain to consider.
- On the other hand, if you're concerned with a power amplifier, the load is the key parameter as it determines both power gain and efficiency, so G_p is very convenient.
- Transducer power gain is most relevant when you have freedom to optimize both the source impedance.
- But as far as optimizing the power gains is concerned, we'll shortly show that all three roads lead to the same result !

Maximum Power Gain and the Bi-Conjugate Match

Comparison of Power Gains

- In general, $P_L \leq P_{av,L}$, with equality for a matched load. Thus we can say that

$$G_T \leq G_a$$

- The maximum transducer gain as a function of the load impedance thus occurs when the load is conjugately matched to the two-port output impedance

$$G_{T,max,L} = \frac{P_L(Y_L = Y_{out}^*)}{P_{av,S}} = G_a$$

Comparison of Power Gains (cont)

- Likewise, since $P_{in} \leq P_{av,S}$, again with equality when the the two-port is conjugately matched to the source, we have

$$G_T \leq G_p$$

- The transducer gain is maximized with respect to the source when

$$G_{T,max,S} = G_T(Y_{in} = Y_S^*) = G_p$$

Bi-Conjugate Match

- When the input and output are simultaneously conjugately matched, or a *bi-conjugate match* has been established, we find that the transducer gain is maximized with respect to the source and load impedance

$$G_{T,max} = G_{p,max} = G_{a,max}$$

- This is thus the recipe for calculating the optimal source and load impedance in to maximize gain

$$Y_{in} = Y_{11} - \frac{Y_{12}Y_{21}}{Y_L + Y_{22}} = Y_S^*$$

$$Y_{out} = Y_{22} - \frac{Y_{12}Y_{21}}{Y_S + Y_{11}} = Y_L^*$$

- Solution of the above four equations (real/imag) results in the optimal $Y_{S,opt}$ and $Y_{L,opt}$.

Calculation of Optimal Source/Load

- Another approach is to simply equate the partial derivatives of G_T with respect to the source/load admittance to find the maximum point

$$\frac{\partial G_T}{\partial G_S} = 0$$

$$\frac{\partial G_T}{\partial B_S} = 0$$

$$\frac{\partial G_T}{\partial G_L} = 0$$

$$\frac{\partial G_T}{\partial B_L} = 0$$

- Again we have four equations. But we should be smarter about this and recall that the maximum gains are all equal. Since G_a and G_p are only a function of the source or load, we can get away with only solving two equations.

Calculation of Optimal Source/Load

- Working with available gain

$$\frac{\partial G_a}{\partial G_S} = 0$$

$$\frac{\partial G_a}{\partial B_S} = 0$$

- This yields $Y_{S,opt}$ and by setting $Y_L = Y_{out}^*$ we can find the $Y_{L,opt}$.
- Likewise we can also solve

$$\frac{\partial G_p}{\partial G_L} = 0$$

$$\frac{\partial G_p}{\partial B_L} = 0$$

- And now use $Y_{S,opt} = Y_{in}^*$.

Optimal Power Gain Derivation

- Let's outline the procedure for the optimal power gain. We'll use the power gain G_p and take partials with respect to the load. Let

$$Y_{jk} = g_{jk} + jb_{jk} \qquad Y_{12}Y_{21} = P + jQ = Le^{j\phi}$$

$$Y_L = G_L + jB_L \qquad G_p = \frac{|Y_{21}|^2}{D} G_L$$

$$\Re\left(Y_{11} - \frac{Y_{12}Y_{21}}{Y_L + Y_{22}}\right) = m_{11} - \frac{\Re(Y_{12}Y_{21}(Y_L + Y_{22})^*)}{|Y_L + Y_{22}|^2}$$

$$D = g_{11}|Y_L + Y_{22}|^2 - P(G_L + g_{22}) - Q(B_L + b_{22})$$

$$\frac{\partial G_p}{\partial B_L} = 0 = -\frac{|Y_{21}|^2 G_L}{D^2}\frac{\partial D}{\partial B_L}$$

Optimal Load (cont)

- Solving the above equation we arrive at the following solution

$$B_{L,opt} = \frac{Q}{2g_{11}} - b_{22}$$

- In a similar fashion, solving for the optimal load conductance

$$G_{L,opt} = \frac{1}{2g_{11}}\sqrt{(2g_{11}b_{22} - P)^2 - L^2}$$

- If we substitute these values into the equation for G_p (lot's of algebra ...), we arrive at

$$G_{p,max} = \frac{|Y_{21}|^2}{2g_{11}g_{22} - P + \sqrt{(2g_{11}g_{22} - P)^2 - L^2}}$$

Final Solution

- Notice that for the solution to exists, G_L must be a real number. In other words

$$(2g_{11}g_{22} - P)^2 > L^2 \qquad (2g_{11}g_{22} - P) > L$$

$$K = \frac{2g_{11}g_{22} - P}{L} > 1$$

- This factor K plays an important role as we shall show that it also corresponds to an unconditionally stable two-port. We can recast all of the work up to here in terms of K

$$Y_{S,opt} = -j\Im(Y_{11}) + \frac{Y_{12}Y_{21} - 2\Re(Y_{11})\Re(Y_{22}) + |Y_{12}Y_{21}|(K + \sqrt{K^2 - 1})}{2\Re(Y_{22})}$$

$$Y_{L,opt} = -j\Im(Y_{22}) + \frac{Y_{12}Y_{21} - 2\Re(Y_{11})\Re(Y_{22}) + |Y_{12}Y_{21}|(K + \sqrt{K^2 - 1})}{2\Re(Y_{11})}$$

$$G_{p,max} = G_{T,max} = G_{a,max} = \frac{Y_{21}}{Y_{12}} \frac{1}{K + \sqrt{K^2 - 1}}$$

Stability of a Two-Port

Stability of a Two-Port

- A two-port is unstable if the admittance of either port has a negative conductance for a passive termination on the second port. Under such a condtion, the two-port can oscillate.
- Consider the input admittance:

$$Y_{in} = G_{in} + jB_{in} = Y_{11} - \frac{Y_{12}Y_{21}}{Y_{22} + Y_L}$$

- Using the following definitions:

$$Y_{11} = g_{11} + jb_{11} \qquad Y_{12}Y_{21} = P + jQ = L\angle\phi$$

$$Y_{22} = g_{22} + jb_{22} \qquad Y_L = G_L + jB_L$$

- Now substitute real/imag parts of the above quantities into Y_{in}

$$Y_{in} = g_{11} + jb_{11} - \frac{P + jQ}{g_{22} + jb_{22} + G_L + jB_L}$$

Input Conductance

- Taking the real part, we have the input conductance

$$\Re(Y_{in}) = G_{in} = g_{11} - \frac{P(g_{22}+G_L)+Q(b_{22}+B_L)}{(g_{22}+G_L)^2+(b_{22}+B_L)^2}$$

$$= \frac{(g_{22}+G_L)^2+(b_{22}+B_L)^2 - \frac{P}{g_{11}}(g_{22}+G_L) - \frac{Q}{g_{11}}(b_{22}+B_L)}{D}$$

- Since $D > 0$ if $g_{11} > 0$, we can focus on the numerator. Note that $g_{11} > 0$ is a requirement since otherwise oscillations would occur for a short circuit at port 2.
- The numerator can be factored into several positive terms

$$N = (g_{22}+G_L)^2+(b_{22}+B_L)^2 - \frac{P}{g_{11}}(g_{22}+G_L) - \frac{Q}{g_{11}}(b_{22}+B_L)$$

$$= \left(G_L + \left(g_{22} - \frac{P}{2g_{11}}\right)\right)^2 + \left(B_L + \left(b_{22} - \frac{Q}{2g_{11}}\right)\right)^2 - \frac{P^2+Q^2}{4g_{11}^2}$$

Input Conductance (cont)

- Now note that the numerator can go negative only if the first two terms are smaller than the last term. To minimize the first two terms, choose $G_L = 0$ and $B_L = -\left(b_{22} - \frac{Q}{2g_{11}}\right)$ (reactive load)

$$N_{min} = \left(g_{22} - \frac{P}{2g_{11}}\right)^2 - \frac{P^2 + Q^2}{4g_{11}^2}$$

- And thus the above must remain positive, $N_{min} > 0$, so

$$\left(g_{22} + \frac{P}{2g_{11}}\right)^2 - \frac{P^2 + Q^2}{4g_{11}^2} > 0$$

$$g_{11}g_{22} > \frac{P + L}{2} = \frac{L}{2}(1 + \cos\phi)$$

Linvill/Llewellyn Stability Factors

- Using the above equation, we define the Linvill stability factor

$$L < 2g_{11}g_{22} - P$$

$$C = \frac{L}{2g_{11}g_{22} - P} < 1$$

- The two-port is stable if $0 < C < 1$.
- It's more common to use the inverse of $K = 1/C$ as the stability measure

$$K = \frac{2g_{11}g_{22} - P}{L} > 1$$

Stability (cont)

- The above definition of stability is perhaps the most common

$$K = \frac{2\Re(Y_{11})\Re(Y_{22}) - \Re(Y_{12}Y_{21})}{|Y_{12}Y_{21}|} > 1$$

- The above expression is identical if we interchange ports 1/2. Thus it's the general condition for stability.
- Note that $K > 1$ is the same condition for the maximum stable gain derived earlier. The connection is now more obvious. If $K < 1$, then the maximum gain is infinity!

Maximum Gain

- The maximum gain is usually written in the following insightful form

$$G_{max} = \frac{Y_{21}}{Y_{12}}(K - \sqrt{K^2 - 1})$$

- For a reciprocal network, such as a passive element, $Y_{12} = Y_{21}$ and thus the maximum gain is given by the second factor

$$G_{r,max} = K - \sqrt{K^2 - 1}$$

- Since $K > 1$, $|G_{r,max}| < 1$. The reciprocal gain factor is known as the efficiency of the reciprocal network.
- The first factor, on the other hand, is a measure of the non-reciprocity.

Maximum Stable Gain

- We have seen that for a stable two port, $K > 1$ and the maximum stable gain is well defined:

$$G_{max} = \frac{Y_{21}}{Y_{12}}(K - \sqrt{K^2 - 1})$$

- The peak value occurs when $K = 1$, which is the boundary of stability. At this value, define

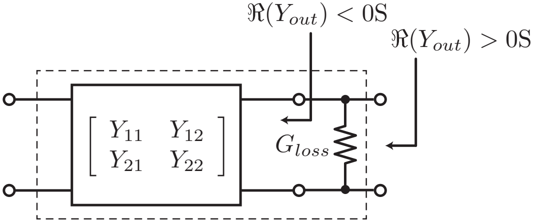
$$G_{MSG} = G_{max}|_{K=1} = \frac{Y_{21}}{Y_{12}}$$

- The larger K, the more gain we give up compared to G_{MSG}. If the two-port in inherently stable, then this is not something under our control. But what if the two-port is unstable?
- Since $K < 1$, the above equation is not valid. We actually know that for an unstable two-port, the maximum gain is infinity. In this case the output can be non-zero for a zero source, which implies infinite gain (it's basically an oscillator).

Stabilizing An Amplifier

- When a two-port is not unconditionally stable, we can still build an amplifier with it by carefully choosing the source and load. This is something we will learn about in 242B. For now, suppose that we wish to make an amplifier unconditionally stable. How can we do this ? It's simple, you just need to add loss to the system. This can come in the form of series or shunt resistors, or even feedback resistors used to stabilize the bias.
- Even in the stable case, if K is close to unity, we may find that over process/temperature variations there's a chance for instability. In this case, we may intentionally add loss to the two-port to make it more stable and robust, giving up some gain.

Stabilizing An Amplifier (cont)



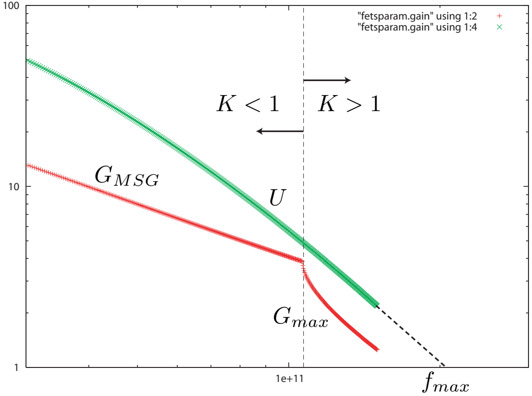
- Note that if Y_{in} has an imaginary part, we simply can add a shunt resistor to make it stable and to create a new unconditionally stable two-port:

$$\Re(Y'_{in}) = \Re(Y_{in}) + G_{loss} > 0\text{S}$$

$$G_{loss} > |\Re(Y_{in})|$$

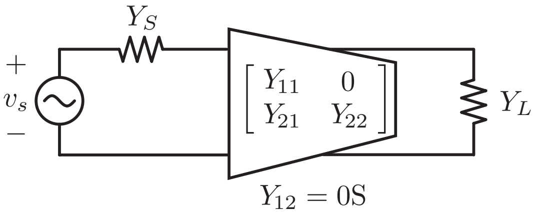
- Keep in mind that the K factor (and the Y parameters) are a function of bias, temperature, and frequency. An unconditionally stable amplifier needs to be stable under all conditions. You need to plot K versus frequency to ensure this is true.

G_{max} Plot



- The maximum gain of a device is then dependent on the value of K. For $K < 1$, the maximum gain is obtained by stabilizing the device and is given by MSG. For $K > 1$, the maximum gain is given by the expression presented above.
- U is the maximum unilateral gain, something we'll soon cover.

Unilateral Maximum Gain



- For a unilateral network, the design for maximum gain is trivial. For a bi-conjugate match

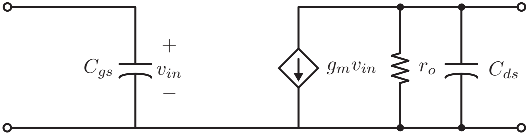
$$Y_S = Y_{11}^*$$

$$Y_L = Y_{22}^*$$

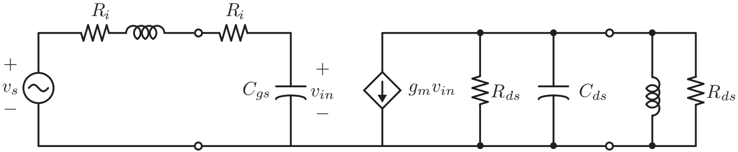
$$G_{T,max} = \frac{|Y_{21}|^2}{4 g_{11} g_{22}}$$

- An interesting idea is to somehow make a two-port unilateral by using feedback/feedforward, in other words a feed a signal that can cancel the feedback signal. We'll return to this concept later.

Ideal MOSFET



- The AC equivalent circuit for a MOSFET at low to moderate frequencies is shown above. Since $|S_{11}| = 1$, this circuit has infinite power gain. This is a trivial fact since the gate capacitance cannot dissipate power whereas the output can deliver real power to the load.



- A more realistic equivalent circuit is shown above. If we make the unilateral assumption, then the input and output power can be easily calculated. Assume we conjugate match the input/output
- The most common figure of merit for a transistor high frequency performance is the maximum frequency of oscillation, f_{max}, or equivalently, the highest frequency that we can obtain power gain from a transistor.
- Include gate resistance and neglect C_μ (for simplicity), and due to the unilateral nature ($Y_{12} = 0$) we can simply find the maximum gain by impedance matching the source and load directly

The Maximum Unity Power Gain Frequency (f_{max})

$$Z_S = Z_{in}^* = \left(R_g + \frac{1}{j\omega C_{gs}}\right)^* = R_g + j\omega L_s \tag{1}$$

$$Y_L = Y_{out}^* = (G_o + j\omega C_{ds})^* = G_o + j\omega L_d \tag{2}$$

- Under such bi-conjugate matched conditions, we have

$$P_{avs} = \frac{|V_S|^2}{8R_i}$$

$$P_L = \Re(\tfrac{1}{2} I_L V_L^*) = \tfrac{1}{2}\left|\frac{g_m V_1}{2}\right|^2 R_{ds}$$

$$G_{TU,max} = g_m^2 R_{ds} R_i \left|\frac{V_1}{V_S}\right|^2$$

Real MOSFET (cont)

- At the center resonant frequency, the voltage at the input of the FET is given by

$$V_1 = \frac{1}{j\omega C_{gs}} \frac{V_S}{2R_i}$$

$$G_{TU,max} = \frac{R_{ds}}{R_i} \frac{(g_m/C_{gs})^2}{4\omega^2}$$

- This can be written in terms of the device unity gain frequency f_T

$$G_{TU,max} = \frac{1}{4} \frac{R_{ds}}{R_i} \left(\frac{f_T}{f} \right)^2$$

- The above expression is very insightful. To maximum power gain we should maximize the device f_T and minimize the input resistance while maximizing the output resistance.

- Taking the ratio, and noting that $g_m/C_{gs} \approx \omega_T$, we have a simple expression for the power gain

$$P_L = \left(\frac{\omega_T}{\omega}\right)^2 \frac{R_{ds}}{R_i^2} \frac{1}{32} v_s^2 \tag{3}$$

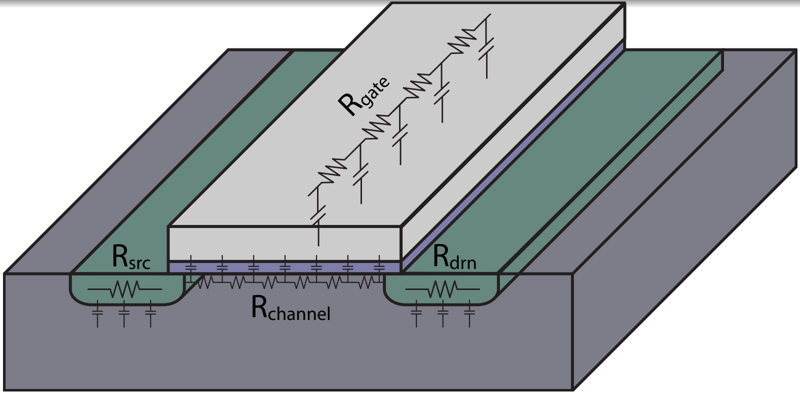
$$G_p = \frac{P_L}{P_{in}} = \frac{1}{4} \left(\frac{R_{ds}}{R_i}\right) \left(\frac{\omega_T}{\omega}\right)^2 \tag{4}$$

- Now we are in a position to find the maximum frequency f_{max} for which the transistor provides power gain

$$G_p = 1 = \frac{1}{4} \left(\frac{R_{ds}}{R_i}\right) \left(\frac{f_T}{f_{max}}\right)^2 \tag{5}$$

$$f_{max} = \frac{f_T}{2} \sqrt{\frac{R_{ds}}{R_i}} = \frac{f_T}{2} \sqrt{\frac{r_o}{R_i}} \tag{6}$$

Gate Resistance



- Despite the simplicity of our transistor model, this equation is very insightful as it connects the maximum power gain to the device f_T, or unity gain frequency. This affirms our faith in f_T as an important metric for RF and analog circuits, but it also shows that f_T is not the complete story. The device f_{max} may in fact be larger than f_T if the ratio of output resistance r_o is a large factor bigger than the device input resistance R_i.

Limit for f_{max}

- R_i depends on the gate polysilicon resistance and the channel resistance as seen from the gate terminal (gate AC current travels through the gate oxide and flows out through the channel and into the source and drain terminals). In the limit, if we layout the device with many fingers to minimize the physical gate resistance, only the channel resistance contributes to R_i

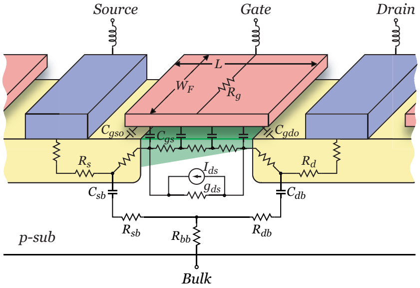
$$R_i \approx \frac{1}{5g_m} \tag{7}$$

- An ideal FET with no C_μ and gate resistance coming only from the channel:

$$f_{max} = \frac{f_T}{2}\sqrt{\frac{r_o}{\frac{1}{5g_m}}} = \frac{f_T}{2}\sqrt{5g_m r_o} = \frac{f_T}{2}\sqrt{5A_0} \tag{8}$$

Since for any respectable transistor $A_0 > 1$, we have that $f_{max} > f_T$.

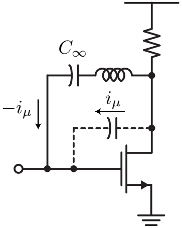
f_{max} for Practical Transistor

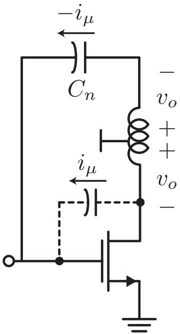


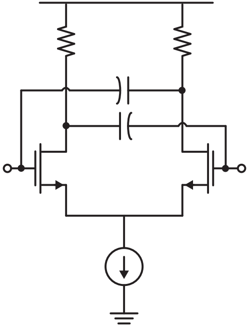
$$f_{max} \approx \frac{f_T}{2\sqrt{R_g\left(g_m C_{gd}/C_{gg}\right) + \left(R_g + r_{ch} + R_s\right) g_{ds}}} \qquad (9)$$

- This equation highlights the importance of minimizing the loss in the device, such as the drain/source resistance R_s, R_d, and the gate resistance R_g (part of R_i in our hybrid-pi model, the rest coming from r_{ch}). Check limit as $C_{gd} = 0$F.

Can we just cancel C_μ with an Inductor?

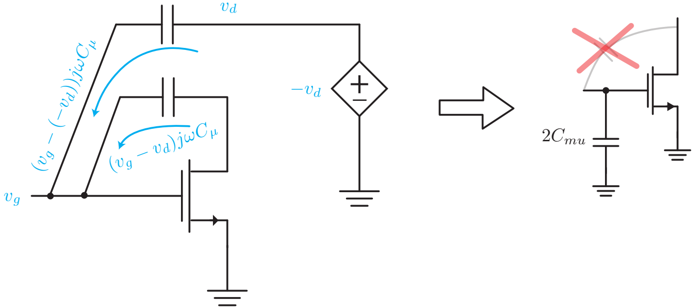




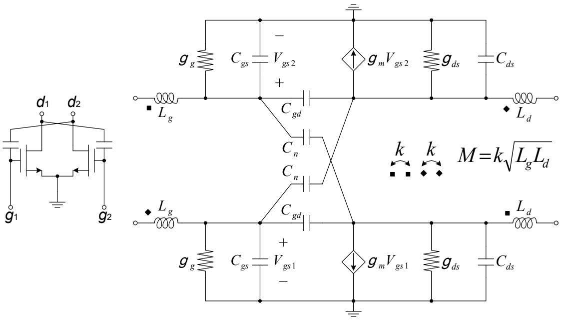


- In RF circuits we can neutralize this capacitance by connecting an inductor L_μ in parallel to resonate it out at any given frequency and over a narrow bandwidth. In differential circuits, we can neutralize the capacitance with a cross-coupled matching capacitor. In fact, is the optimal L_μ the value that completely cancels the capacitance? This question can be answered in a very general way by consideration of the Unilateral Gain.

Differential Neutralization



- Notice that in a differential circuit, the drain voltage with opposite polarity is available for "free" using the other side of the diff pair. In a single-ended amplifier, a transformer can invert the polarity of the input.
- The additional feedback current cancels out with the original internal feedback due to C_μ. From this perspective, it's a feedforward current.
- The net result is an increase in the input capacitance to $2C_\mu$, but a cancellation of the feedback capacitor C_μ.



- In practice, the mutual inductance between the lines is small and can be ignored. With this simplifying assumption, the *MSG* is given by

$$MSG = \sqrt{\frac{g_m^2}{\omega^2(C_{gd} - C_n)^2} + 1}$$

Ref: Zhiming Deng's dissertation.

Stability of Neutralization

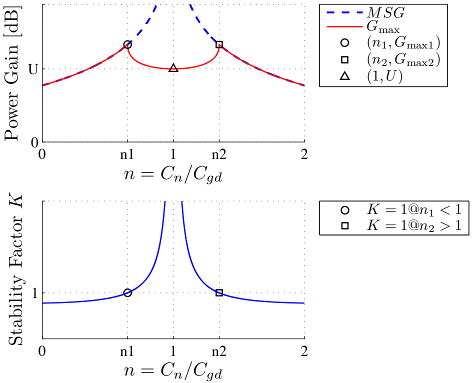
- Since neutralization is a feedback operation, we have to examine the stability carefully. The stability factor is given by

$$K = \left[1 + \frac{2g_g g_{ds}}{\omega^2 (C_{gd} - C_n)^2}\right] \cdot MSG^{-1}$$

- For $K < 1$, the maximum gain is just MSG. If we completely neutralize, we eliminate the feedback and the system becomes unilateral. Let's call this gain U

$$U = G_{max|C_n = C_{gd}} = \frac{g_m^2}{4 g_g g_{ds}}$$

Maximum Obtainable Gain with Neutralization



- The plots above show the power gain relative to the amount of neutralization, $n = C_n/C_{gd}$. As discussed previously, when $n = 1$, we unilaterize the device and obtain a gain of U. But it seems that we can obtain even higher gain, in particular two values of n result in $K = 1$ and higher gain.

Neutralization and G_{max}

- Therefore U is not the maximum gain (this is a common mistake people make). Recall that with some positive feedback (loop gain less than 1), we can obtain higher gain.
- The peak gain happens when $K = 1$ (conditionally stable) and this occurs for two values of $n = C_n/C_{gd}$

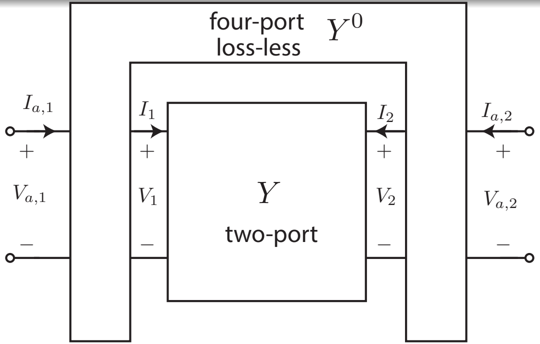
$$n = 1 \pm \frac{1}{\omega C_{gd}} \sqrt{\frac{g_g g_{ds}}{U-1}}$$

- For these values of n, which we call the optimal under- and over-neutralized designed, the maximum gain is given by

$$G_{max} = 2U - 1$$

- Can we do even better ?

Mason's Unilateral Gain U



- One of the most important metrics in high frequency transistor characterization is U, or Mason's Unilateral Gain. The definition of U is the power gain for a two-port under a general 4-port lossless embedding shown. The idea is to provide lossless "feedback" to unilaterize the two-port, and then under these conditions U is the gain we obtain from the two-port

$$U = \frac{|Y_{21} - Y_{12}|}{4(\Re(Y_{11})\Re(Y_{22}) - \Re(Y_{12})\Re(Y_{21}))} \tag{10}$$

Properties of U

- The U function has several important properties:
  1. If $U > 1$, the two-port is active. Otherwise, if $U \leq 1$, the two-port is passive.
  2. U is the maximum unilateral power gain of a device under a lossless reciprocal embedding.
  3. U is the maximum gain of a three-terminal device regardless of the common terminal.

  These properties have contributed to the widespread use of U as a metric to test a transistor power gain
- We find that the less loss there is in a device, the more gain we can extract, in stark contrast to MSG, which in fact requires us to *add* loss to stabilize the device when $K < 1$.

Is U the gain limit?

- Is U the maximum possible power gain we can obtain?
- In fact it is not the maximum, and as we've seen, through neutralization we can get a power gain as high as $\sim 2U$, and in general one can show that the gain can be as high as $\sim 4U$.

$$G_{max} = 2U - 1 + 2\sqrt{U(U-1)} \approx 4U$$

- Link: Niknejad, Ali, and Siva Thyagarajan. "Maximum Achievable Gain of a Two Port Network." (2016). Berkeley Reports (UCB/EECS-2016-15)