Integrated Circuits for Communication

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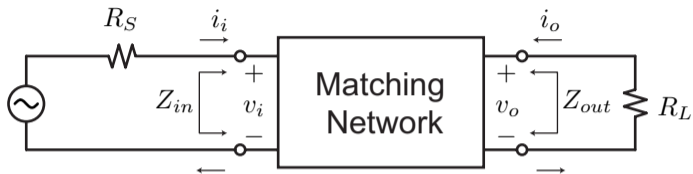
Matching Networks

Prof. Ali M. Niknejad

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- "RF design is all about impedance matching." Inductors and capacitors are handy elements at impedance matching.
- Viewed as a black-box, an impedance matcher changes a given load resistance R_L to a source resistance R_S. Without loss of generality, assume $R_S > R_L$, and a power match factor of $m = R_S/R_L$ is desired. In fact any matching network that boosts the resistance by some factor can be flipped over to do the opposite matching.

Why Play the Matchmaker?

- Optimal Power Transfer: Maximize the power transfer from the source (say an antenna) and the load (say an amplifier). Most amplifiers have a capacitive input impedance and a small resistive part.

- Optimal Noise Figure: Build amplifiers that add the least amount of noise to a signal while performing amplification. We'l see that this depends on the source impedance, so you'll need to transform the source.

- Minimum Reflections in Transmission Lines: Reflections cause dispersion/inter-symbol interference ("ghost" in analog TV), and result in a sensitive input impedance when looking in the transmission line (changes with distance).

- Optimal Efficiency: Power amplifiers obtain maximum efficiency when we utilize the largest possible voltage swing at the drain (collector) node, requiring us to match the load to a value that satisfies the conditions on load power and load swing.

- Since $R_L = v_o/i_o$ and $R_S = v_i/i_i$, we can see that this transformation can be achieved by a voltage gain, $v_i = k v_o$. Assuming the black box is realized with passive elements without memory, power conservation implies
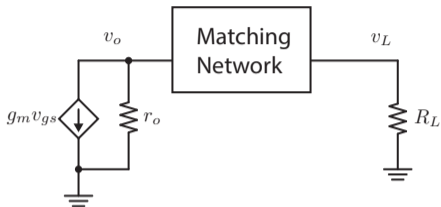
$$i_i v_i = i_o v_o$$

  thus the current must drop by the same factor, $i_i = k^{-1} i_o$, resulting in

$$Z_{in} = \frac{v_i}{i_i} = \frac{k v_o}{k^{-1} i_o} = k^2 \frac{v_o}{i_o} = k^2 R_L$$

  which means that $k = \sqrt{m}$ to achieve an impedance match. There are many ways to realize such a circuit block. Transformers are a natural choice but in this section we'll explore techniques employing inductors and capacitors.

- Since the dawn of the RF age, people have argued about matching. Is it really necessary, for example, when you're designing a voltage amplifier and you don't care about T-line effects (circuit is small relative to wavelength) ?
- Consider the scenario above, what looks like the output of a FET or BJT driving a load through an optional matching network M.
- If we don't match, we get a voltage gain of $g_m r_o || R_L$. Can we do better?
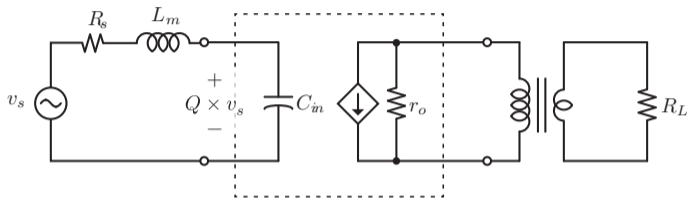
- Recall that the input voltage across the gate can be boosted by the Q factor of the network ! This can provide a lot of gain.
- The Q factor is determined by the driving R_s and the component Q values (including the gate capaitor).

$m{:}1$ $R_L < r_o$
Step Down !
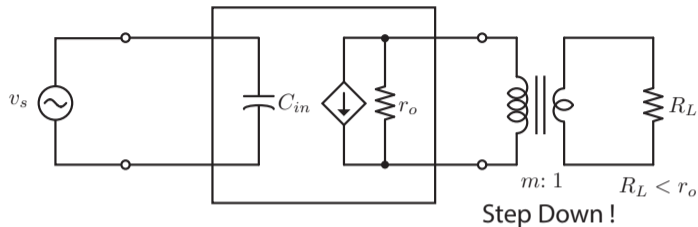
- It's easy to imagine that for voltage gain, you should not match at all, since it seems that matching reducing the load resistance by a factor of 2 compared to an open circuit load.
- This is somewhat non-intuitive but it actually helps to match because if you drop the voltage across the load with a transformer or matching network by a factor of m, the impedance seen by the amplifier is larger by m^2, so the net gain in voltage is potentially m ! Of course, the output of the amplifier loads and you cannot do this for any arbitrarily large m.

- Suppose the matching network transforms the load R_L by a factor of m^2 (since output stages are usually higher impedance than the load, usually $m > 1$ but for now let's just keep it general).

$$v_o = g_m v_{gs} r_o \| m^2 R_L = g_m v_{gs} \frac{r_o m^2 R_L}{r_o + m^2 R_L}$$

- But that means the voltage at the load is smaller by a factor of m

$$v_L = g_m v_{gs} \frac{r_o m R_L}{r_o + m^2 R_L} = g_m r_o R_L \cdot v_{gs} \cdot \left(\frac{m}{r_o + m^2 R_L} \right)$$

Optimal Voltage Gain

- The optimum value of m can be found easily

$$\frac{d}{dm}\left(\frac{m}{r_o + m^2 R_L}\right) = \frac{(r_o + m^2 R_L) - m^2 2 R_L}{(r_o + m^2 R_L)^2} = 0$$

- Solving for m, we find the optimal matching network for voltage is the same as the one we found for power gain

$$m_{opt} = \sqrt{r_o/R_L}$$

- Which leads to the matched voltage gain of G_m

$$G_m = \frac{v_L}{v_{gs}} = \frac{g_m r_o R_L}{r_o + \frac{r_o}{R_L} R_L}\sqrt{\frac{r_o}{R_L}}$$

$$G_m = \frac{g_m}{2}\sqrt{R_L r_o}$$

Voltage Gain: Match vs No Match

- If we didn't do any matching at all, recall the gain would be

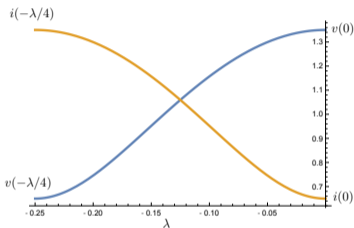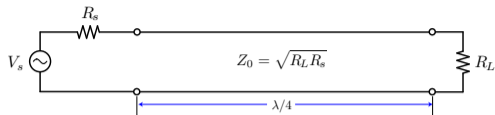$$G_u = g_m r_o || R_L = g_m \frac{r_o R_L}{r_o + R_L}$$

- Taking the ratio we have

$$\frac{G_m}{G_u} = \frac{\frac{1}{2}\sqrt{R_L r_o}}{\frac{r_o R_L}{r_o + R_L}} = \frac{r_o + R_L}{2} \cdot \frac{1}{\sqrt{R_L r_o}}$$

$$\frac{G_m}{G_u} = \frac{AM}{GM}$$

$\frac{r_o}{R_L}$	$\frac{G_m}{G_u}$
1	1
2	1.06
4	1.25
10	1.74
100	5.05

- Since the Geometric Mean (GM) is always less than the Arithmetic Mean (AM), matching always wins. The benefit of matching diminishes as r_o and R_L become comparable as shown in the table :

- Note that a transmission line has the desired properties of voltage/current gain if there's a standing wave on the line.

- For example, if the source and load are both real impedances, then we can move from high/low impedance to low/high impedance by adding a quarter wave line.

- The voltage is maximimum at one end and minimum at the other end, and the opposite is true for the current. So in effect the transmission line is a voltage/current multiplier (resonator).
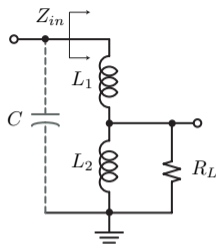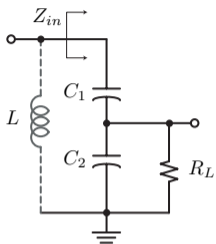
- Perhaps the simplest matching networks are simple voltage dividers. Consider the capacitive voltage divider. At RF frequencies, if $R_L \gg X_2$, then we can see that the circuit will work as advertised. Assuming that negligible current flows into R_L, the current flowing into the capacitors is given by

$$i = \frac{v_i}{j(X_1 + X_2)}$$

- The voltage across the is therefore
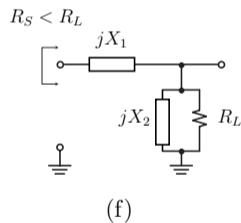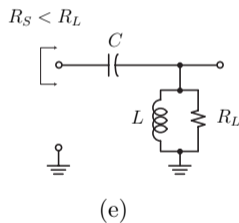
$$v_o = v_{C_2} = jX_2 \times i = v_i \frac{X_2}{X_1 + X_2} = v_i \frac{1}{1 + \frac{C_2}{C_1}} = k v_i$$

which means that the load resistance is boosted by a factor of k^2

$$R_{in} \approx \left(1 + \frac{C_2}{C_1}\right)^2 R_L$$

An L-Match

(a) (b) (c)

(d) (e) (f)
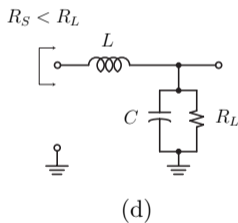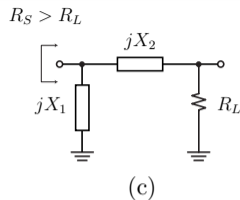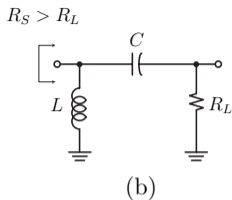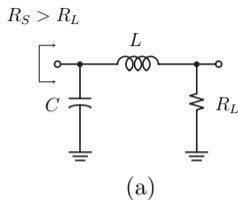
- Consider the L-Matching networks, named due to the topology of the network. We shall see that one direction of the L-match boosts the load impedance (in series with load) whereas the other lowers the load impedance (in shunt with the load).

- Let's focus on the first two networks shown. Here, in absence of the source, we have a simple series RLC circuit.

- Recall that in resonance, the voltage across the reactive elements is Q times larger than the voltage on the load! In essence, that is enough to perform the impedance transformation.

- Without doing any calculations, you can immediately guess that the impedance seen by the source is about Q^2 larger than R_L. Furthermore, since the circuit is operating in resonance, the net impedance seen by the source is purely real. To be sure, let's do the math.
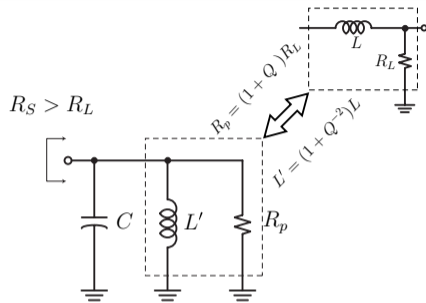
- A quick way to accomplish this feat is to begin with the series to parallel transformation, where the load resistance in series with the inductor is converted to an equivalent parallel load equal to

$$R_p = (1 + Q^2)R_L$$

- where $Q = X_L/R_L$, and $X'_L = X_L(1 + Q^{-2})$.

- The circuit is now nothing but a parallel *RLC* circuit and it's clear that at resonance the source will see only R_p, or a boosted value of R_L.
- The boosting factor is indeed equal to $Q^2 + 1$, very close to the value we guessed from the outset.
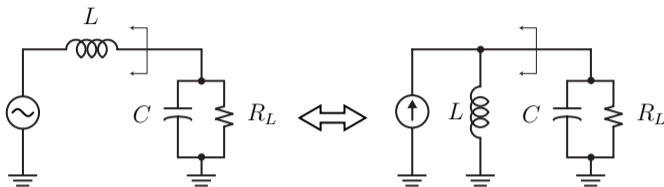
Norton Equiv.

- To gain further insight into the operation, consider an Norton equivalent of the same circuit.
- Now the circuit is easy to understand since it's simply a parallel resonant circuit. We known that at resonance the current through the reactances is Q times larger than the current in the load.
- Since the current in the series element is controlled by the source voltage, we can immediately see that $i_s = Qi_L$, thus providing the required current gain to lower the load resistance by a factor of Q^2.

- As you may guess, the mathematics will yield a similar result. Simply do a parallel to series transformation of the load to obtain

$$R_s = \frac{R_p}{1 + Q^2}$$

$$X_p' = \frac{X_p}{1 + Q^{-2}}$$

- The resulting circuit is a simple series RLC circuit. At resonance, the source will only see the reduced series resistance R_s.

- The following design procedure applies to an *L*-match using the generic forms. The actual choice between the forms depends on the application. For instance some provide AC coupling (DC isolation) which may be required in many applications. In other applications a common DC voltage may be needed, making the networks with DC coupling the obvious choice.

L-Match Design Equations

Let $R_{hi} = \max(R_S, R_L)$ and $R_{lo} = \min(R_S, R_L)$. The L-matching networks are designed as follows:

1. Calculate the boosting factor $m = \frac{R_{hi}}{R_{lo}}$.

2. Compute the required circuit Q by $(1 + Q^2) = m$, or $Q = \sqrt{m-1}$.

3. Pick the required reactance from the Q. If you're boosting the resistance, e.g. $R_S > R_L$, then $X_s = Q \cdot R_L$. If you're dropping the resistance, $X_p = \frac{R_L}{Q}$.

4. Compute the effective resonating reactance. If $R_S > R_L$, calculate $X_s' = X_s(1 + Q^{-2})$ and set the shunt reactance in order to resonate, $X_p = -X_s'$. If $R_S < R_L$, then calculate $X_p' = \frac{X_p}{1+Q^{-2}}$ and set the series reactance in order to resonate, $X_s = -X_p'$.

5. For a given frequency of operation, pick the value of L and C to satisfy these equations.

Insertion Loss of an *L*-Match

- We'd like to include the losses in our passive elements into the design of the matching network. The most detrimental effect of the component Q is the insertion loss which reduces the power transfer from source to load.

- Let's begin by using our intuition to derive an approximate expression for the loss. Note that the power delivered to the input of the matching network P_{in} can be divided into two components

$$P_{in} = P_L + P_{diss}$$

- where P_L is the power delivered to the load and P_{diss} is the power dissipated by the non-ideal inductors and capacitors.

- The insertion loss is therefore given by

$$IL = \frac{P_L}{P_{in}} = \frac{P_L}{P_L + P_{diss}} = \frac{1}{1 + \frac{P_{diss}}{P_L}}$$

- Recall that for the equivalent series RLC circuit in resonance, the voltages across the reactances are Q times larger than the voltage across R_L. We can show that the reactive power is also a factor of Q larger. For instance the energy in the inductor is given by

$$W_m = \frac{1}{4} L i_s^2 = \frac{1}{4} \frac{v_s^2}{4 R_S^2} L$$

or

$$\omega_0 \times W_m = \frac{1}{4} \frac{v_s^2}{4 R_S} \frac{\omega_0 L}{R_S} = \frac{1}{2} \frac{v_s^2}{8 R_S} Q = \frac{1}{2} P_L \times Q$$

where P_L is the power to the load at resonance

$$P_L = \frac{v_L^2}{2 R_S} = \frac{v_s^2}{4 \cdot 2 \cdot R_S} = \frac{v_s^2}{8 R_S}$$

- The total reactive power is thus exactly Q times larger than the power in the load

$$\omega_0(W_m + W_e) = Q \times P_L \tag{1}$$

By the definition of the component Q_c factor, the power dissipated in the non-ideal elements of net quality factor Q_c is simply

$$P_{diss} = \frac{P_L \cdot Q}{Q_c} \tag{2}$$

which by using the original forms of the equation immediately leads to the following expression for the insertion loss

$$IL = \frac{1}{1 + \frac{Q}{Q_c}} \tag{3}$$

- The above equation is very simple and insightful. Note that using a higher network Q, e.g. a higher matching ratio, incurs more insertion loss with the simple single stage matching network. Furthermore, the absolute component Q is not important but only the component Q_c normalized to the network Q. Thus if a low matching ratio is needed, the actual components can be moderately lossy without incurring too much insertion loss.

- Also note that the the actual inductors and capacitors in the circuit can be modeled with very complicated sub-circuits, with several parasitics to model distributed and skin effect, but in the end, at a given frequency, one can calculate the equivalent component Q_c factor and use it in the above equation.
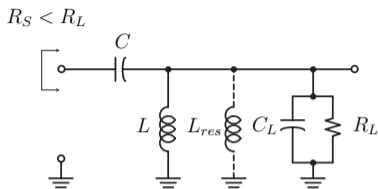
- In most situations the load and source impedances are often complex and our discussion so far only applies to real load and source impedances. An easy way to handle complex loads is to simply absorb them with reactive elements.
- For example, for the complex load shown, to apply an *L*-matching circuit, we can begin by simply resonating out the load reactance at the desired operating frequency. For instance, we add an inductance L_{res} in shunt with the capacitor to produce a real load.
- From here the design procedure is identical. Note that we can absorb the inductor L_{res} into the shunt *L*-matching element.
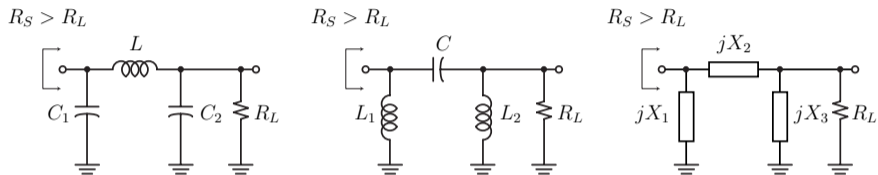
- The *L*-Match circuit is simple and elegant but is somewhat constrained. In particular, we cannot freely choose the Q of the circuit since it is fixed by the required matching factor m. This restriction is easily solved with the Π-Matching circuit, also named from its topology.
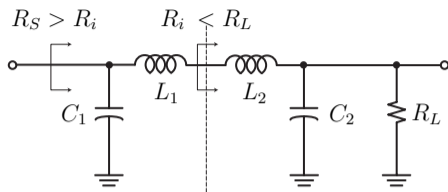
- The idea behind the Π match can be easily understood by studying the cascade of two back-to-front L matches.
- In this circuit the first L match will lower the load impedance to an intermediate value R_i

$$R_i = \frac{R_L}{1 + Q_1^2} \tag{4}$$

or

$$Q_1 = \sqrt{\frac{R_L}{R_i} - 1} \tag{5}$$
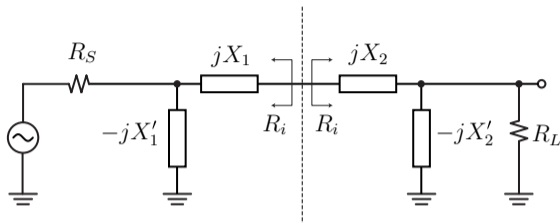
- Since $R_i < R_L$, the second L match needs to boost the value of R_i up to R_s. The Q of the second L network is thus

$$Q_2 = \sqrt{\frac{R_S}{R_i} - 1} > \sqrt{\frac{R_S}{R_L} - 1} \tag{6}$$

- The reflected input and output impedance are both equal to R_i at the center of the Π network.

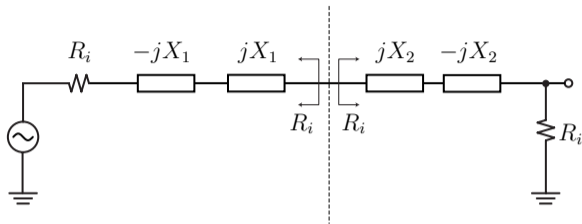- When we combine the two *L* networks, we obtain a Π network with a higher *Q* than possible with a single stage transformation. In general the *Q*, or equivalently the bandwidth $B = \frac{\omega_0}{Q}$, is a free parameter that can be chosen at will for a given application.
- Note that when the source is connected to the input, the circuit is symmetric about the center. Now it's rather easy to compute the network *Q* by drawing a series equivalent circuit about the center of the structure.

- If the capacitors and inductors in series are combined, the result is a simple RLC circuit with Q given by

$$Q = \frac{X_1 + X_2}{2R_i} = \frac{Q_1 + Q_2}{2}$$

- It's important to note the inclusion of the source resistance when calculating the network Q as we are implicitly assuming a power match. In a power amplifier, the source impedance may be different and the above calculation should take that into consideration.

- For instance, if the PA is modeled as a high impedance current source (Class A/B operation), then the factor of 2 disappears.

A *T*-Match
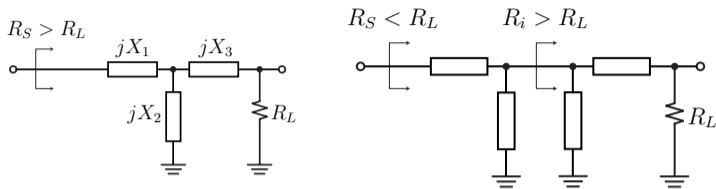
- The *T*-matching network is the dual of the Π network.
- The *T* network can also be decomposed into a cascade of two back-to-front *L* networks. The first *L* transforms the resistance up to some intermediate value $R_i > R_S$, and the second *L* transforms the resistance back down to R_S. Thus the net Q is higher than a single stage match.
- The network Q can be derived in an analogous fashion and yields the same solution

$$Q = \tfrac{1}{2} \left(\sqrt{\frac{R_i}{R_L} - 1} + \sqrt{\frac{R_i}{R_S} - 1} \right)$$
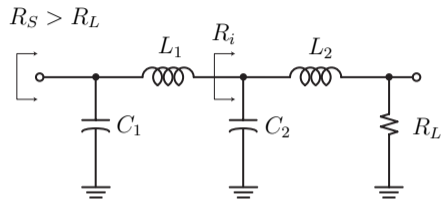
- We have seen that the Π and T matching networks are essentially two stage networks which can boost the network Q. In many applications we actually would like to achieve the opposite effect, e.g. low network Q is desirable in broadband applications.
- Furthermore, a low Q design is less susceptible to process variations. Also, a lower Q network lowers the loss of the network (see IL equation).

Lower Q Networks

- To lower the Q of an L matching network, we can employ more than one stage to change the impedance in smaller steps. Recall that $Q = \sqrt{m-1}$, and a large m factor requires a high Q match.

- If we simply change the impedance by a factor $k < m$, the Q of the first L section is reduced. Likewise, a second L section will further change the resistance to the desired R_S with a step size $l < m$, where $l \cdot k = m$.

- Reflecting all impedances to the center of the network, the real part of the impedance looking left or right is R_i at resonance. Thus the power dissipation is equal for both networks. The overall Q is thus given by

$$Q = \frac{\omega(W_{s1} + W_{s2})}{P_{d1} + P_{d2}} = \frac{\omega W_{s1}}{2P_d} + \frac{\omega W_{s2}}{2P_d} = \frac{Q_1 + Q_2}{2}$$

$$Q = \tfrac{1}{2}\left(\sqrt{\frac{R_i}{R_L} - 1} + \sqrt{\frac{R_S}{R_i} - 1}\right)$$
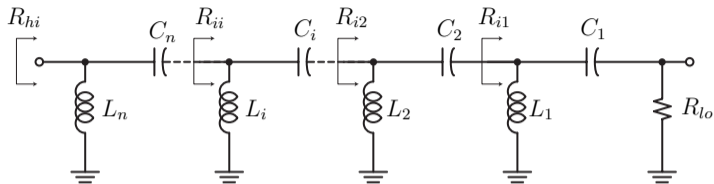
Optimally Low Q

- Note the difference between the above and Eq. 32. The R_i term appears once in the denominator and once in the numerator since it's an intermediate value. What's the lowest Q achievable? To find out, take the derivative with respect to R_i and solve for the minimum

$$R_{i,\text{opt}} = \sqrt{R_L R_S}$$

which results in a Q approximately lower by a square root factor

$$Q_{opt} = \sqrt{\sqrt{\frac{R_S}{R_L}} - 1} \approx m^{1/4}$$

It's clear that the above equations apply to the opposite case when $R_L > R_S$ by simply interchanging the role of the source and the load.

Multi-Section L

- To even achieve a lower Q, we can keep adding sections. The optimally low Q value is obtained when the intermediate impedances are stepped in geometric progression

$$\frac{R_{i1}}{R_{lo}} = \frac{R_{i2}}{R_{i1}} = \frac{R_{i3}}{R_{i2}} = \cdots = \frac{R_{hi}}{R_{in}} = 1 + Q^2$$

where $R_{hi} = \max(R_S, R_L)$ and $R_{lo} = \min(R_S, R_L)$. In the limit that $n \to \infty$, we take very small "baby" steps from R_{lo} to R_{hi} and the circuit starts to look like a tapered transmission line.

- Multiplying each term in the above equation

$$\frac{R_{i1}}{R_{lo}} \cdot \frac{R_{i2}}{R_{i1}} \cdot \frac{R_{i3}}{R_{i2}} \cdot \ldots \cdot \frac{R_{hi}}{R_{in}} = \frac{R_{hi}}{R_{lo}} = (1 + Q^2)^N$$

which results in the optimally Q factor for the overall network

$$Q = \sqrt{\left(\frac{R_{hi}}{R_{lo}}\right)^{1/N} - 1}$$

The loss in the optimal multi-section line can be calculated as follows. Using the same approach as before, note that the total power dissipated in the matching network is given by

$$P_{diss} = \frac{NQP_L}{Q_u}$$

where N section are used, each with equal Q.

- This leads to the following expression

$$IL = \frac{1}{1 + N\frac{Q}{Q_u}}$$

or

$$IL = \frac{1}{1 + \frac{N}{Q_u}\sqrt{\left(\frac{R_{hi}}{R_{lo}}\right)^{1/N} - 1}}$$

- It's interesting to observe that this expression has an optimum for a particular value of N. It's easy enough to plot *IL* for a few values of N to determine the optimal number of sections. Intuitively adding sections can decrease the insertion loss since it also lowers the network Q factor. Adding too many sections, though, can counterbalance this benefit.

PA Example

- Suppose a power amplifier delivering 100 W of power has an optimal load resistance of $.5\Omega$, but needs to drive a 50Ω antenna. Design a matching network assuming that the component Q's of 30 are available.

- First note that a matching factor of $m = 50/.5 = 100$ is needed. The table below shows the network Q and insertion loss as a function of the number of sections N. Clearly three sections yields the optimal solution. But since a three section filter is more expensive, and has only marginally better performance, a two section matching network may be preferable.

N	Q	IL (dB)
1	9.95	-1.24
2	3	-0.79
3	1.91	-0.76

N	Q	IL (dB)
4	1.47	-0.78
5	1.23	-0.81
6	1.07	-0.85

Multi-Stage Match: Exact Derivation
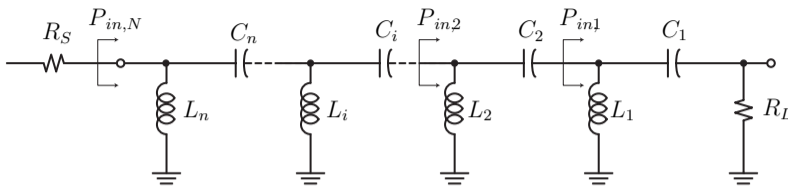
- The power into each stage can be written in the following form

$$P_{in,1} = P_L + P_{diss,1} = \left(1 + \frac{Q_1}{Q_{u1}}\right) P_L$$

- where Q_1 is the quality factor of the first stage matching network (form the load side), and Q_{u1} is the qualify factor of the components in the first stage matching network

$$\frac{1}{Q_{u1}} = \frac{1}{Q_{c1}} + \frac{1}{Q_{L1}}$$

- For the second stage, we can say that

$$P_{in,2} = P_{in,1} + P_{diss,2} = \left(1 + \frac{Q_2}{Q_{u2}}\right) P_{in,1}$$

$$= \left(1 + \frac{Q_1}{Q_{u1}}\right) \left(1 + \frac{Q_2}{Q_{u2}}\right) P_L$$

- And so for the Nth stage, we have

$$P_{in,N} = \left(1 + \frac{Q_1}{Q_{u1}}\right) \left(1 + \frac{Q_2}{Q_{u2}}\right) \cdots \left(1 + \frac{Q_N}{Q_{uN}}\right) P_L$$

$$IL = \frac{P_L}{P_{in,N}} = \frac{1}{\left(1 + \frac{Q_1}{Q_{u1}}\right) \left(1 + \frac{Q_2}{Q_{u2}}\right) \cdots \left(1 + \frac{Q_N}{Q_{uN}}\right)}$$

Insertion Loss for Optimal Case

- If $Q_1 = Q_2 = \cdots Q_N$, and also all components have the same Q, $Q_{u1} = Q_{u2} = \cdots = Q_u$, we have[1]

$$IL = \frac{1}{\left(1 + \frac{Q}{Q_u}\right)^N}$$

- If $Q/Q_u \ll 1$, then we have the following approximate expression

$$IL = \frac{1}{1 + N\frac{Q}{Q_u}}$$

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[1]Derivation courtesy of Hossein Shirinabadi (a former 142 student and group member).