Integrated Circuits for Communication

Berkeley

Introduction to Receivers and Mixers

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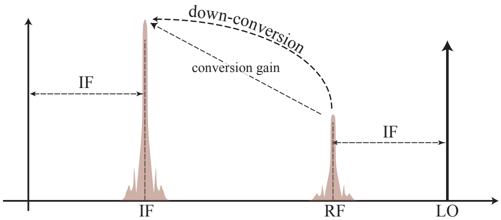
Mixers

- An ideal mixer is usually drawn with a multiplier symbol
- A real mixer cannot be driven by arbitrary inputs. Instead one port, the "LO" port, is driven by an *local oscillator* with a fixed amplitude sinusoid, and the other port is driven by a signal.
- In a *down-conversion* mixer, the other input port is driven by the "RF" signal, and the output is at a lower IF *intermediate frequency*
- In an *up-coversion* mixer, the other input is the IF signal and the output is the RF signal

Frequency Down-Conversion



- As shown above, an ideal mixer translates the modulation around one carrier to another. In a receiver, this is usually from a higher RF frequency to a lower IF frequency.
- We know that an LTI circuit cannot perform frequency translation. Mixers can be realized with either time-varying circuits or non-linear circuits

Ideal Multiplier

- Suppose that the input of the mixer is the RF and LO signal

$$v_{RF} = A(t)\cos\left(\omega_0 t + \phi(t)\right)$$

$$v_{LO} = A_{LO}\cos\left(\omega_{L0} t\right)$$

- Recall the trigonometric identity

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

- Applying the identity, we have

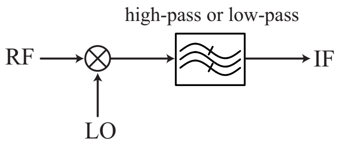
$$\begin{aligned} v_{out} &= v_{RF} \times v_{LO} \\ &= \frac{A(t)A_{LO}}{2}\{\cos\phi\left(\cos(\omega_{LO}+\omega_0)t + \cos(\omega_{LO}-\omega_0)t\right) \\ &\quad - \sin\phi\left(\sin(\omega_{LO}+\omega_0)t + \sin(\omega_{LO}-\omega_0)t\right)\} \end{aligned}$$

Ideal Multiplier (cont)

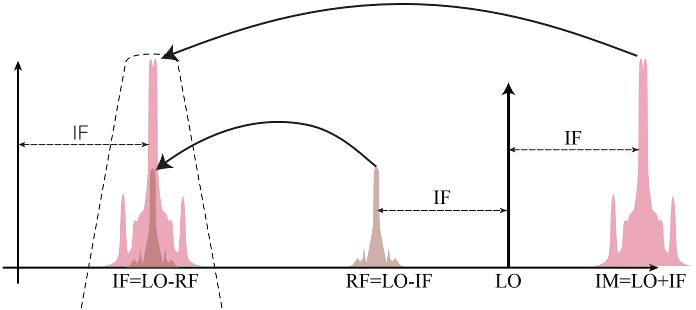
- Grouping terms we have

$$v_{out} = \frac{A(t)A_{LO}}{2} \quad \{\cos\left((\omega_{LO} + \omega_0)t + \phi(t)\right) + \cos\left((\omega_{LO} - \omega_0)t + \phi(t)\right)\}$$

- We see that the modulation is indeed translated to two new frequencies, $LO + RF$ and $LO - RF$. We usually select either the upper or lower "sideband" by filtering the output of the mixer



Mixer + Filter



- Note that the LO can be below the RF (lower side injection) or above the RF (high side injection)
- Also note that for a given LO, energy at $LO \pm IF$ is converted to the same IF frequency. This is a potential problem!

Upper/Lower Injection and Image

- Example: Downconversion Mixer

$$RF = 1\mathrm{GHz} = 1000\mathrm{MHz}$$

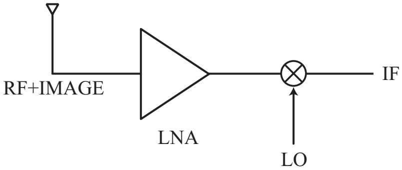
$$IF = 100\mathrm{MHz}$$

Let's say we choose a low-side injection:

$$LO = 900\mathrm{MHz}$$

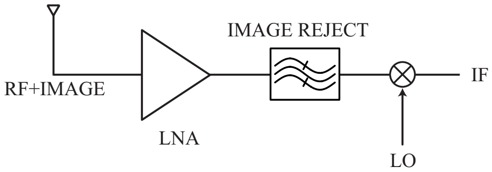
That means that any signals or noise at $800\mathrm{MHz}$ will also be downconverted to the same IF

Receiver Application



- The image frequency is the second frequency that also down-converts to the same IF. This is undesirable becuase the noise and interferance at the image frequency can potentially overwhelm the receiver.
- One solution is to filter the image band. This places a restriction on the selection of the IF frequency due to the required filter Q

Image Rejection



- Suppose that $RF = 1000\mathrm{MHz}$, and $IF = 1\mathrm{MHz}$. Then the required filter bandwidth is much smaller than $2\mathrm{MHz}$ to knock down the image.
- In general, the filter Q is given by

$$Q = \frac{\omega_0}{BW} = \frac{RF}{BW}$$

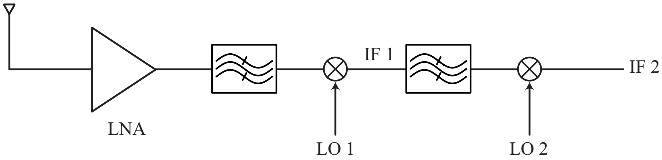
Image Reject Filter

- In our example, $RF = 1000\mathrm{MHz}$, and $IF = 1\mathrm{MHz}$. The Imagine is only $2IF = 2\mathrm{MHz}$ away.
- Let's design a filter with $f_0 = 1000\mathrm{MHz}$ and $f_1 = 1001\mathrm{MHz}$.
- A fifth-order Chebyshev filter with 0.2 dB ripple is down about 80 dB at the IF frequency.
- But the Q for such a filter is

$$Q = \frac{10^3\mathrm{MHz}}{1\mathrm{MHz}} = 10^3$$

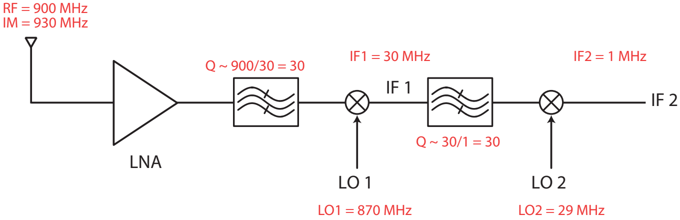
- Such a filter requires components with $Q > 10^3$!

Dual Conversion



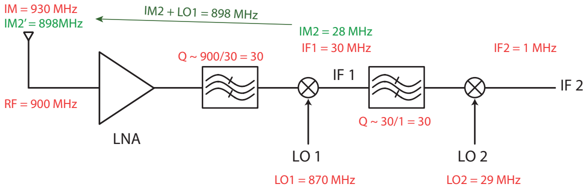
- The fact that the required filter Q is so high is related to the problem of filtering interferers. The very reason we choose the superheterodyne architecture is to simplify the filtering problem. It's much easier to filter a fixed IF than filter a variable RF.
- The image filtering problem can be relaxed by using multi-IF stages. Instead of moving to such a low IF where the image filtering is difficult (or expensive and bulky), we down-convert twice, using successively lower IF frequencies.

Dual Conversion Example



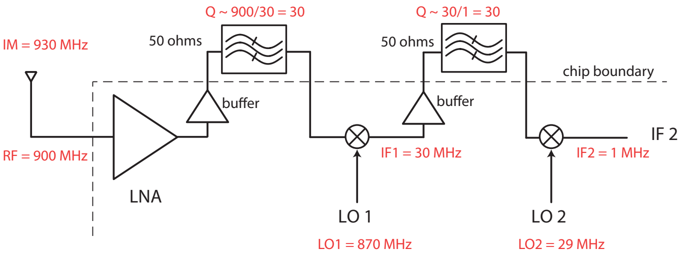
- Let's say the RF band is at 900 MHz and the IF is at 1 MHz. Let's choose $IF_1 = 30$MHz. Note that the Q has dropped from $Q = 900/1 = 900$ for a single stage downconversion to $Q = 900/30 = 30$ for a two stage design
- If we select the LO at 870 MHz, the first image band is at 840 MHz. A BPF is used to attenuate this image band.

Second Image



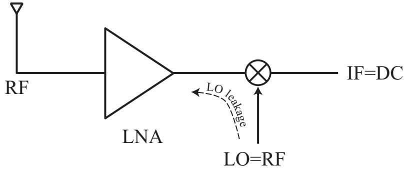
- The second mixer also has an image band. To convert from 30 MHz to 1 MHz, let's use an LO of 29 MHz. This places the image band at 28 MHz. It's very likely that this band is heavily attenuated by the LNA/filtering in the first stage, so do we need to worry?
- Due to the mixer, we need to up-convert the image band to the antenna port to see the issue. The first LO is at 870 MHz, so we have the effective second image at 870 MHz + 28 MHz = 898 MHz. This is very close to the RF band! So likely this band is not attenuated at all by our first filter and therefore we need a second image reject filter to null it out.

Dual Conversion: Issues



- Need two independent PLL's to frequency translate input bands to the same IF frequencies. "Sliding IF" is a variation that does not have this requirement (See 290C or 242B).
- Filter Q is pretty high and probably will be realized off-chip. Going off-chip means driving 50 ohms or substantial capacitance, which is power hungry. Going off-chip is also an excellent chance to pickup interference !

Direct Conversion Receiver

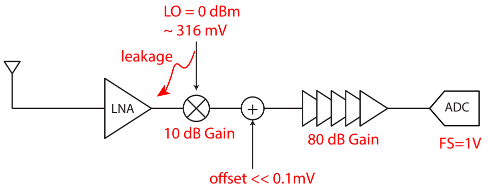


- A mixer will frequency translate two frequencies, $LO \pm IF$
- Why not simply down-convert directly to DC? In other words, why not pick a zero IF?
- This is the basis of the direct conversion architecture. There are some potential problems...

Direction Conversion

- First, note that we must down-convert the desired signal and all the interfering signals. In other words, the LNA and mixer must be extremely linear.
- Since IF is at DC, all *even* order distortion now plagues the system, because the distortion at DC can easily swamp the desired signal.
- Furthermore, CMOS circuits produce a lot of flicker noise. Before we ignored this source of noise because it occurs at low frequency. Now it also competes with our signal.
- Another issue is with LO leakage. If any of the LO leaks into the RF path, then it will self-mix and produce a DC offset. The DC offset can rail the IF amplifier stages.
- Finally, if the modulation is complex, a simple mixer will garble the upper and lower side-band, a point we'll cover soon.

LO Leakage / DC Offset



- Example: If the IF amplifier has 80 dB of gain, and the mixer has 10 dB of gain, let's estimate the allowed LO leakage. Assume the ADC uses a $1\mathrm{V}$ reference (Full Scale, FS).
- Model the mixer as a multiplier with gain G and note that the input leakage self-mixes and produces DC at the output

$$v_{out} = Gv_{in}(t) \times V_L O(t) = V_{leak} \cos(\omega t) \times V_{LO} \cos(\omega t)$$

$$= GV_{leak} V_{LO} \left(\frac{1 + \cos(2\omega t)}{2} \right)$$

LO Leakage / DC Offset (cont)

- Now assume $V_{leak} = \alpha V_{LO}$ where α is a small number:

$$v_{out}^{DC} = \frac{G}{2} V_{leak} V_{LO} = G\alpha V_{LO}^2$$

- To rail the output, we require a DC offset less than $10^{-4}\mathrm{V}$. If the LO power is $0\,\mathrm{dBm}$ ($316\mathrm{mV}$), we require an isolation α to be

$$\alpha < \frac{10^{-4}\mathrm{V}}{GV_{LO}^2} = -80dBc$$

- A better solution is to high-pass filter (if the modulation format allows it) or to cancel the offset voltage with a DAC in the baseband.

Phase of LO

- In a direction conversion system, the LO frequency is equal to the RF frequency.
- Consider an input voltage $v(t) = A(t)\cos(\omega_0 t)$. Since the LO is generated "locally", it's phase is random relative to the RF input:

$$v_{LO} = A_{LO}\cos(\omega_0 t + \phi_0)$$

- If we are so unlucky that $\phi_0 = 90^\circ$, then the low frequency output mixer will be zero (assuming modulation bandwidth is much lower than the carrier)

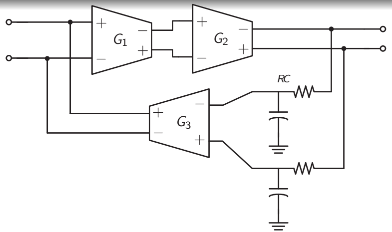
$$A(t)A_{LO}\sin(\omega_0 t)\cos(\omega_0 t) = A(t)A_{LO}\sin(2\omega_0 t)$$

$$LPF\left(v_{LO}v(t)\right) \approx 0$$

Direct Conversion in Practice

- Despite all the issues with Direct Conversion, the simplicity in the hardware and elimination of the filtering or extra PLLs has made it very popular.
- The LO feedthrough can be minimized with good balanced design and good reverse isolation in the LNA.
- The DC offset and flicker noise can be eliminated by high-pass filtering, which can become physically large (large RC time constants). This assumes that cutting out the low band of the signal has a negligible impact on the receiver sensitivity.
- Alternatively, a DC feedback loop or a digital DC cancellation loop can be used to eliminate the DC.

DC Feedback Loop



- The transfer function of the amplifier is given by:

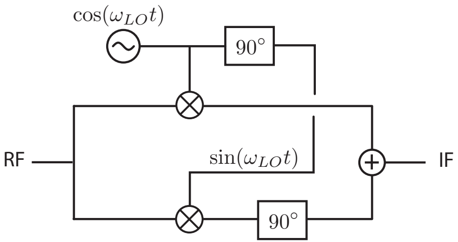
$$\frac{V_o}{V_i} = \frac{A}{1 + Af} = \frac{G_1 G_2}{1 + G_1 G_2 \frac{G_3}{RCs}}$$

- For simplicity we model the RC feedback filter as an integrator:

$$\frac{V_o}{V_i} = \frac{A}{1 + Af} = \frac{G_1 G_2 s}{s + G_1 G_2 \frac{G_3}{RC}}$$

- This is a high-pass response with a corner frequency determined by the product $\omega_{-3dB} = G_1 G_2 G_3 / RC$
- It's important to note that if the gain stages are variable gain, the corner frequency moves, so G_3 should be adjusted to compensate to keep the corner constant.
- At high-frequencies, the transfer function is essentially the same as an open loop amplifier with gain $G_1 G_2$.

I/Q Hartley Mixer



- An I/Q mixer implemented as shown above is known as a Hartley Mixer.
- We will also show that such a mixer can perform image rejection. The reason is fairly obvious if you consider that this is essentially the inverse of a single-sideband mixer that we studied already ! A mixer that only generates a single-sideband in the up-converter can be used to build an image reject mixer in the receiver just as well.

Delay Operation

- Consider the action of a 90° delay on an arbitrary signal. Clearley $\sin(x - 90^\circ) = -\cos(x)$. Even though this is obvious, consider the effect on the complex exponentials

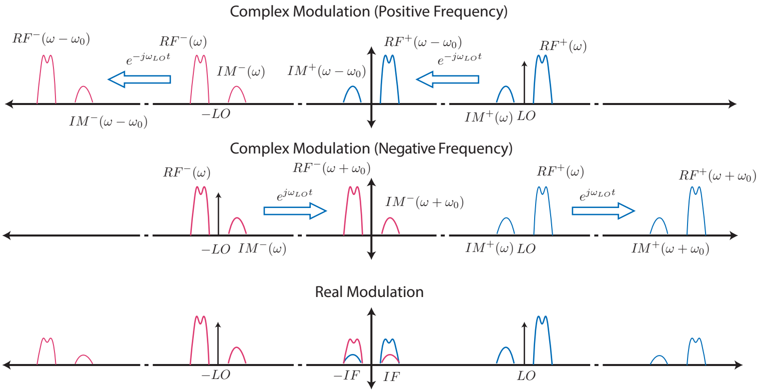
$$\sin(x - \frac{\pi}{2}) = \frac{e^{jx - j\pi/2} - e^{-jx + j\pi/2}}{2j}$$

$$= \frac{e^{jx} e^{-j\pi/2} - e^{-jx} e^{j\pi/2}}{2j} = \frac{e^{jx}(-j) - e^{-jx}(j)}{2j}$$

$$= -\frac{e^{jx} + e^{-jx}}{2} = -\cos(x)$$

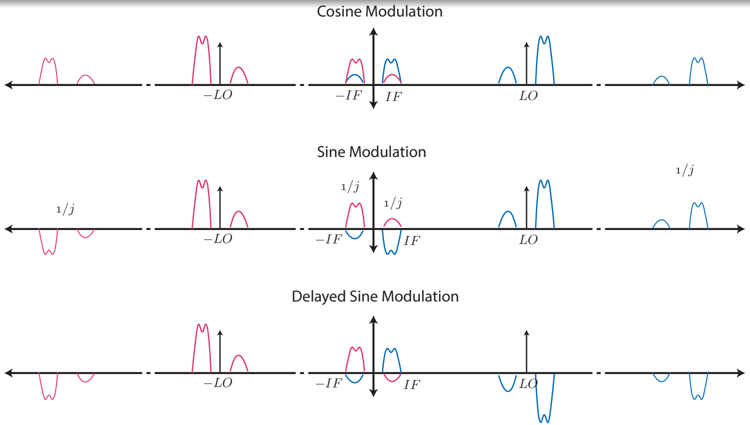
- Positive frequencies get multiplied by $-j$ and negative frequencies by $+j$. This is true for a narrowband signal when it is delayed by 90°.

Image Problem (Again)



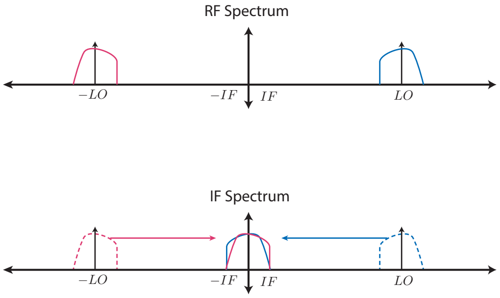
- We see that the image problem is due to to multiplication by the sinusoid and not a complex exponential. If we could synthesize a complex exponential, we would not have the image problem.

Sine/Cosine Modulation



- Using the same approach, we can find the result of multipling by sin and cos as shown above. If we delay the sin portion, we have a very desirable situation! The image is inverted with respect to the cos and can be cancelled.

Direct Down-Conversion with Complex Modulated Waveform



- Note that if the signal is a complex modulated signal up-converted, then if we simply downconvert it with a tone (sin or cos), the image reject probelm get translated into a spectrum mangling problem.
- For this reason, a complex down-converter is required, which explains why most modern communication sytems use a complex I and Q mixer.

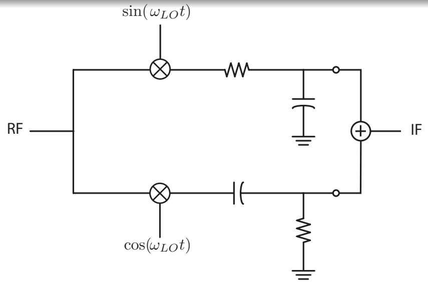
Image Rejection Matching Requirements

- The image rejection scheme just described is very sensitive to phase and gain match in the I/Q paths. Any mismatch will produce only finite image rejection.
- The image rejection for a given gain/phase match is approximately given by

$$IRR(\mathrm{dB}) = 10 \cdot \log \frac{1}{4}\left(\left(\frac{\delta A}{A}\right)^2 + (\delta\theta)^2\right)$$

- For typical gain mismatch of $0.2 - 0.5\,\mathrm{dB}$ and phase mismtach of $1^\circ - 4^\circ$, the image rejection is about $30\,\mathrm{dB}$ - $40\,\mathrm{dB}$. We usually need about $60 - 70\,\mathrm{dB}$ of total image rejection.

$\pm 45^\circ$ Delay Element



- The passive R/C and C/R lowpass and highpass filters are a nice way to implement the delay. Note that their relative phase difference is always 90°.

$$\angle H_{lp} = \angle \frac{1}{1 + j\omega RC} = -\arctan \omega RC$$

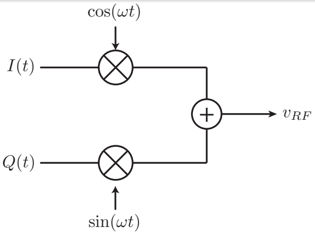
$$\angle H_{hp} = \angle \frac{j\omega RC}{1 + j\omega RC} = \frac{\pi}{2} - \arctan \omega RC$$

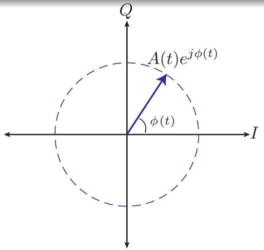
- Also note that the gain through both paths is the same at the corner frequency:

$$|H_{LP}(\omega_0)| = |H_{HP}(\omega_0)| = \sqrt{\frac{1}{2}}$$

- But to have equal gain, the circuit must operate at the $1/RC$ frequency. This restricts the circuit to relatively narrowband systems. Multi-stage polyphase circuits remedy the situation but add insertion loss to the circuit.
- The I/Q LO signal is usually generated directly rather than through an high-pass and low-pass network.
- Two ways to generate the I/Q LO is through a divide-by-two circuit (requires $2 \times LO$) or a quadrature oscillator (requires two tanks).

Complex I/Q Modulator

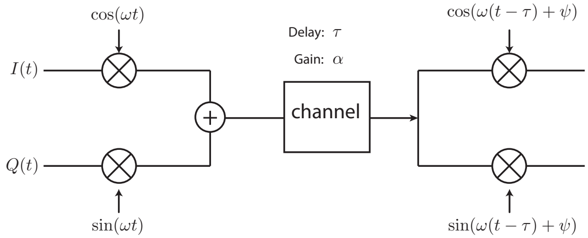




- Most modern receivers are realized as direct conversion "complex" receivers. Instead of performing image rejection like the Hartley, we perform the signal processing in the digital domain
- To see how this works, consider a signal modulated by a "universal" modulator shown above:

$$v_{RF}(t) = I(t)\cos(\omega t) + Q(t)\sin(\omega t) = A(t)\cos(\omega t + \phi(t))$$

Complex I/Q Receiver



- Note that at the receiver, we have a delayed copy of this signal (assuming there is only one dominant path for simplicity)

$$v_{Rx}(t) = \alpha \cdot I(t-\tau)\cos(\omega(t-\tau)+\psi) + \alpha \cdot Q(t-\tau)\sin(\omega(t-\tau)+\psi)$$

- where α is the path loss. Here we assume that the receiver is frequency coherent with the transmitter.

Complex I/Q Receiver

- There is also a random phase mismatch ψ between the LO on the Rx and Tx. This phase reflects the fact that the LO at the Rx has an arbitrary phase relation compared to the Tx. We can absorb this phase shift into the phase delay introduced by the path delay

$$\cos(\omega(t-\tau)+\psi) = \cos(\omega t + \psi')$$

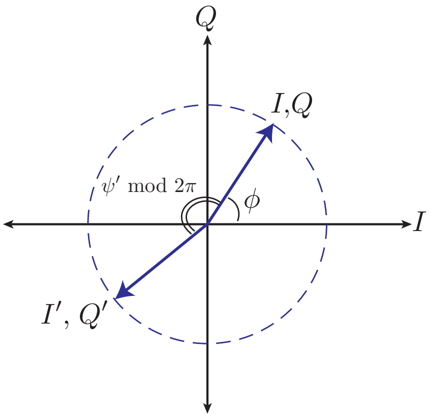
$$\sin(\omega(t-\tau)+\psi) = \sin(\omega t + \psi')$$

- Expanding, we have

$$\cos(\omega t + \psi') = \cos(\omega t)\cos(\psi') - \sin(\omega t)\sin(\psi')$$

$$\sin(\omega t + \psi') = \sin(\omega t)\cos(\psi') + \cos(\omega t)\sin(\psi')$$

Rotated I/Q Constellation



$$\begin{pmatrix} I' \\ Q' \end{pmatrix} = \begin{pmatrix} \cos\psi' & -\sin\psi' \\ \sin\psi' & \cos\psi' \end{pmatrix} \begin{pmatrix} I \\ Q \end{pmatrix}$$

- The effect of the static (or relatively slow) phase offset is just a rotation of the constellation as shown.

Rotated I/Q Constellation

- So we have
$$v_{Rx}(t) = \alpha \cdot I'(t)\cos(\omega t) + \alpha \cdot Q'(t)\sin(\omega t)$$
- Where I' and Q' are related to I and Q by scaling, a time-delay and a phase rotation of ψ' (which includes the static phase offset between the source and destination LO and also the propagation path delay induced offset.

$$I'(t) = \alpha \cdot \left(I(t-\tau)\cos(\psi') - Q(t-\tau)\sin(\phi')\right)$$

$$Q'(t) = \alpha \cdot \left(I(t-\tau)\sin(\phi') + Q(t-\tau)\cos(\phi')\right)$$

Demodulation

- We assume that the receiver can correct for the static phase offset. So let's use a complex mixer to recover the transmitted symbols. On the I-path, the output is given by

$$\hat{I}(t+\tau) = I'(t)\cos^2(\omega t) + Q'(t)\cos(\omega t)\sin(\omega t)$$

$$\hat{I}(t+\tau) = I'(t)\frac{1+\cos(2\omega t)}{2} + Q'(t)\frac{1}{2}\sin(2\omega t)$$

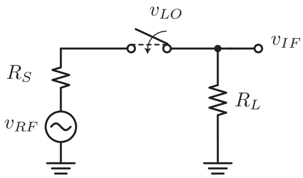
- After low-pass filtering, we are able to detect the symbol transmitter

$$\hat{\hat{I}}(t+\tau) \approx \frac{I'(t)}{2}$$

- Likewise, on the Q path, we have

$$\hat{\hat{Q}}(t+\tau) \approx \frac{Q'(t)}{2}$$

Practical Mixer Realization



- Real mixers are realized not as "multipliers" but using switches. The above schematic is in the core building block for a mixer: a switch !
- The control port of the switch is driven by the periodic LO signal (square or sine wave), and hence the transfer function varies periodically:

$$v_{IF}(t) = v_{RF}(t)\frac{R_L}{R_L + R_{SW}(t)}$$

$$R_{SW}(t) = f(v_{LO}(t))$$

Mixer Analysis: Time Domain

- A generic mixer operates with a periodic transfer function $h(t+T) = h(t)$, where $T = 1/\omega_0$, or T is the LO period. We can thus expand $h(t)$ into a Fourier series

$$y(t) = h(t)x(t) = \sum_{-\infty}^{\infty} c_n e^{j\omega_0 n t} x(t)$$

- For a sinusoidal input, $x(t) = A(t)\cos\omega_1 t$, we have

$$y(t) = \sum_{-\infty}^{\infty} \frac{c_n}{2} A(t) \left(e^{j(\omega_1 + \omega_0 n)t} + e^{j(-\omega_1 + \omega_0 n)t} \right)$$

- Since $h(t)$ is a real function, the coefficients $c_{-k} = c_k$ are even. That means that we can pair positive and negative frequency components.

Time Domain Analysis (cont)

$$= c_1 \frac{e^{j(\omega_1+\omega_0)t} + e^{j(-\omega_1+\omega_0)t}}{2} A(t) + c_{-1} \frac{e^{j(\omega_1-\omega_0)t} + e^{j(-\omega_1-\omega_0)t}}{2} A(t) + \cdots$$

- Take c_1 and c_{-1} as an example ($c_1 = c_{-1}$)

$$= c_1 A(t) \cos(\omega_1 + \omega_0)t + c_1 A(t) \cos(\omega_1 - \omega_0)t + \cdots$$

- Summing together all the components, we have

$$y(t) = \sum_{-\infty}^{\infty} c_n \cos(\omega_1 + n\omega_0)t$$

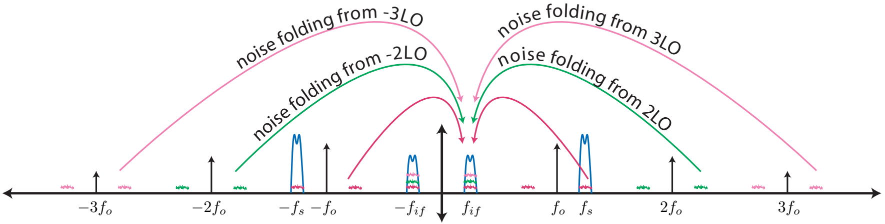
- Unlike a perfect multiplier, we get an infinite number of frequency translations up and down by harmonics of ω_0.

- Since multiplication in time, $y(t) = h(t) \cdot x(t)$, is convolution in the frequency domain, we have
$$Y(f) = H(f) * X(f)$$
- The transfer function $H(f) = \sum_{-\infty}^{\infty} c_n \delta(f - nf_0)$ has a discrete spectrum. So the output is given by

$$Y(f) = \int_{-\infty}^{\infty} \sum_{-\infty}^{\infty} c_n \delta(\sigma - nf_0) X(f - \sigma) d\sigma$$

$$= \sum_{-\infty}^{\infty} c_n \int_{-\infty}^{\infty} \delta(\sigma - nf_0) X(f - \sigma) d\sigma$$

Frequency Domain (cont)



- By the frequency sifting property of the $\delta(f-\sigma)$ function, we have

$$Y(f) = \sum_{-\infty}^{\infty} c_n X(f - nf_0)$$

- Thus, the input spectrum is shifted by all harmonics of the LO up and down in frequency.

Mixer Noise Definition

- By definition we have the fundamental definition of noise figure

$$F = \frac{SNR_i}{SNR_o}$$

- If we apply this to a receiving mixer, the input signal S_i is at the "RF" and the output signal S_o is at "IF".

$$F = \frac{\frac{S_i}{N_i}}{\frac{S_o}{N_o}}$$

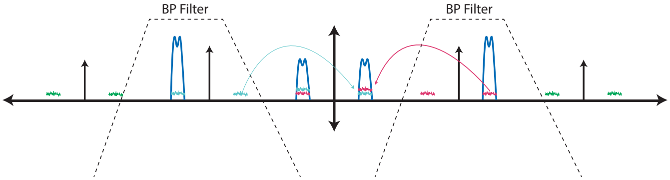
- The input noise is usually broadband, and so due to noise folding, the output noise N_o is not simply the input noise gained from the input band, but the input noise from many different bands. There is also additive noise from the mixer itself. Input referring the mixer noise, we have

$$N_o = \sum_{-\infty}^{\infty} c_n (N_i(f - nf_0) + N_{mix}(f - nf_0))$$

Noise/Image Problem

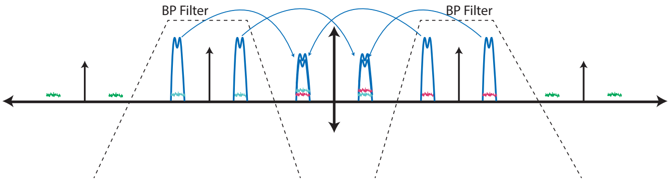
- Previously we examined the "image" problem. Any signal energy a distance of *IF* from the LO gets downconverted in a perfect multiplier. But now we see that for a general mixer, any signal energy with an IF of any harmonic of the LO will be downconverted !
- These other images are easy to reject because they are distant from the desired signal and a band-pass filter will be able to attenuate them significantly.
- The noise power, though, in all image bands will fold onto the IF frequency. Note that the noise is generated by the mixer source resistance itself and has a white spectrum. Even though the noise of the antenna is filtered, new noise is generated by the filter itself!

Single-Side Band (SSB) Noise Figure



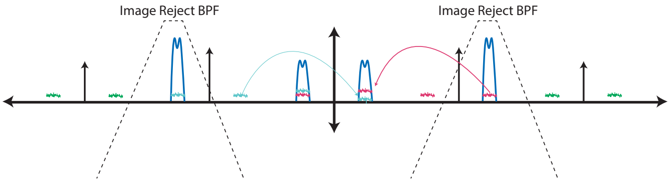
- There is some ambiguity to this definition because we have to specify if the RF signal is a single (upper or lower) or double sideband modulated waveform.
- For a single-sideband modulated waveform, the noise from the image band adds, therefore doubling the IF noise relative to RF. Thus the $F = 2$.

Double-Side Band (DSB) Noise Figure



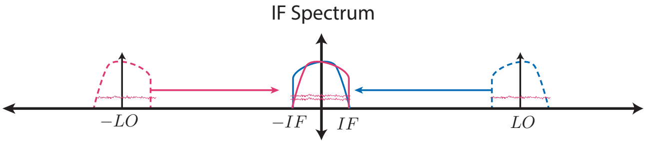
- For a double sideband modulated waveform, though, there is signal energy in both sidebands and so for a perfect multiplying mixer, $F = 1$ since the IF signal is twice as large since energy from both sidebands fall onto the IF.
- If an image reject filter is used, the noise in the image band can be suppressed (to some extent) and thus $F = 1$ (approaches) for a cascade of a sharp image reject filter followed by a multiplier.

Image Reject Mixer and Direct Conversion NF



- If an image reject filter is used, the noise in the image band can be suppressed and thus $F = 1$ (approaches) for a cascade of a sharp image reject filter followed by a multiplier or an ideal image reject mixer.
- Since an direct-conversion system occupies both the RF and image band, the DSB applies
- The de-mangling of the complex spectrum is an issue, so only a DSB signal would actually work unless we used two separate mixers like I and Q to de-mangle the spectrum

Direct Conversion and Complex Receiver



- Each arm of the complex mixer performs direct down-conversion and thus the DSB applies to each branch.
- The signals in the I and Q branch have independent information and each one is modulated and demodulated independently. Therefore, the DSB noise figure applies to the entire receiver when the complex signal from both branches is considered.
- Note that an image rejection mixer does not help because the signal occupies both the image and RF band.