Effect of Feedback on Distortion

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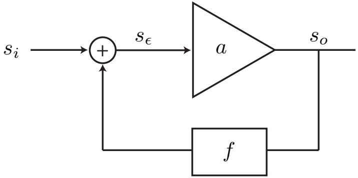


- We usually implement the feedback with a passive network
- Assume that the only distortion is in the forward path a

$$s_o = a_1 s_\epsilon + a_2 s_\epsilon^2 + a_3 s_\epsilon^3 + \cdots$$

$$s_\epsilon = s_i - f s_o$$

$$s_o = a_1(s_i - f s_o) + a_2(s_i - f s_o)^2 + a_3(s_i - f s_o)^3 + \cdots$$

Feedback and Disto (cont)

- We'd like to ultimately derive an equation as follows

$$s_o = b_1 s_i + b_2 s_i^2 + b_3 s_i^3 + \cdots$$

- Substitute this solution into the equation to obtain

$$\begin{aligned} b_1 s_i + b_2 s_i^2 + b_3 s_i^3 + \cdots = {} & a_1(s_i - f b_1 s_i - f b_2 s_i^2 - f b_3 s_i^3 + \cdots) \\ & + a_2(s_i - f b_1 s_i - f b_2 s_i^2 - f b_3 s_i^3 + \cdots)^2 \\ & + a_3(s_i - f b_1 s_i - f b_2 s_i^2 - f b_3 s_i^3 + \cdots)^3 + \cdots \end{aligned}$$

- Solve for the first order terms

$$b_1 s_i = a_1(s_i - f b_1 s_i)$$

$$b_1 = \frac{a_1}{1 + a_1 f} = \frac{a_1}{1 + T}$$

Feedback and Disto (square)

- The above equation is the same as linear analysis (loop gain $T = a_1 f$)
- Now let's equate second order terms

$$b_2 s_i^2 = -a_1 f b_2 s_i^2 + a_2 (s_i - f b_1 s_i)^2$$

$$b_2 (1 + a_1 f) = a_2 \left(1 - \frac{f a_1}{1 + T}\right)^2$$

$$b_2 (1 + T)^3 = a_2 (1 + T - T)^2 = a_2$$

$$b_2 = \frac{a_2}{(1 + T)^3}$$

- Same equation holds at high frequency if we replace with $T(j\omega)$

Feedback and Disto (cube)

- Equating third-order terms

$$b_3 s_i^3 = a_1(-fb_3 s_i^3) + a_2(-fb_2 2s_i^3) + a_3(s_i - fb_1 s_i)^3 + \cdots$$

$$b_3(1 + a_1 f) = -2a_2 b_2 f \frac{1}{1+T} + \frac{a_3}{(1+T)^3}$$

$$b_3(1 + T) = \frac{-2a_2 f}{1+T} \frac{a_2}{(1+T)^3} + \frac{a_3}{(1+T)^3}$$

$$b_3 = \frac{a_3(1 + a_1 f) - 2a_2^2 f}{(1 + a_1 f)^5}$$

Second Order Interaction

- The term $2a_2^2 f$ is the second order interaction
- Second order disto in fwd path is fed back and combined with the input linear terms to generate third order disto
- Can get a third order null if

$$a_3(1 + a_1 f) = 2a_2^2 f$$

HD_2 in Feedback Amp

$$HD_2 = \frac{1}{2}\frac{b_2}{b_1^2} s_{om}$$

$$= \frac{1}{2}\frac{a_2}{(1+T)^3}\frac{(1+T)^2}{a_1^2} s_{om}$$

$$= \frac{1}{2}\frac{a_2}{a_1^2}\frac{s_{om}}{1+T}$$

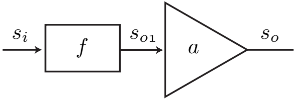
- Without feedback $HD_2 = \frac{1}{2}\frac{a_2}{a_1^2} s_{om}$
- For a *given output* signal, the negative feedback reduces the second order distortion by $\frac{1}{1+T}$

HD_3 in Feedback Amp

$$HD_3 = \frac{1}{4}\frac{b_3}{b_1^3}s_{om}^2$$

$$= \frac{1}{4}\frac{a_3(1+T) - 2a_2^2 f}{(1+T)^5}\frac{(1+T)^3}{a_1^3}s_{om}^2$$

$$= \underbrace{\frac{1}{4}\frac{a_3}{a_1^3}s_{om}^2}_{\text{disto with no fb}} \quad \frac{1}{(1+T)}\left[1 - \frac{2a_2^2 f}{a_3(1+T)}\right]$$



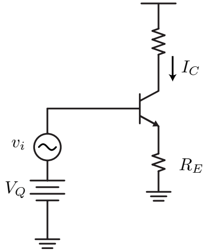
- Notice that the distortion is improved for a given output signal level. Otherwise we can see that simply decreasing the input signal level improves the distortion.
- Say $s_{o1} = fs_i$ with $f < 1$. Then

$$s_o = a_1 s_{o1} + a_2 s_{o1}^2 + a_3 s_{o1}^3 + \cdots = \underbrace{a_1 f}_{b_1} s_i + \underbrace{a_2 f^2}_{b_2} s_i^2 + \underbrace{a_3 f^3}_{b_3} s_i^3 + \cdots$$

- But the distortion is unchanged for a given output signal

$$HD_2 = \frac{1}{2}\frac{b_2}{b_1^2} s_{om} = \frac{1}{2}\frac{a_2}{a_1^2} s_{om}$$

BJT With Emitter Degeneration



The total input signal applied to the base of the amplifier is

$$v_i + V_Q = V_{BE} + I_E R_E$$

- The V_{BE} and I_E terms can be split into DC and AC currents (assume $\alpha \approx 1$)

$$v_i + V_Q = V_{BE,Q} + v_{be} + (I_Q + i_c)R_E$$

- Subtracting bias terms we have a separate AC and DC equation

$$V_Q = V_{BE,Q} + I_Q R_E$$

$$v_i = v_{be} + i_C R_E$$

Feedback Interpretation

- The AC equation can be put into the following form

$$v_{be} = v_i - i_c R_E$$

- Compare this to our feedback equation

$$s_\epsilon = s_i - f s_o$$

- The equations have the same form with the following substitutions

$$s_\epsilon = v_{be}$$

$$s_o = i_c$$

$$s_i = v_i$$

$$f = R_E$$

BJT with Emitter Degen (II)

- Now we know that

$$i_c = a_1 v_{be} + a_2 v_{be}^2 + a_3 v_{be}^3 + \cdots$$

- where the coefficients $a_{1,2,3,\ldots}$ come from expanding the exponential into a Taylor series

$$a_1 = g_m \quad a_2 = \frac{1}{2}\frac{I_Q}{V_t^2} \quad \cdots$$

- With feedback we have

$$i_c = b_1 v_i + b_2 v_i^2 + b_3 v_i^3 + \cdots$$

Emitter Degeneration (cont)

- The loop gain $T = a_1 f = g_m R_E$

$$b_1 = \frac{g_m}{1 + g_m R_E}$$

$$b_2 = \frac{\frac{1}{2}\left(\frac{q}{kT}\right)^2 I_Q}{(1 + g_m R_E)^3}$$

$$b_3 = \frac{g_m}{6(1 + g_m R_E)^4 \left(\frac{kT}{q}\right)^2}\left(1 - \frac{3 g_m R_E}{(1 + g_m R_E)}\right)$$

- For large loop gain $g_m R_E \to \infty$

$$b_1 = \frac{1}{R_E}$$

$$b_3 = \frac{1}{6}\frac{\left(\frac{q}{kT}\right)^2 g_m}{(1 + g_m R_E)^4}$$

Harmonic Distortion with Feedback

- Using our previously derived formulas we have

$$HD_2 = \frac{1}{2}\frac{b_2}{b_1^2}s_{om}$$

$$= \frac{1}{4}\frac{\hat{i}_c}{I_Q}\frac{1}{1+g_m R_E}$$

$$HD_3 = \frac{1}{4}\frac{b_3}{b_1^3}s_{om}^2$$

$$= \frac{1}{24}\left(\frac{\hat{i}_c}{I_Q}\right)^2 \frac{1-\frac{3g_m R_E}{1+g_m R_E}}{1+g_m R_E}$$

Harmonic Distortion Null

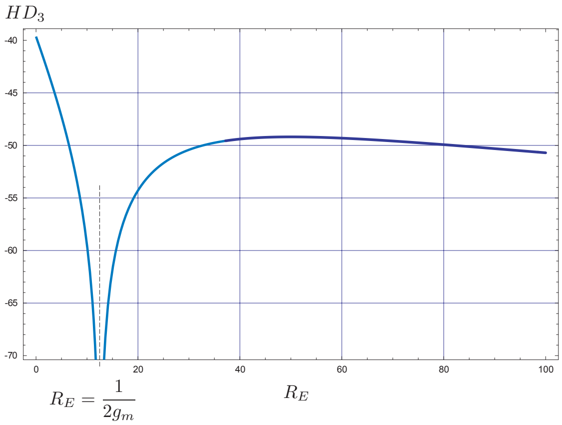
- We can adjust the feedback to obtain a null in HD_3
- $HD_3 = 0$ can be achieved with

$$\frac{3g_m R_E}{1 + g_m R_E} = 1$$

or

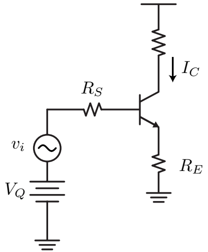
$$R_E = \frac{1}{2g_m}$$

HD_3 Null Example



- Example: For $I_Q = 1\text{mA}$, $R_E = 13\Omega$

BJT with Finite Source Resistance



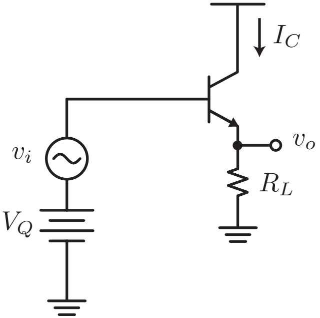
$$v_i + V_Q - I_B R_B = V_{BE} + I_E R_E$$

- Assuming that $\alpha \approx 1$, $\beta = \beta_0$ (constant). Let $R_B = R_S + r_b$ represent the total resistance at the base.

$$v_i + V_Q = V_{BE} + I_C \left(R_E + \frac{R_B}{\beta_0} \right)$$

- The formula is the same as the case of a BJT with emitter degeneration with $R'_E = R_E + R_B/\beta_0$

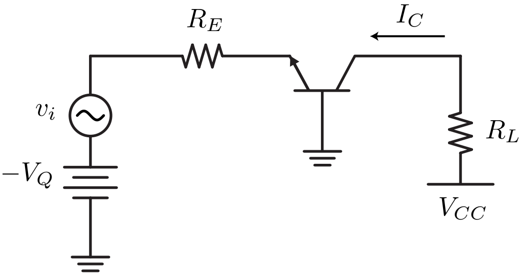
Emitter Follower



- The same equations as before with $R_E = R_L$

- In an cascade, the output stage often dominates the distortion (since it sees the largest signals). Make it as linear as possible. The inherent feedback in an emitter follower makes it very linear (for large loop gain, which may cost power).

Common Base



- Same equation as CE with R_E feedback

$$v_i - V_Q + I_C R_E = -V_{BE}$$

Calculation Tools: Multi-Tone Excitation

N Tones in One Shot

- Consider the effect of an m'th order non-linearity on an input of N tones

$$y_m = \left(\sum_{n=1}^{N} A_n \cos \omega_n t \right)^m$$

$$y_m = \left(\sum_{n=1}^{N} \frac{A_n}{2} \left(e^{\omega_n t} + e^{-\omega_n t} \right) \right)^m$$

$$y_m = \left(\sum_{n=-N}^{N} \frac{A_n}{2} e^{\omega_n t} \right)^m$$

- where we assumed that $A_0 \equiv 0$ and $\omega_{-k} = -\omega_k$.
-

Product of sums...

- The product of sums can be written as lots of sums...

$$= \underbrace{\sum () \times \sum () \times \sum () \cdots \times \sum ()}_{m-\text{times}}$$

$$= \sum_{k_1=-N}^{N} \sum_{k_2=-N}^{N} \cdots \sum_{k_m=-N}^{N} \frac{A_{k_1} A_{k_2} \cdots A_{k_m}}{2^m} \times e^{j(\omega_{k_1}+\omega_{k_2}+\cdots+\omega_{k_m})t}$$

- Notice that we generate frequency component $\omega_{k_1} + \omega_{k_2} + \cdots + \omega_{k_m}$, sums and differences between m non-distinct frequencies.
- There are a total of $(2N)^m$ terms.

Example

- Let's take a simple example of $m = 3$, $N = 2$. We already know that this cubic non-linearity will generate harmonic distortion and IM products.
- We have $(2N)^m = 4^3 = 64$ combinations of complex frequencies. $\omega \in \{-\omega_2, -\omega_1, \omega_1, \omega_2\}$. There are 64 terms that looks like this (HD_3)

$$\omega_1 + \omega_1 + \omega_1 = 3\omega_1$$

$$\omega_1 + \omega_1 + \omega_2 = 2\omega_1 + \omega_2$$

($IM3$)

$$\omega_1 + \omega_1 - \omega_2 = 2\omega_1 - \omega_2$$

(Gain compression or expansion)

$$\omega_1 + \omega_1 - \omega_1 = \omega_1$$

Frequency Mix Vector

- Let the vector $\vec{k} = (k_{-N}, \cdots, k_{-1}, k_1, \cdots, k_N)$ be a $2N$-vector where element k_j denotes the number of times a particular frequency appears in a given term.
- As an example, consider the frequency terms

$$\left.\begin{array}{c} \omega_2 + \omega_1 + \omega_2 \\ \omega_1 + \omega_2 + \omega_2 \\ \omega_2 + \omega_2 + \omega_1 \end{array}\right\} \vec{k} = (0, 0, 1, 2)$$

Properties of $\vec{k}$

- First it's clear that the sum of the k_j must equal m

$$\sum_{j=-N}^{N} k_j = k_{-N} + \cdots + k_{-1} + k_1 + \cdots + k_N = m$$

- For a fixed vector $\vec{k_0}$, how many different sum vectors are there?
- We can sum m frequencies $m!$ ways. But the order of the sum is irrelevant. Since each k_j coefficient can be ordered $k_j!$ ways, the number of ways to form a given frequency product is given by

$$(m;\vec{k}) = \frac{m!}{(k_{-N})!\cdots(k_{-1})!(k_1)!\cdots(k_N)!}$$

Extraction of Real Signal

- Since our signal is real, each term has a complex conjugate present. Hence there is another vector $\vec{k_0'}$ given by

$$\vec{k_0'} = (k_N, \cdots, k_1, k_{-1}, \cdots, k_{-N})$$

- Notice that the components are in reverse order since $\omega_{-j} = -\omega_j$. If we take the sum of these two terms we have

$$2\Re\left\{e^{j(\omega_{k_1}+\omega_{k_2}+\cdots+\omega_{k_m})t}\right\} = 2\cos(\omega_{k_1}+\omega_{k_2}+\cdots+\omega_{k_m})t$$

- The amplitude of a frequency product is thus given by

$$\frac{2 \times (m; \vec{k})}{2^m} = \frac{(m; \vec{k})}{2^{m-1}}$$

Example: IM_3 Again

- Using this new tool, let's derive an equation for the IM_3 product more directly.
- Since we have two tones, $N = 2$. IM_3 is generated by a $m = 3$ non-linear term.
- A particular IM_3 product, such as $(2\omega_1 - \omega_2)$, is generated by the frequency mix vector $\vec{k} = (1, 0, 2, 0)$.

$$(m; \vec{k}) = \frac{3!}{1! \cdot 2!} = 3 \qquad\qquad 2^{m-1} = 2^2 = 4$$

- So the amplitude of the IM_3 product is $3/4 a_3 s_i^3$. Relative to the fundamental

$$IM_3 = \frac{3}{4} \frac{a_3 s_i^3}{a_1 s_i} = \frac{3}{4} \frac{a_3}{a_1} s_i^2$$

Harder Example: Pentic Non-Linearity

- Let's calculate the gain expansion/compression due to the 5th order non-linearity. For a one tone, we have $N = 1$ and $m = 5$.
- A pentic term generates fundamental as follows

$$\omega_1 + \omega_1 + \omega_1 - \omega_1 - \omega_1 = \omega_1$$

- In terms of the $\vec{k}$ vector, this is captured by $\vec{k} = (2, 3)$. The amplitude of this term is given by $$(m; \vec{k}) = \frac{5!}{2! \cdot 3!} = \frac{5 \cdot 4}{2} = 10 \qquad 2^{m-1} = 2^4 = 16$$
- So the fundamental amplitude generated is $a_5 \frac{10}{16} S_i^5$.

Apparent Gain Due to Pentic

- The apparent gain of the system, including the 3rd and 5th, is thus given by

$$\mathrm{AppGain} = a_1 + \frac{3}{4} a_3 S_i^2 + \frac{10}{16} a_5 S_i^4$$

- At what signal level is the 5th order term as large as the 3rd order term?

$$\frac{3}{4} a_3 S_i^2 = \frac{10}{16} a_5 S_i^4 \qquad\qquad S_i = \sqrt{1.2 \frac{a_3}{a_5}}$$

- For a bipolar amplifier, we found that $a_3 = 1/(3! V_t^3)$ and $a_5 = 1/(5! V_t^5)$. Solving for S_i, we have

$$S_i = V_t \sqrt{1.2 \times 5 \times 4} \approx 127\mathrm{mV}$$