Integrated Circuits for Communication

Berkeley

Distortion Metrics

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Gain Compression

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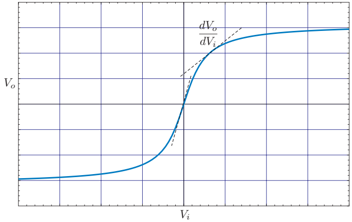
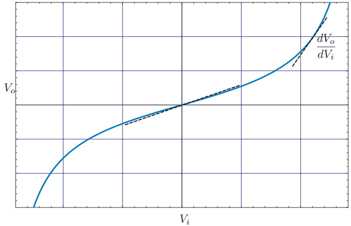
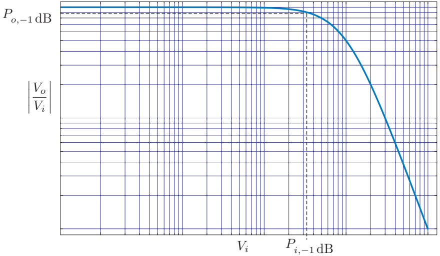




- The large signal input/output relation can display gain compression or expansion. Physically, most amplifier experience gain compression for large signals.
- The small-signal gain is related to the slope at a given point. For the graph on the left, the gain decreases for increasing amplitude.

$1\,\mathrm{dB}$ Compression Point



- Gain compression occurs because eventually the output signal (voltage, current, power) limits, due to the supply voltage or bias current.
- If we plot the gain (log scale) as a function of the input power, we identify the point where the gain has dropped by $1\,\mathrm{dB}$. This is the $1\,\mathrm{dB}$ compression point. It's a very important number to keep in mind.

Apparent Gain

- Recall that around a small deviation, the large signal curve is described by a polynomial

$$s_o = a_1 s_i + a_2 s_i^2 + a_3 s_i^3 + \cdots$$

- For an input $s_i = S_1 \cos(\omega_1 t)$, the cubic term generates

$$S_1^3 \cos^3(\omega_1 t) = S_1^3 \cos(\omega_1 t) \frac{1}{2} (1 + \cos(2\omega_1 t))$$

$$= S_1^3 \left(\frac{1}{2} \cos(\omega_1 t) + \frac{2}{4} \cos(\omega_1 t) \cos(2\omega_1 t) \right)$$

- Recall that $2 \cos a \cos b = \cos(a+b) + \cos(a-b)$

$$= S_1^3 \left(\frac{1}{2} \cos(\omega_1 t) + \frac{1}{4} (\cos(\omega_1 t) + \cos(3\omega_1 t)) \right)$$

Apparent Gain (cont)

- Collecting terms

$$= S_1^3 \left(\frac{3}{4} \cos(\omega_1 t) + \frac{1}{4} \cos(3\omega_1 t) \right)$$

- The apparent gain of the system is therefor

$$G = \frac{S_{o,\omega_1}}{S_{i,\omega_1}} = \frac{a_1 S_1 + \frac{3}{4} a_3 S_1^3}{S_1}$$

$$= a_1 + \frac{3}{4} a_3 S_1^2 = a_1 \left(1 + \frac{3}{4} \frac{a_3}{a_1} S_1^2 \right) = G(S_1)$$

- If $a_3/a_1 < 0$, the gain compresses with increasing amplitude.

1-dB Compression Point

- Let's find the input level where the gain has dropped by $1\,\mathrm{dB}$

$$20\log\left(1+\frac{3}{4}\frac{a_3}{a_1}S_1^2\right)=-1\,\mathrm{dB}$$

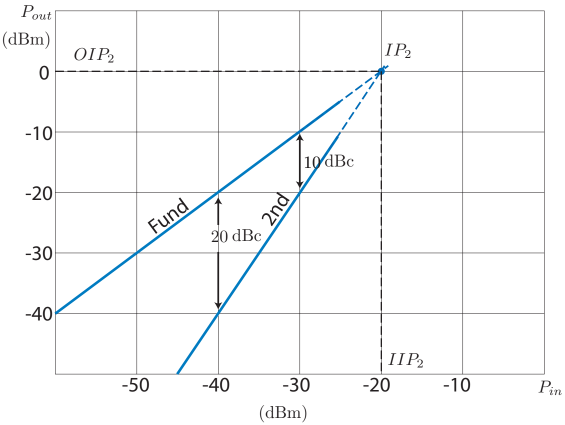
$$\frac{3}{4}\frac{a_3}{a_1}S_1^2=-0.11$$

$$S_1=\sqrt{\frac{4}{3}\left|\frac{a_1}{a_3}\right|}\times\sqrt{0.11}=IIP3-9.6\,\mathrm{dB}$$

- The term in the square root is called the third-order intercept point (see next few slides).

Intermodulation Distortion Intercept Points

Intercept Point IP_2

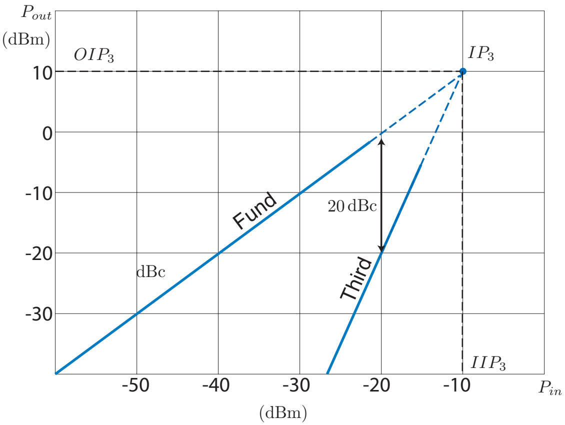


- The extrapolated point where $IM_2 = 0\,\mathrm{dBc}$ is known as the second order intercept point IP_2.

Properties of Intercept Point IP_2

- Since the second order IM distortion products increase like s_i^2, we expect that at some power level the distortion products will overtake the fundamental signal.
- The extrapolated point where the curves of the fundamental signal and second order distortion product signal meet is the Intercept Point (IP_2).
- At this point, then, by definition $IM_2 = 0\,\mathrm{dBc}$.
- The input power level is known as IIP_2, and the output power when this occurs is the OIP_2 point.
- Once the IP_2 point is known, the IM_2 at any other power level can be calculated. Note that for a dB back-off from the IP_2 point, the IM_2 improves dB for dB

Intercept Point IP_3



- The extrapolated point where $IM_3 = 0\,\mathrm{dBc}$ is known as the third-order intercept point IP_3.

Properties of Intercept Point IP_3

- Since the third order IM distortion products increase like s_i^3, we expect that at some power level the distortion products will overtake the fundamental signal.
- The extrapolated point where the curves of the fundamental signal and third order distortion product signal meet is the Intercept Point (IP_3).
- At this point, then, by definition $IM_3 = 0\,\mathrm{dBc}$.
- The input power level is known as IIP_3, and the output power when this occurs is the OIP_3 point.
- Once the IP_3 point is known, the IM_3 at any other power level can be calculated. Note that for a $10\,\mathrm{dB}$ back-off from the IP_3 point, the IM_3 improves $20\,\mathrm{dB}$.

Intercept Point Example

- From the previous graph we see that our amplifier has an $IIP_3 = -10\,\mathrm{dBm}$.
- What's the IM_3 for an input power of $P_{in} = -20\,\mathrm{dBm}$?
- Since the IM_3 improves by $20\,\mathrm{dB}$ for every $10\,\mathrm{dB}$ back-off, it's clear that $IM_3 = 20\,\mathrm{dBc}$
- What's the IM_3 for an input power of $P_{in} = -110\,\mathrm{dBm}$?
- Since the IM_3 improves by $20\,\mathrm{dB}$ for every $10\,\mathrm{dB}$ back-off, it's clear that $IM_3 = 200\,\mathrm{dBc}$

Calculated $IIP2/IIP3$

- We can also calculate the IIP points directly from our power series expansion. By definition, the $IIP2$ point occurs when

$$IM_2 = 1 = \frac{a_2}{a_1} S_i$$

- Solving for the input signal level

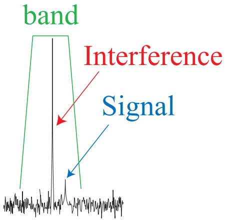
$$IIP_2 = S_i = \frac{a_1}{a_2}$$

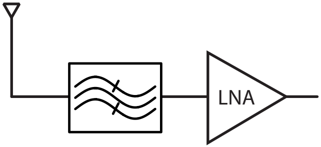
- In a like manner, we can calculate IIP_3

$$IM_3 = 1 = \frac{3}{4}\frac{a_3}{a_1} S_i^2 \qquad IIP_3 = S_i = \sqrt{\frac{4}{3}\left|\frac{a_1}{a_3}\right|}$$

Blockers or Jammers

Blocker or Jammer





- Consider the input spectrum of a weak desired signal and a "blocker"

$$S_i = \underbrace{S_1 \cos \omega_1 t}_{\text{Blocker}} + \underbrace{s_2 \cos \omega_2 t}_{\text{Desired}}$$

- We shall show that in the presence of a strong interferer, the gain of the system for the desired signal is reduced. This is true even if the interference signal is at a substantially different frequency. We call this interference signal a "jammer".

Blocker (II)

- Obviously, the linear terms do not create any kind of desensitization. The second order terms, likewise, generate second harmonic and intermodulation, but not any fundamental signals.
- In particular, the cubic term $a_3 S_i^3$ generates the jammer desensitization term

$$S_i^3 = S_1^3 \cos^3 \omega_1 t + s_2^3 \cos^3 \omega_2 t + 3S_1^2 s_2 \cos^2 \omega_1 t \cos \omega_2 t +$$

$$3S_1 s_2^2 \cos^2 \omega_2 t \cos \omega_1 t$$

- The first two terms generate cubic and third harmonic.
- The last two terms generate fundamental signals at ω_1 and ω_2. The last term is much smaller, though, since $s_2 \ll S_1$.

Blocker (III)

- The blocker term is therefore given by

$$a_3 3 S_1^2 s_2 \frac{1}{2} \cos \omega_2 t$$

- This term adds or *subtracts* from the desired signal. Since $a_3 < 0$ for most systems (compressive non-linearity), the effect of the blocker is to reduce the gain

$$\text{App Gain} = \frac{a_1 s_2 + a_3 \frac{3}{2} S_1^2 s_2}{s_2}$$

$$= a_1 + a_3 \frac{3}{2} S_1^2 = a_1 \left(1 + \frac{3}{2} \frac{a_3}{a_1} S_1^2 \right)$$

Out of Band 3 dB Desensitization

- Let's find the blocker power necessary to desensitize the amplifier by 3 dB. Solving the above equation

$$20 \log \left(1 + \frac{3}{2}\frac{a_3}{a_1} S_1^2\right) = -3\,\mathrm{dB}$$

- We find that the blocker power is given by

$$P_{OB} = P_{-1\,\mathrm{dB}} + 1.2\,\mathrm{dB}$$

- It's now clear that we should avoid operating our amplifier with any signals in the vicinity of $P_{-1\,\mathrm{dB}}$, since gain reduction occurs if the signals are larger. At this signal level there is also considerable intermodulation distortion.

Distortion of AM Signals

- Consider a simple AM signal (modulated by a single tone)

$$s(t) = S_2(1 + m\cos\omega_m t)\cos\omega_2 t$$

- where the modulation index $m \leq 1$. This can be written as

$$s(t) = S_2\cos\omega_2 t + \frac{m}{2}\cos(\omega_2 - \omega_m)t + \frac{m}{2}\cos(\omega_2 + \omega_m)t$$

- The first term is the RF carrier and the last terms are the modulation sidebands
- Note that the AM modulation can be analog or digital. In a digital case, the actual modulation is likely to be *complex* so that the two sidebands are no longer symmetric, but the analysis that follows still applies.

Cross Modulation

- Cross modulation occurs in AM systems (e.g. video cable tuners, QAM digital modulation)
- The modulation of a large AM signal transfers to another carrier going through the same amplifier (or non-linear system)

$$S_i = \underbrace{S_1 \cos \omega_1 t}_{\text{wanted}} + \underbrace{S_2 (1 + m \cos \omega_m t) \cos \omega_2 t}_{\text{interferer}}$$

- CM occurs when the output contains a term like

$$K (1 + \delta \cos \omega_m t) \cos \omega_1 t$$

- Where δ is called the transferred modulation index

Cross Modulation (cont)

- For $S_o = a_1 S_i + a_2 S_i^2 + a_3 S_i^3 + \cdots$, the term $a_2 S_i^2$ does not produce any CM
- The term $a_3 S_i^3 = \cdots + 3a_3 S_1 \cos\omega_1 t\, (S_2(1 + m\cos\omega_m t)\cos\omega_2 t)^2$ is expanded to

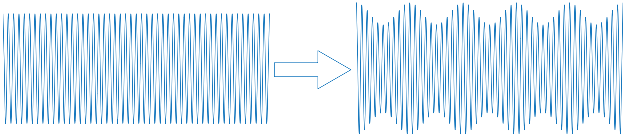
$$= \cdots + 3a_3 S_1 S_2^2 \cos\omega_1 t(1 + 2m\cos\omega_m t + m^2\cos^2\omega_m t)\times$$

$$\frac{1}{2}(1 + \cos 2\omega_2 t)$$

- Grouping terms we have in the output

$$S_o = \cdots + a_1 S_1(1 + 3\frac{a_3}{a_1} S_2^2 m\cos\omega_m t)\cos\omega_1 t$$

CM Definition



unmodulated waveform (input)   modulated waveform due to CM

$$CM = \frac{\text{Transferred Modulation Index}}{\text{Incoming Modulation Index}}$$

$$CM = 3\frac{a_3}{a_1}S_2^2 = 4IM_3$$

$$= IM_3(\text{dB}) + 12\text{dB}$$

$$= 12HD_3 = HD_3(\text{dB}) + 22\text{dB}$$

Calculation Tools

Series Inversion

- Sometimes it's easier to find a power series relation for the input in terms of the output. In other words

$$S_i = a_1 S_o + a_2 S_o^2 + a_3 S_o^3 + \cdots$$

- But we desire the inverse relation

$$S_o = b_1 S_i + b_2 S_i^2 + b_3 S_i^3 + \cdots$$

- To find the inverse relation, we can substitute the above equation into the original equation and equate coefficient of like powers.

$$S_i = a_1(b_1 S_i + b_2 S_i^2 + b_3 S_i^3 + \cdots) + a_2(\ )^2 + a_3(\ )^3 + \cdots$$

Inversion (cont)

- Equating linear terms, we find, as expected, that $a_1 b_1 = 1$, or $b_1 = 1/a_1$.
- Equating the square terms, we have

$$0 = a_1 b_2 + a_2 b_1^2$$

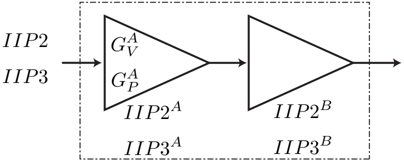
$$b_2 = -\frac{a_2 b_1^2}{a_1} = -\frac{a_2}{a_1^3}$$

- Finally, equating the cubic terms we have

$$0 = a_1 b_3 + a_2 2 b_1 b_2 + a_3 b_1^3 \qquad b_3 = \frac{2a_2^2}{a_1^5} - \frac{a_3}{a_1^4}$$

- It's interesting to note that if one power series does not have cubic, $a_3 \equiv 0$, the inverse series has cubic due to the first term above.

Cascade



- Another common situation is that we cascade two non-linear systems, as shown above. we have

$$y = f(x) = a_1 x + a_2 x^2 + a_3 x^3 + \cdots$$

$$z = g(y) = b_1 y + b_2 y^2 + b_3 y^3 + \cdots$$

- We'd like to find the overall relation

$$z = c_1 x + c_2 x^2 + c_3 x^3 + \cdots$$

Cascade Power Series

- To find $c_1, c_2, \cdots$, we simply substitute one power series into the other and collect like powers.
- The linear terms, as expected, are given by

$$c_1 = b_1 a_1 = a_1 b_1$$

- The square terms are given by

$$c_2 = b_1 a_2 + b_2 a_1^2$$

- The first term is simply the second order distortion produced by the first amplifier and amplified by the second amplifier linear term. The second term is the generation of second order by the second amplifier.

Cascade Cubic

- Finally, the cubic terms are given by

$$c_3 = b_1 a_3 + b_2 2 a_1 a_2 + b_3 a_1^3$$

- The first and last term have a very clear origin. The middle terms, though, are more interesting. They arise due to second harmonic interaction. The second order distortion of the first amplifier can interact with the linear term through the second order non-linearity to produce cubic distortion.
- Even if both amplifiers have negligible cubic, $a_3 = b_3 \equiv 0$, we see the overall amplifier can generate cubic through this mechanism.

Cascade Example

- In the above amplifier, we can decompose the non-linearity as a cascade of two non-linearities, the G_m non-linearity

$$i_d = G_{m1} v_{in} + G_{m2} v_{in}^2 + G_{m3} v_{in}^3 + \cdots$$

- And the output impedance non-linearity

$$v_o = R_1 i_d + R_2 i_d^2 + R_3 i_d^3 + \cdots$$

- The output impedance can be a non-linear resistor load (such as a current mirror) or simply the load of the device itself, which has a non-linear component.

$IIP2$ Cascade

- Commonly we'd like to know the performance of a cascade in terms of the overall $IIP2$. To do this, note that $IIP2 = c_1/c_2$

$$\frac{c_2}{c_1} = \frac{b_1 a_2 + b_2 a_1^2}{b_1 a_1} = \frac{a_2}{a_1} + \frac{b_2}{b_1} a_1$$

- This leads to

$$\frac{1}{V_{IIP2}} = \frac{1}{V_{IIP2^A}} + \frac{a_1}{V_{IIP2^B}}$$

- This is a very intuitive result, since it simply says that we can *input refer* the V_{IIP2} of the second amplifier to the input by the voltage gain of the first amplifier.

$IIP2$ Cascade Example

- Example 1: Suppose the input amplifiers of a cascade has $IIP2^A = +0\,\mathrm{dBm}$ and a voltage gain of $20\,\mathrm{dB}$. The second amplifier has $IIP2^B = +10\,\mathrm{dBm}$.
- The input referred $IIP2_i^B = 10\,\mathrm{dBm} - 20\,\mathrm{dB} = -10\,\mathrm{dBm}$
- This is a much smaller signal than the $IIP2^A$, so clearly the second amplifier dominates the distortion. The overall distortion is given by $IIP2 \approx -12\,\mathrm{dBm}$.

- Example 2: Now suppose $IIP2^B = +20\,\mathrm{dBm}$. Since $IIP2_i^B = 20\,\mathrm{dBm} - 20\,\mathrm{dB} = 0\,\mathrm{dBm}$, we cannot assume that either amplifier dominates.
- Using the formula, we see the actual $IIP2$ of the cascade is a factor of 2 down, $IIP2 = -6\,\mathrm{dBm}$.

IIP3 Cascade

- Using the same approach, let's start with

$$\frac{c_3}{c_1} = \frac{b_1 a_3 + b_2 a_1 a_2 2 + b_3 a_1^3}{b_1 a_1} = \left(\frac{a_3}{a_1} + \frac{b_3}{b_1} a_1^2 + \frac{b_2}{b_1} 2 a_2 \right)$$

- The last term, the second harmonic interaction term, will be neglected for simplicity. Then we have

$$\frac{1}{V_{IIP3}^2} = \frac{1}{V_{IIP3_A}^2} + \frac{a_1^2}{V_{IIP3_B}^2}$$

- Which shows that the V_{IIP3} of the second amplifier is input referred by the voltage gain squared, or the power gain.

LNA/Mixer Example

- A common situation is an LNA and mixer cascade. The mixer can be characterized as a non-linear block with a given $IIP2$ and $IIP3$.
- In the above example, the LNA has an $IIP3^A = -10\,\mathrm{dBm}$ and a power gain of $20\,\mathrm{dB}$. The mixer has an $IIP3^B = -20\,\mathrm{dBm}$.
- If we input refer the mixer, we have $IIP3_i^B = -20\,\mathrm{dBm} - 20\,\mathrm{dB} = -40\,\mathrm{dBm}$.
- The mixer will dominate the overall $IIP3$ of the system.

Frequency Domain Convolution

Multiplication to Convolution

- Calculation of power of sinusoids can be viewed from a different perspective, one that will become fruitful when we examine the interaction between cascades or feedback. Let's consider a cosine wave $\cos(\omega_c t)$. In the frequency domain, it consist of two delta functions

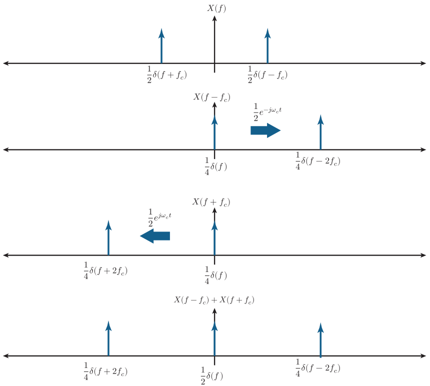
$$\mathcal{F}(\cos(\omega_c t)) = X(f) = \frac{1}{2}\delta(f - f_c) + \frac{1}{2}\delta(f + f_c)$$

- Since multiplication in time is the same as convolution in the frequency domain, we can perform the calculation directly in the frequency domain. For example, if we multiply a spectrum by a complex exponential at frequency f_c, we can view it as convolution with a delta function :

$$Z(f) = \int_{-\infty}^{\infty} X(f)\delta(\sigma - f_c)d\sigma = X(f - f_c)$$

- This shifts the spectrum down by the frequency of the carrier complex exponential. So multiplication a cosine results in both a shift up and shift down by the carrier frequency since it has both positive and negative frequency components.

Cosine Squared Visulaized



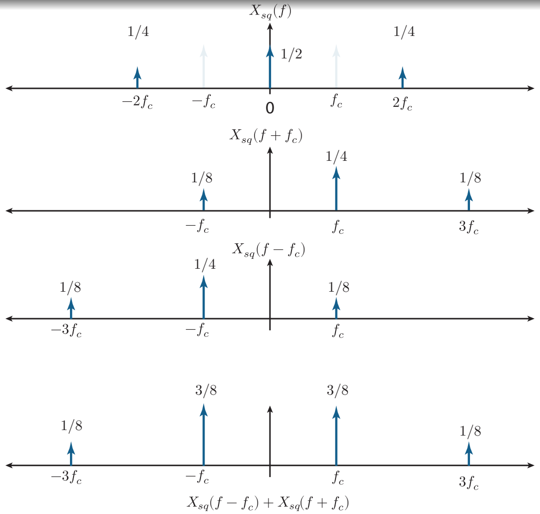
$$\mathcal{F}\left(\cos^2(\omega_c t)\right) = X(f) * X(f) =$$

$$= \frac{1}{4}\delta(f+f_c-f_c)+\frac{1}{2}\delta(f-f_c-f_c)+\frac{1}{4}\delta(f+f_c+f_c)+\frac{1}{2}\delta(f-f_c+f_c)$$

$$= \frac{1}{2}\delta(f) + \frac{1}{4}\delta(f+2f_c) + \frac{1}{4}\delta(f-2f_c) \leftrightarrow \frac{1}{2}\cos(2\omega_c t) + \frac{1}{2}$$

- Applying this result to a cosine, when we multiply a cosine by itself, we should therefore shift the spectrum up by f_c and down by f_c, since a cosine has two delta functions in frequency. This places the output signal at DC (shift down) and at twice the frequency:

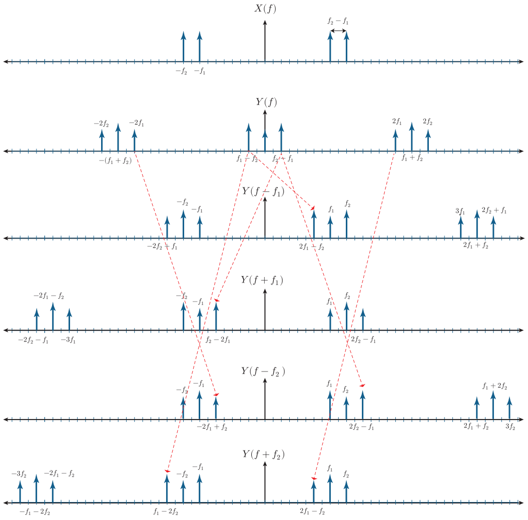
Cosine Cubed



- It's now easy to see that if we start with the cosine squared spectrum, we can shift it up and down to generate cosine cubed.
- As expected, we regenerate fundamental and third harmonic. Note the fundamental is generated from the DC generated from the first stage.
- We can also keep track of the amplitude by noting that every convolution multiplies by a factor of 1/2 due to the cosine Euler expansion.

$$cos^3(\omega_c t) = \frac{1}{4}\cos(\omega_c t) + \frac{3}{4}\cos(3\omega_c t)$$

Two-Tone Intermodulation and Second-Order Interaction



- Beginning with two tones (top), we now can quickly visualize the output of the second-order stage, which produces IM_2 distortion (second graph).
- Now we shift up and down by each tone f_1 and f_2 in the next four graphs. Each one represents the product of the second-order generated spectrum with the original two-tones, which occurs in both cubic and in second-order interactions.
- We can now clearly see that the origin of the IM_3 is indeed from IM_2 mixed with fundamental. This is why filtering out low-frequency and high frequency products eliminates IM_3 due to second-order interaction in a cascade.