Integrated Circuits for Communication

Berkeley

Physical Origin of Electrical Noise

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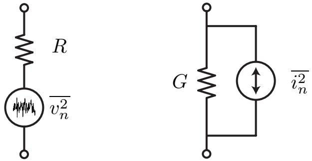
Thermal Noise of a Resistor



- All resistors generate noise. The noise power generated by a resistor R can be represented by a series voltage source with mean square value $\overline{v_n^2}$

$$\overline{v_n^2} = 4k_B TRB$$

- Equivalently, we can represent this with a current source in shunt

$$\overline{i_n^2} = 4k_B TGB$$

Resistor Noise Example

- Here B is the bandwidth of observation and kT is Boltzmann's constant times the temperature of observation
- This result comes from thermodynamic considerations, thus explaining the appearance of kT
- Often we speak of the "spot noise", or the noise in a specific narrowband δf

$$\overline{v_n^2} = 4k_B TR\delta f$$

- Since the noise is white, the shape of the noise spectrum is determined by the external elements (L's and C's)

Resistor Noise Example

- Suppose that $R = 10\mathrm{k}\Omega$ and $T = 20°\mathrm{C} = 293\mathrm{K}$.

$$4k_B T = 1.62 \times 10^{-20}$$

$$\overline{v_n^2} = 1.62 \times 10^{-16} \times B$$

$$v_{n,rms} = \sqrt{\overline{v_n(t)^2}} = 1.27 \times 10^{-8}\sqrt{B}$$

- If we limit the bandwidth of observation to $B = 10\mathrm{MHz}$, then we have

$$v_{n,rms} \approx 13\mu\mathrm{V}$$

- This represents the limit for the smallest voltage we can resolve across this resistor in this bandwidth

Combination of Resistors

- If we put two resistors in series, then the mean square noise voltage is given by

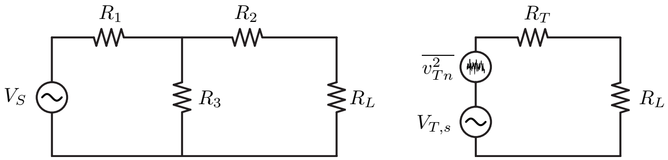
$$\overline{v_n^2} = 4k_B T(R_1 + R_2)B = \overline{v_{n1}^2} + \overline{v_{n2}^2}$$

- The noise powers add, *not* the noise voltages
- Likewise, for two resistors in parallel, we can add the mean square currents

$$\overline{i_n^2} = 4k_B T(G_1 + G_2)B = \overline{i_{n1}^2} + \overline{i_{n2}^2}$$

- This holds for any pair of independent noise sources (zero correlation)

Resistive Circuits

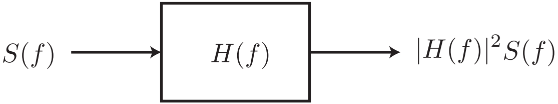


- For an arbitrary resistive circuit, we can find the equivalent noise by using a Thevenin (Norton) equivalent circuit or by transforming all noise sources to the output by the appropriate *power* gain (e.g. voltage squared or current squared)

$$V_{T,s} = V_S \frac{R_3}{R_1 + R_3}$$

$$\overline{v_{Tn}^2} = 4k_B T R_T B = 4k_B T(R_2 + R_1||R_3)B$$

Noise in an LTI System

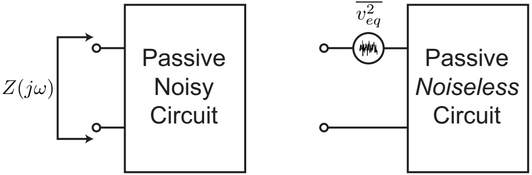


- A fundamental result from Stochastic Systems is that if you inject noise into an LTI system (such as a filter), the output noise is shaped by the magnitude of the transfer function

$$\overline{V}^2 = \int_{-\infty}^{\infty} S(f)|H(f)|^2 df$$

- Note that we can't say anything about the phase, but we know the magnitude response will be filtered.
- Any white noise source, such as a resistor, will be shaped by poles in the system.

Noise for Passive Circuits (I)



- For a general linear circuit, the mean square noise voltage (current) at any port is given by the equivalent input resistance (conductance)

$$\overline{v_{eq}^2} = 4k_B T \Re(Z(f))\delta f$$

Noise for Passive Circuits (II)

- This is the "spot" noise. If the network has a filtering property, then we integrate over the band of interest

$$\overline{v_{T,eq}^2} = 4k_B T \int_B \Re(Z(f))df$$

- Unlike resistors, L's and C's *do not* generate noise. They do shape the noise due to their frequency dependence.

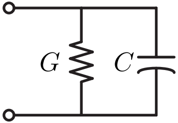
Noise Analysis

- To find the equivalent mean square noise voltage for a circuit, we use the small signal model (noise signals are actually small, so it's a good approximation). For each noise source, invoke superposition and calculate the noise contribution to the desired node

$$\overline{v_{n,o}^2} = |G_{1,o}|^2 \overline{v_{n,1}^2} + |G_{2,o}|^2 \overline{v_{n,2}^2} + \cdots = \sum_k |G_{k,o}|^2 \overline{v_{n,k}^2}$$

- where $\overline{v_{n,k}^2}$ is the kth noise source, and the gain from that noise to the output node is given by $G_{k,o}$. Note that the polarity of the noise sources is irrelevant since we're summing powers (all positive quantities). The above expression assumes that the noise sources are *independent*. Later on we'll see how to handle correlated noise sources.

Example: Noise of an RC Circuit



To find the equivalent mean square noise voltage of an RC circuit, begin by calculating the impedance

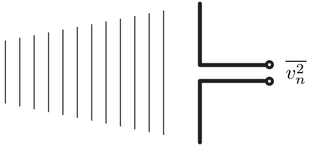
$$Z = \frac{1}{Y} = \frac{1}{G + j\omega C} = \frac{G - j\omega C}{G^2 + \omega^2 C^2}$$

- Integrating the noise over all frequencies, we have

$$\overline{v_n^2} = \frac{4 k_B T}{2\pi} \int_0^\infty \frac{G}{G^2 + \omega^2 C^2} d\omega = \frac{k_B T}{C}$$

- Notice the result is *independent* of R. Since the noise and BW is proportional/inversely proportional to R, its influence cancels out

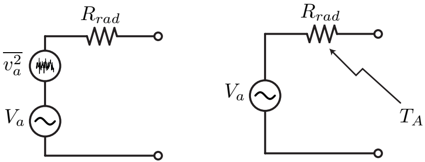
Noise of a Receiving Antenna



- Assume we construct an antenna with ideal conductors so $R_{wire} = 0$
- If we connect the antenna to a spectrum analyzer, though, we will observe noise
- The noise is also "white" but the magnitude depends on where we point our antenna (sky versus ground)

Equivalent Antenna Temperature

$$\overline{v_a^2} = 4 k_B T_A R_{rad} B$$



- T_A is the equivalent antenna temperature and R_{rad} is the radiation resistance of the antenna
- Since the antenna does not generate any of its own thermal noise, the observed noise must be incident on the antenna. In fact, it's "black body" radiation.
- Physically T_A is related to the temperature of the external bodies radiating into space (e.g. space or the ground)

Diode Shot Noise

- A forward biased diode exhibits noise called *shot noise*. This noise arises due to the quantized nature of charge.
- The noise mean square current is given by

$$\overline{i_{d,n}^2} = 2qI_{DC}B$$

- The noise is white and proportional to the DC current I_{DC}
- Reversed biased diodes exhibit excess noise not related to shot noise.

Noise in a BJT

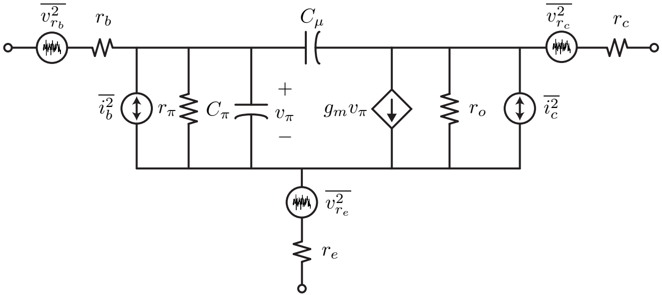
- All physical resistors in a BJT produce noise (r_b, r_e, r_c). The output resistance r_o, though, is *not* a physical resistor. Likewise, r_π, is not a physical resistor. Thus these resistances do not generate noise
- The junctions of a BJT exhibit shot noise

$$\overline{i_{b,n}^2} = 2qI_B B$$

$$\overline{i_{c,n}^2} = 2qI_C B$$

- At low frequencies the transistor exhibits "Flicker Noise" or $1/f$ Noise.

BJT Hybrid-Π Model



- The above equivalent circuit includes noise sources. Note that a small-signal equivalent circuit is appropriate because the noise perturbation is very small

FET Noise

- In addition to the extrinsic physical resistances in a FET (r_g, r_s, r_d), the channel resistance also contributes thermal noise
- The drain current noise of the FET is therefore given by

$$\overline{i_{d,n}^2} = 4k_B T\gamma g_{ds0}\delta f + K\frac{I_D^a}{C_{ox}L_{eff}^2 f^e}\delta f$$

- The first term is the thermal noise due to the channel resistance and the second term is the "Flicker Noise", also called the $1/f$ noise, which dominates at low frequencies.
- The factor $\gamma = \frac{2}{3}$ for a long channel device.
- The constants K, a, and e are usually determined empirically.

FET Channel Resistance

- Consider a FET with $V_{DS} = 0$. Then the channel conductance is given by

$$g_{ds,0} = \frac{\partial I_{DS}}{\partial V_{DS}} = \mu C_{ox} \frac{W}{L}(V_{GS} - V_T)$$

- For a long-channel device, this is also equal to the device transconductance g_m in saturation

$$g_m = \frac{\partial I_{DS}}{\partial V_{GS}} = \mu C_{ox} \frac{W}{L}(V_{GS} - V_T)$$

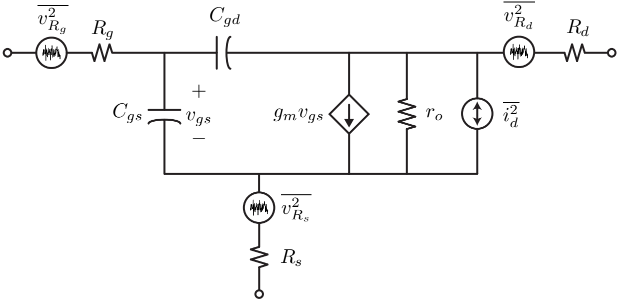
- For short-channel devices, this relation is not true, but we can define

$$\alpha = \frac{g_m}{g_{d0}} \neq 1$$

- With this definition of α, we can write the drain noise in terms of g_m:

$$\overline{i_{d,n}^2} = 4k_B T \gamma \frac{g_m}{\alpha} \delta f + K \frac{I_D^a}{C_{ox} L_{eff}^2 f^e} \delta f$$

FET Noise Equivalent Circuit



- The resistance of the substrate also generates thermal noise. In most circuits we will be concerned with the noise due to the channel $\overline{i_d^2}$ and the input gate noise $\overline{v_{R_g}^2}$

Physical Origin of Noise

- An elegant derivation of the physical origin of the noise of a resistor is due to van der Ziel. Consider an *RC* circuit where as a result of thermal agitation of electrons, the capacitor is charged and discharged constantly. On average, the energy stored is given by the equipartition theorem:

$$\frac{1}{2}C\overline{V^2} = \frac{1}{2}k_B T$$

$$\overline{V^2} = \frac{k_B T}{C}$$

- This result can be derived rigorously assuming a Boltzmann distribution for the energy (see later slides).

Noise Voltage Due to Resistor

- Let's say that we don't know the power spectral density of the noise of the resistor. Whatever it's noise is, though, we know that noise voltage at the capacitor can be computed from

$$\overline{V}^2 = \int_{-\infty}^{\infty} S_V^2(\omega)|H(\omega)|^2 d\omega$$

- where $H(\omega)$ is the transfer function from the resistor noise to the capacitor

$$H(\omega) = \frac{1}{1 + j\omega RC}$$

$$|H(\omega)|^2 = \frac{1}{1 + \omega^2(RC)^2}$$

Noise Bandwidth

- Integrating the noise we have

$$\overline{V}^2 = \int_{-\infty}^{\infty} S_V^2(\omega) \frac{1}{1+\omega^2(RC)^2} d\omega$$

- If we assume that the voltage noise density does not depend on frequency (experimental fact), then we have

$$\overline{V}^2 = \overline{S}_V^2 \int_{-\infty}^{\infty} \frac{1}{1+\omega^2(RC)^2} d\omega = \frac{S_V^2}{2RC}$$

- Now applying the Equipartition Theorem

$$\frac{\overline{S_V}^2}{2RC} = \frac{k_B T}{C}$$

$$\overline{S_V}^2 = 2k_B TR$$

Double Sideband Noise Spectral Density

- In most noise calculations, we integrate noise over positive frequencies, which means we should double the result of our previous calculation to properly account for noise

$$\overline{S_V}^2 = 4k_B TR$$

If you like Physics...

- For a capacitor, the energy stored $E = CV^2/2$ due to a noise resistor should be proportional to the Boltzmann distribution $exp(-E/k_B T)$ (assume thermal equilibrium at temperature T).
- To find the proportionality constant, note that integrating this quantity over all energy values should be unity

$$\int_{-\infty}^{\infty} Kexp\left(\frac{-CV^2}{2k_B T}\right) dV = 1$$

$$K = \sqrt{\frac{C}{2\pi k_B T}}$$

Thermodynamics Leads the Way

- Now we can compute the mean squared value of the voltage

$$\overline{V^2} = \sqrt{\frac{C}{2\pi k_B T}} \int_{-\infty}^{\infty} V^2 exp\left(\frac{-CV^2}{2k_B T}\right) dV$$

$$\overline{V^2} = \frac{k_B T}{C}$$

- The last step follows after performing the integral.