SOLUTIONS 1

University of California, Berkeley
EECS 117

Spring 2004
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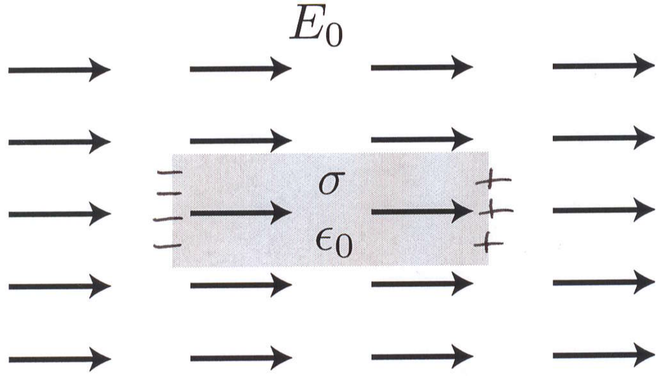
Exam II (closed book)
Tuesday, March 30, 2004

Guidelines: Closed book and notes. You may use a calculator. Do not unstaple the exam.
Warning: Illustrations not to scale.

1. (*25 points*) A conductor with $\sigma = 10^7$S/m and dielectric constant $\epsilon = \epsilon_0$ is placed into a uniform electric field $E_0 = \hat{x}50$V/m.



55 AV
13 STD
58 MED
21 MIN
77 MAX

(a) Find the charge density everywhere. Show the location of the charge distribution schematically on the figure above.

INSIDE THE CONDUCTOR, $E=0$, $\rho=0$

AT SURFACE, $\rho_s = \hat{n}\cdot\vec{D} = \varepsilon_0 E_0$

$$\rho_s = 50 \times 8.854 \times 10^{-12}\ C/m^2 = 443\ \frac{pC}{m^2}$$

(b) Estimate the time scale to reach the steady-state charge distribution.

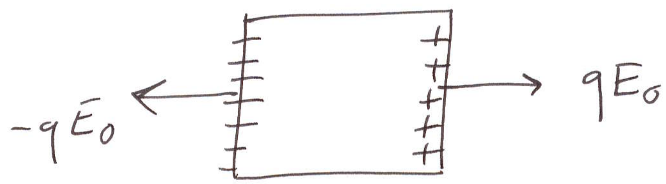
SEVERAL TIME CONSTANTS

$$\tau = \varepsilon/\sigma \cong 9 \times 10^{-19}$$

THE EVOLUTION IS EXPONENTIAL

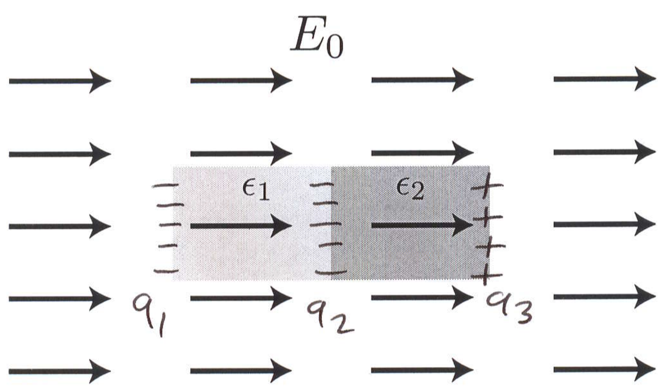
(c) Is there a force on the conductor? If so, draw the direction of the force and calculate the magnitude.

NET FORCE IS ZERO



$q = \rho_s \cdot A$

(d) Now a dielectric object is placed in the field as shown below. Half of the material has dielectric constant $\epsilon_1 = 2$ while the other half has $\epsilon_2 = 4$. Is there a force on the object? If so, calculate the magnitude and indicate the direction of the force.



NO!

~~YES~~, THE BOUND POLARIZATION CHARGE IS NOT THE SAME IN THE TWO REGIONS.

$$\rho_{s1} = -\varepsilon_0 E_0 \left(1 - \frac{\varepsilon_0}{\varepsilon_1}\right) = -\frac{1}{2}\varepsilon_0 E_0$$

$$\rho_{s3} = +\varepsilon_0 E_0 \left(1 - \frac{\varepsilon_0}{\varepsilon_2}\right) = \frac{3}{4}\varepsilon_0 E_0$$

$$\rho_{s2} = -\left[\varepsilon_0 E_0 \left(1 - \frac{\varepsilon_0}{\varepsilon_1}\right) - \left\{-\varepsilon_0 E_0 \left(1 - \frac{\varepsilon_0}{\varepsilon_2}\right)\right\}\right]$$

$$= -\rho_{s3} - \rho_{s1} = -\frac{1}{4}\varepsilon_0 E_0$$

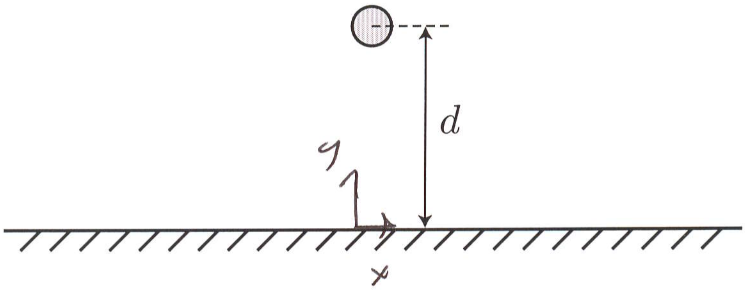
$$F_1 = -\frac{1}{2}\varepsilon_0 E_0^2 \cdot A$$

$$F_2 = -\frac{1}{4}\varepsilon_0 E_0^2 \cdot A \qquad \sum F = 0$$

$$F_3 = \frac{3}{4}\varepsilon_0 E_0^2 A$$

SOLUTIONS 5

3. (*30 points*) A transmission line is formed by running a single wire of radius a parallel to earth. The cross-section of the transmission line is shown below. We wish to find the transmission line parameters. You may assume that the region above the wire is air and thus $\epsilon \approx \epsilon_0$, $\mu \approx \mu_0$.



10

(a) Find the potential in the charge-free region for $y > 0$. You may assume that the wire radius is negligible. *Hint:* Use the method of images.

TAKE ORIGIN ON PLANE BELOW WIRE

SINCE $E \propto \frac{1}{r}$ GAUSS' LAW, $\phi \propto \ln r$

$$\Delta l \, 2\pi r' \varepsilon_0 E_r = \rho_l \cdot \Delta l \qquad E_r = \frac{\rho_l}{2\pi r' \varepsilon_0}$$

$$\phi = -\int E_r \cdot dr = \frac{-\rho_l}{2\pi \varepsilon_0} \ln r'$$

SHIFT ORIGIN & USE METHOD OF IMAGES

$$\phi = -\frac{\rho_l}{2\pi\varepsilon_0} \left\{ \ln |r - d\hat{y}| - \ln |r + d\hat{y}| \right\}$$

10

(b) Find the capacitance per unit length of the line.

$$\text{NOTE} \quad \phi(y=0) = -\frac{\rho_l}{2\pi\varepsilon_0} \ln \left| \frac{x^2 + d^2}{x^2 + d^2} \right| = 0$$

$$V_0 = \phi(x=0, y=d-a) = \frac{-\rho_l}{2\pi\varepsilon_0} \ln \left| \frac{a}{2d-a} \right|$$

$$= \frac{\rho_l}{2\pi\varepsilon_0} \ln \left| \frac{2d}{a} - 1 \right|$$

$$\Rightarrow C' = \frac{\rho_l}{V_0} = \frac{2\pi\varepsilon_0}{\ln \left| \frac{2d}{a} - 1 \right|}$$

(c) Find the inductance per unit length of the line.

SINCE $L'C' = \mu_0 \varepsilon_0$

$$L' = \frac{\mu_0}{2\pi} \ln\left|\frac{2d}{a} - 1\right|$$

(d) Calculate the shunt conductive loss G' per unit length.

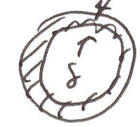
$$RC = \frac{\varepsilon_0}{\sigma} = \frac{C}{G}$$

$$G = \frac{C\sigma}{\varepsilon_0} = \frac{2\pi\sigma}{\ln\left|\frac{2d}{a} - 1\right|}$$

(e) Estimate the series resistance per unit length R'. You may assume that current flows uniformly in a layer of thickness δ along the outer edges of the conductors. Assume earth has a conductivity of σ_2. *Hint:*

$$\int_{-\infty}^{\infty} \frac{y}{x^2 + y^2} = \pi$$

CURRENT ON "SKIN"



$$R' = R'_{WIRE} + R'_{GND}$$

$$R'_{WIRE} = \frac{1}{\sigma A} \cong \frac{1}{\sigma\, 2\pi a \delta}$$

$$J_{GND} = \rho_{GND} \cdot v_p$$

$$\rho_{GND} = \hat{n} \cdot D_n \Big|_{y=0}$$

(f) Calculate the line characteristic impedance.

$$Z_0 = \sqrt{\frac{j\omega L' + R'}{j\omega C' + G'}}$$

$$E = -\nabla \phi$$

$$E_y(y=0) = \frac{\rho_\ell}{\pi \varepsilon_0} \frac{-d\, \hat{y}}{x^2 + d^2}$$

$$\Rightarrow \quad \rho_{GND}(x) = \frac{\rho_\ell}{\pi} \frac{d}{x^2 + d^2}$$

$$J(x) = \frac{I\, d}{\pi (x^2 + d^2)} \Rightarrow \frac{I}{W_{eff}} = J_{eff}$$

$$\cancel{J_{eff} = \int_{-\infty}^{\infty} \frac{I\, d}{\pi (x^2+d^2)}\, dx}$$

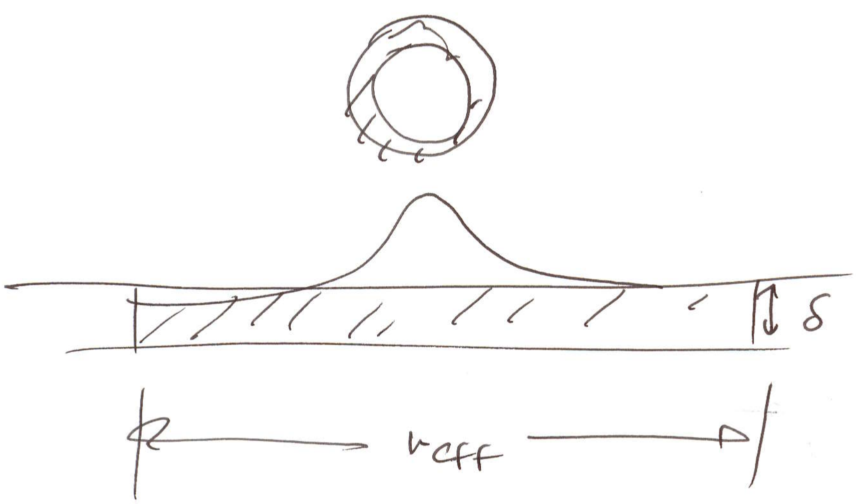
$$P_L = \int J \cdot E \, dV \overset{?}{=} \sigma \int |J|^2 \, dV$$

↓ BACK

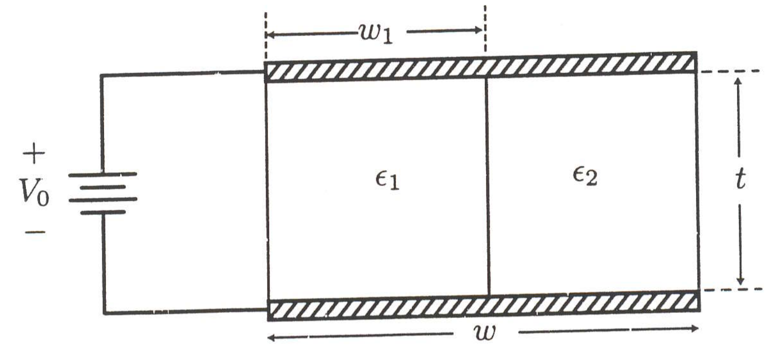
$$\int_{-\infty}^{\infty} \left(\frac{d}{\pi (x^2 + d^2)} \right)^2 dx = \frac{1}{2\pi d} = \frac{1}{W_{eff}}$$

$$R'_{GND} = \frac{1}{\sigma_{GND} \underbrace{2\pi d \, \delta}_{\text{Effective Area}}}$$

$$R'_T = R'_{GND} + R'_{WIRE} = \frac{1}{\sigma_{GND} 2\pi d \delta} + \frac{1}{\sigma \, 2\pi a \delta}$$



4. (*10 points*) Using the fact that a capacitor stores energy equal to $\frac{1}{2}CV^2$, and the electrostatic field has energy density of $\frac{1}{2}\mathbf{E}\cdot\mathbf{D}$, calculate the capacitance of the following structure by equating the total energy stored in the cap to the energy contained in the electrostatic field. *Neglect* fringing fields.



$$E_1 = E_2 = \frac{V_o}{t} \qquad D_1 = \varepsilon_1 E_1$$
$$D_2 = \varepsilon_2 E_2$$

$$E = \frac{1}{2}\int_{V_1} E\cdot D \, dV + \frac{1}{2}\int_{V_2} E\cdot D \, dV$$

$$= \frac{1}{2}\,\frac{w_1 \cancel{t} \cdot \varepsilon_1 V_o^2}{t^{\cancel{2}}} + \frac{1}{2}\,\frac{(w-w_1)\cancel{t}\,\varepsilon_2 V_o^2}{t^{\cancel{2}}}$$

$$= \frac{1}{2} C' V_o^2$$

$$C' = \frac{w_1 \varepsilon_1}{t} + \frac{(w-w_1)\varepsilon_2}{t}$$