University of California, Berkeley
EECS 117

Spring 2004
Prof. A. Niknejad

Exam I (closed book)
Tuesday, February 17, 2004

Guidelines: Closed book and notes. One 8.5" by 11" page (both sides) of *your own notes* is allowed. You may use a calculator. Do not unstaple the exam. *Warning:* Illustrations not to scale.

You may find the following formulas useful:
For a distributed circuit with impedance per unit length Z' and admittance per unit length Y', the voltage and current on a transmission line take the following form in harmonic steady-state

$$v(z) = v^+ e^{-\gamma z} + v^- e^{\gamma z}$$

$$i(z) = \frac{v^+}{Z_0} e^{-\gamma z} - \frac{v^-}{Z_0} e^{\gamma z}$$

where $\gamma = \sqrt{Y'Z'} = \alpha + j\beta$ is the propagation constant and $Z_0 = \sqrt{Z'/Y'}$ is the characteristic impedance of the line. The reflection coefficient from a termination Z_L is given by

$$\rho = \frac{Z_L - Z_0}{Z_L + Z_0}$$

The impedance on the transmission line at an arbitrary distance from the load is given by

$$Z_{in}(-\ell) = Z_0 \frac{Z_L + Z_0 \tanh(\gamma \ell)}{Z_0 + Z_L \tanh(\gamma \ell)}$$

For a lossless line, the equation take on the form

$$Z_{in}(-\ell) = Z_0 \frac{Z_L + jZ_0 \tan(\beta \ell)}{Z_0 + jZ_L \tan(\beta \ell)}$$

The standing-wave on the transmission line is given by

$$SWR = \frac{|v_{max}|}{|v_{min}|} = \frac{1 + |\rho|}{1 - |\rho|}$$

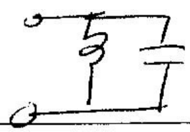
1. (*10 points*) Answer each question briefly.

(a) What's the SWR for a lossless transmission line terminated in a reactive load?

$|\rho| = 1 \Rightarrow SWR = \infty$

(b) Draw an equivalent circuit for a shorted transmission line of length $3\lambda/4$ for a small offset around the frequency $f = c/\lambda$. You *do not* need to specify the component values.

AT MULTIPLES OF $\lambda/4$ A SHORTED LINE LOOKS OPEN. AT SMALL FREQ OFFSETS IT BEHAVES LIKE A PARALLEL LC CIRCUIT



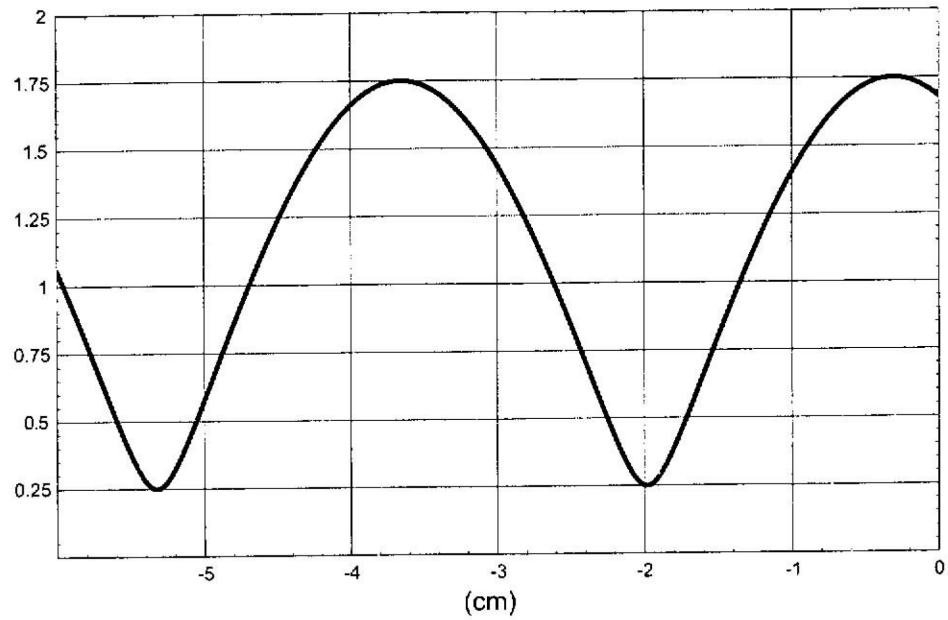
(c) Derive the input impedance of a short section of an open lossy transmission line of length ℓ and propagation constant γ ($\gamma\ell \ll 1$).

$$Z_{in} = Z_0 \frac{Z_L + Z_0 \tanh \gamma\ell}{Z_0 + Z_L \tanh \gamma\ell} \simeq \frac{Z_0}{\tanh \gamma\ell}$$

$$\tanh \gamma\ell \approx \gamma\ell$$

$$Z_{in} = \frac{Z_0}{\gamma\ell}$$

2. (*10 points*) The plot below is the magnitude of the voltage $|v(z)|$ (linear scale) plotted along a transmission line excited by a sinusoidal signal. The transmission line has a propagation velocity of $.75c$. The load appears at position $z = 0$.



(a) Compute the SWR?

$SWR = \frac{V_{max}}{V_{min}} = 7$

(b) Is the load inductive or capacitive? Explain

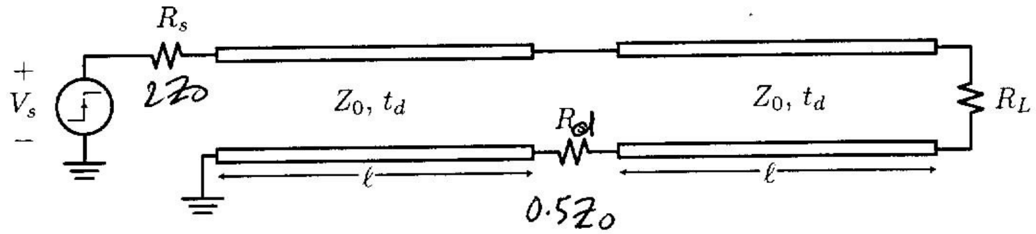
Inductive since the maximum is hit first. That means we must be on the upper half of the Smith chart since we intersect with the real pos. axis first.

(c) What is the source excitation frequency?

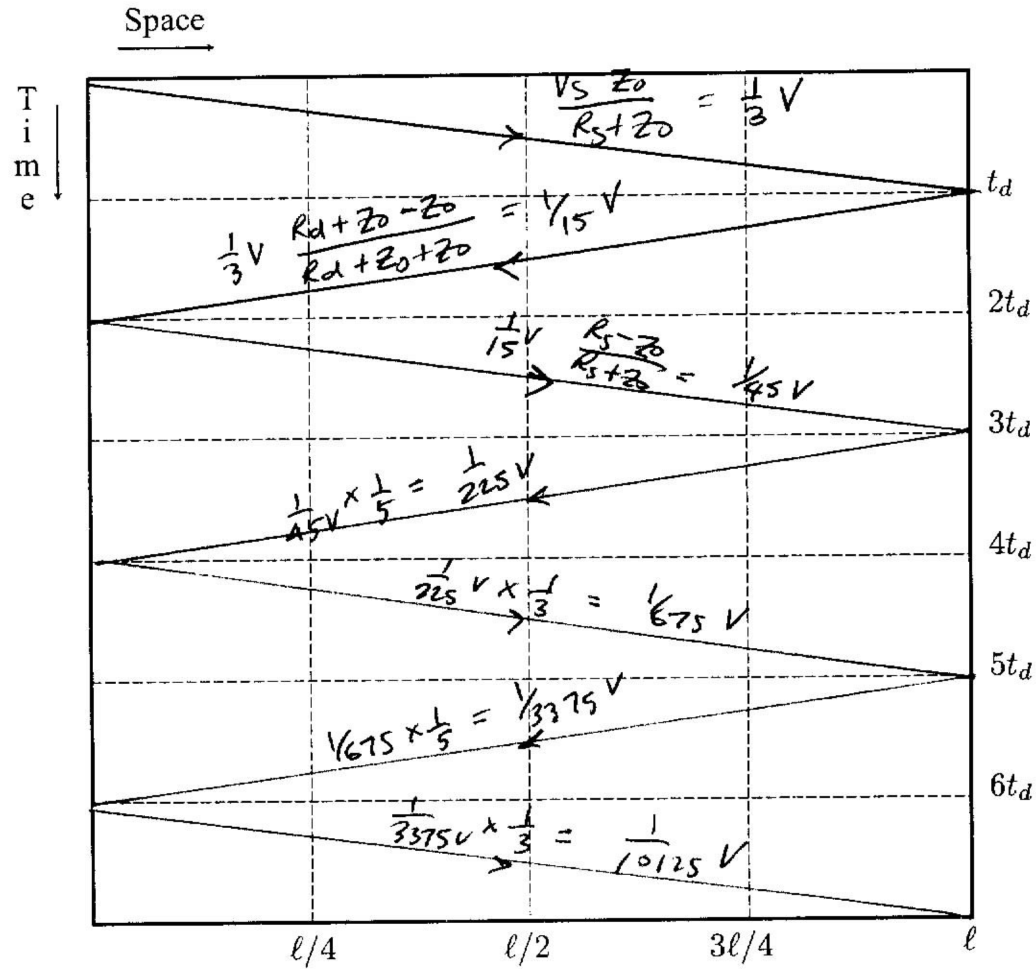
TWO POINTS WHERE $|V(z)|$ IS MIN: -2 cm, -5.3 cm

$\frac{\lambda}{2} = 3.3$ cm $\quad f = \frac{0.75c}{\lambda} = 3.4$ GHz

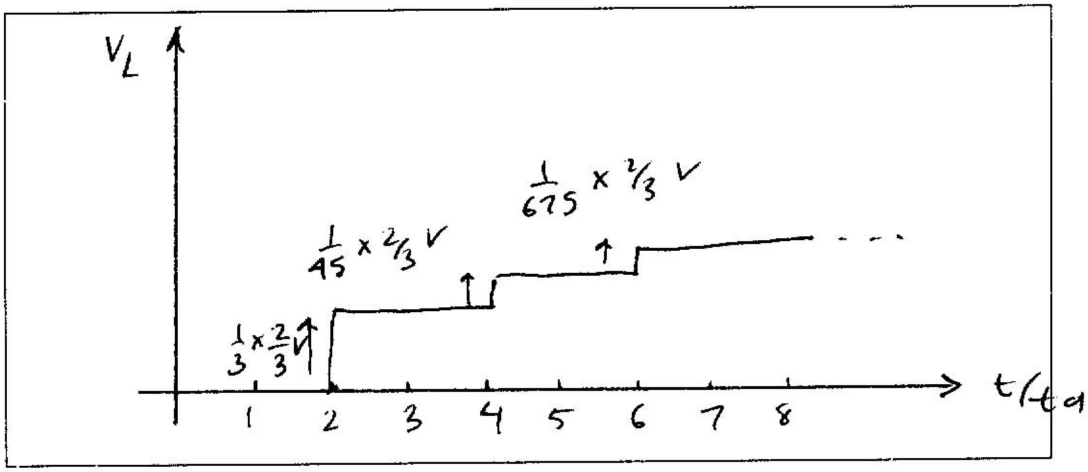
3. (*20 points*) Consider the following circuit consisting of two transmission lines of characteristic impedance Z_0. Due to a layout error, the ground connection is not good and presents a resistance of $R_d = .5Z_0$. The circuit is excited by a pulse at the generator with amplitude 1V and source impedance $R_s = 2Z_0$ (zero rise-time). The load is matched $R_L = Z_0$.



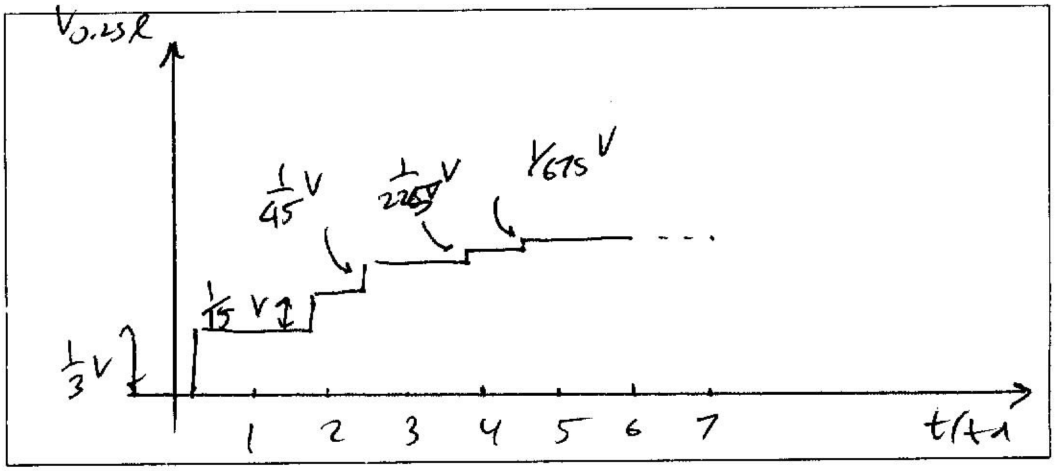
(a) Using the provided graph, draw the bounce diagram for the circuit. Only include the action on the first transmission line.



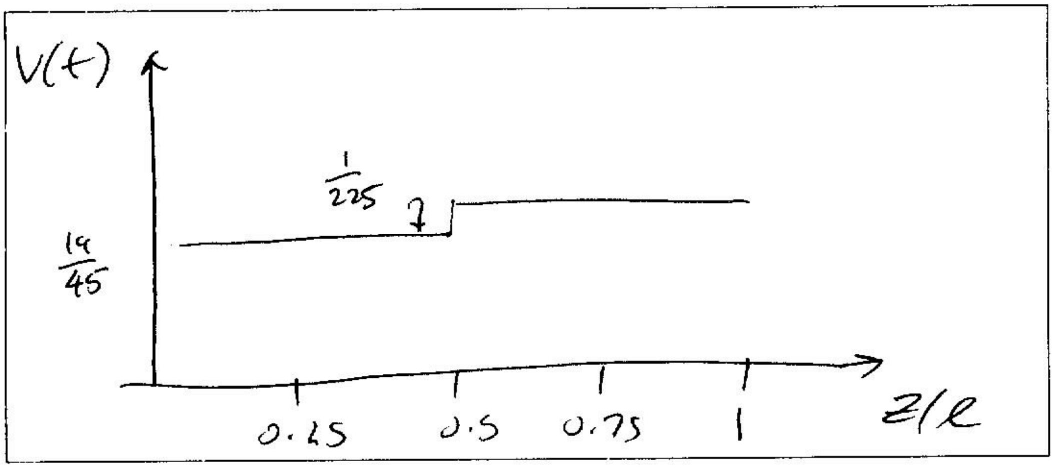
(b) Sketch the voltage waveform at the load.



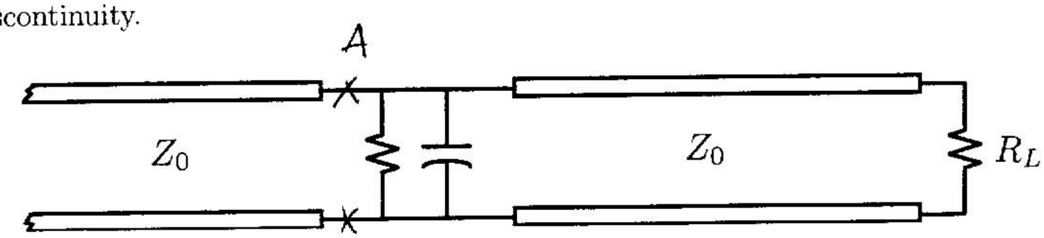
(c) Sketch the voltage waveform at the point $z/\ell = .25$ as a function of time on the first transmission line.



(d) Sketch the voltage waveform on the first transmission line at $t = 3.5t_d$.



4. (*20 points*) The transmission line below has a discontinuity connected in shunt as shown below. The discontinuity consists of a capacitor and a parallel resistor. The load is matched to the line. Derive the equations governing the reflection v^- at the discontinuity.



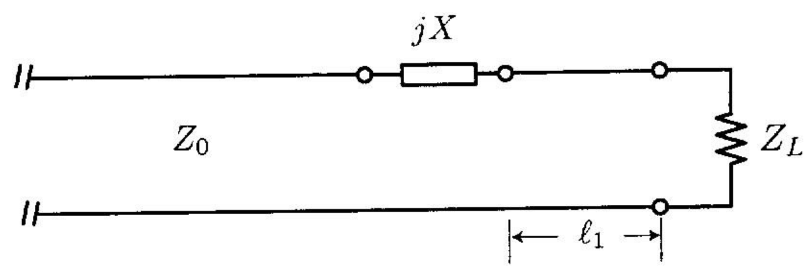
SINCE $R_L = Z_0$, THE LOAD OF THE FIRST T-LINE IS $R \| C \| Z_0$

ρ AT POINT A $= \dfrac{Z_L - Z_0}{Z_L + Z_0}$ $\qquad Z_L = \left(\dfrac{1}{R} + \dfrac{1}{Z_0} + j\omega C\right)^{-1} = \dfrac{R Z_0}{R + Z_0 + j\omega R Z_0 C}$

$$= \frac{-(Z_0^2 + j\omega R Z_0^2 C)}{2RZ_0 + Z_0^2 + j\omega R Z_0^2 C}$$

$V^- = \rho V^+ \Rightarrow$ USE FOURIER OR LAPLACE TRANSFORM TO FIND TIME BEHAVIOR

5. (*20 points*) A antenna is mounted on a tower 100m above ground. The impedance of the antenna is $Z_L = 200 + j60\Omega$ at 600MHz. It is desired to match the antenna impedance to the transmission line feed which has a characteristic impedance of $Z_0 = 100\Omega$ and propagation velocity $.7c$. The schematic of the matching network is shown below and consists of a section of feed transmission line with a series reactance jX.



(a) Using the Smith Chart below, find ℓ_1 and X to perform an impedance match. It should be noted that the stub must be placed at ground level, in other words a distance of at least 100m from the load.

$\frac{Z_L}{Z_0} = 2 + j0.6$ (POINT A)

$\omega = 2\pi f = 3.77$ G rad/s

$\beta = \frac{\omega}{v} = 17.9\ m^{-1}$

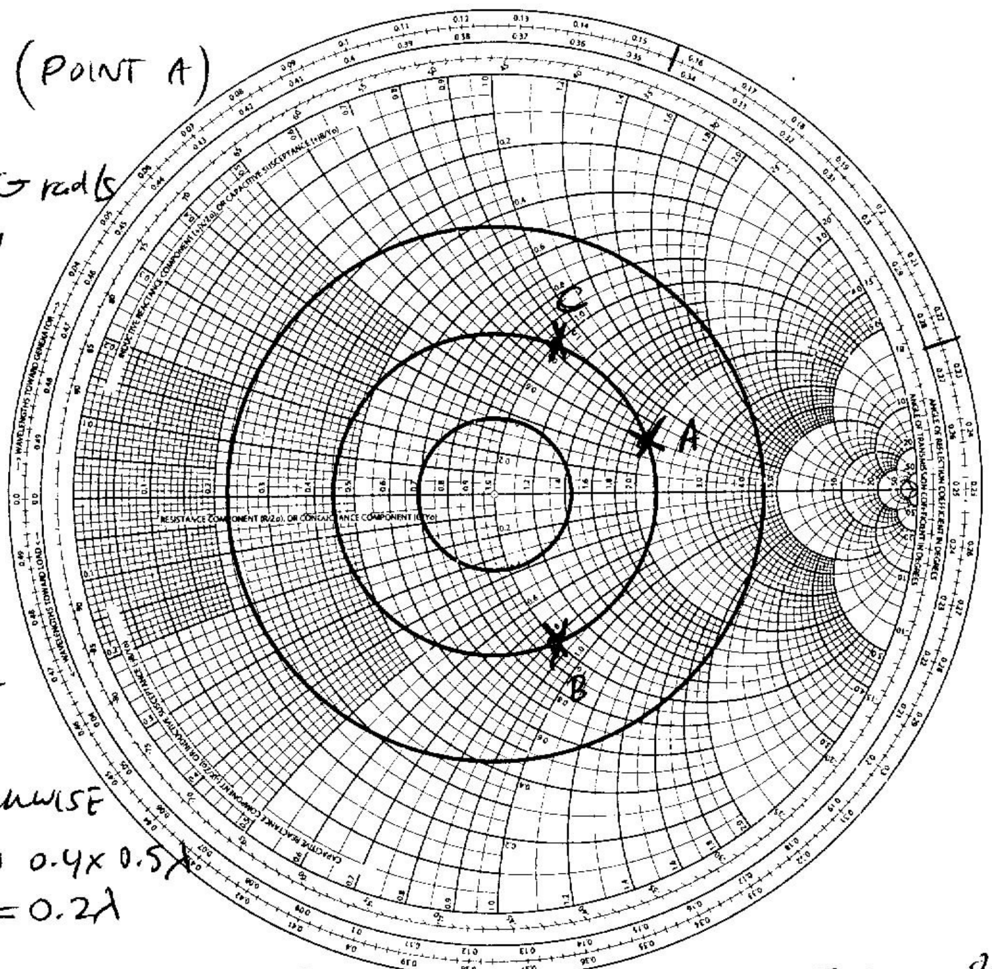
$\lambda = 35$ cm

$\ell_1 = 100$ m

$\beta\ell = 571.4\pi$

SO POINT A NEEDS TO MOVE AROUND 571 FULL CIRCLES + 0.4 FULL CIRCLE CLOCKWISE

0.4 FULL CIRCLE $\Rightarrow 0.4 \times 0.5\lambda = 0.2\lambda$



POINT A HAS A READING OF 0.223λ

POINT C HAS A READING OF 0.156λ

AT POINT C $r = 1$ $x = 0.825$

$\Rightarrow X = 82.5\Omega$

$\beta\ell = 571\pi + \frac{0.5 - 0.223 + 0.156}{0.5}\pi$

$\ell = 100.07$ m

PLACE $X = -82.5\Omega$ IN SERIES

(b) If the load is transmitting 100kW (note this is the power dissipated in the real part of the load), what is the peak voltage on the line?

THE SWR CIRCLE CROSSES THE REAL AXIS AT $r = 2.2$

$$\frac{V_{max}^2}{2r\cdot 100} = 100kW$$

$$\Rightarrow V_{max} = 6.6\ kV$$

SOL 2: $100kW = \frac{|V^+|^2}{2Z_0}(1-|\rho_L|^2) \Rightarrow |V^+| = 4837$

$V_{max} = V^+(1+|\rho_L|) \cong 6.6\ kV$

(c) If the feed line lossy with $\alpha = 0.1$dB/m, estimate the power lost to the transmission line when transmitting 100kW.

$$P(z) = \frac{1}{2} Re\{V(z)\,\overline{I(z)}\}$$

$$= \frac{1}{2} Re\{\underbrace{e^{-2\alpha z} + \rho_L e^{2j\beta z} - \bar{\rho}_L e^{-2j\beta z}}_{\text{IMAG.}} - |\rho_L|^2 e^{2\alpha z}\} \times \frac{|V^+|^2}{2Z_0}$$

$$= \frac{1}{2}\{e^{-2\alpha z} - |\rho_L|^2 e^{2\alpha z}\}$$

$$P(-\ell) = \frac{|V^+|^2}{2Z_0}\left(e^{-2\alpha z} - |\rho_L|^2 e^{-2\alpha z}\right)$$

$$P(0) = 100kW = \frac{|V^+|^2}{2Z_0}(1-|\rho_L|^2) \Rightarrow \frac{|V^+|^2}{2Z_0} = 117kW$$

$P_{diss} = P(-\ell) - P(0) \qquad |\rho_L| = 0.381$

$$\alpha = 0.1\,dB/m \Rightarrow \frac{P_2}{P_1} = e^{-2\alpha\,1m}$$

$$10\log\frac{P_2}{P_1} = -0.1dB = 10\log e^{-2\alpha}$$

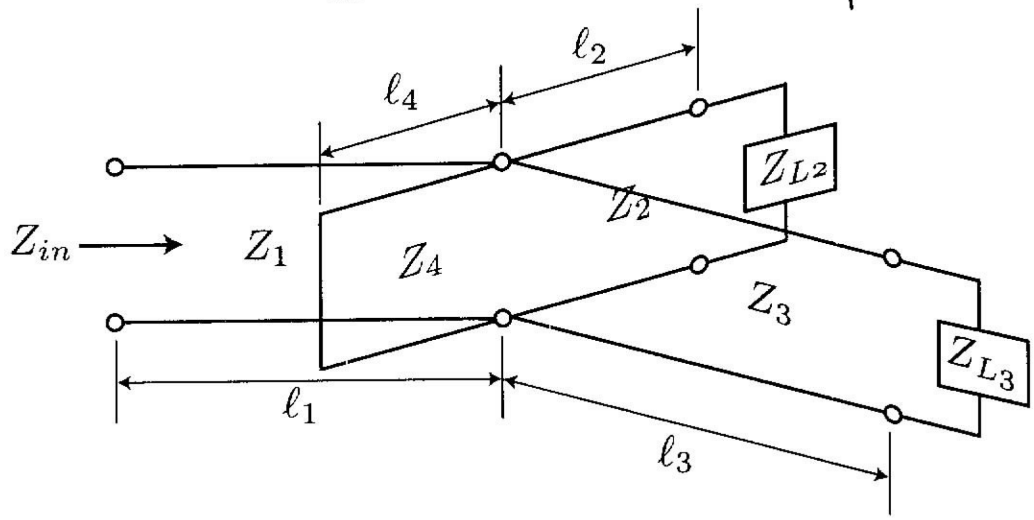
$$\alpha = 0.0115\ m^{-1}$$

$P_{in} = P(-\ell) = 1.165\ MW$

$P_{diss} = 1.065\ MW$

6. (*20 points*) Calculate the input impedance at 1 GHz at the location shown below. The table below summarizes the characteristic of each transmission line. The load impedances are $Z_{L2} = 2Z_1$ and $Z_{L3} = .75Z_3$. *Hint:* This problem is easier than it looks.

	Z_0 (Ω)	ℓ (cm)	v/c	$\beta\ell$	Z_{in}
Z_1	50	5.25	.7	0.5π	
Z_2	75	1.875	.25	0.5π	56.25
Z_3	300	3.75	.25	π	225
Z_4	150	6	.8	0.5π	∞



USE $\frac{\omega}{v} = \beta \Rightarrow \frac{2\pi \times 1\text{GHz}}{v} \times \ell = \beta\ell$

SEE TABLE ABOVE AND ADD COLUMNS FOR $\beta\ell$ AND Z_{in}

Z_{eq} OF LINE 1 : $56.25\Omega \,||\, 225\Omega$

$Z_{eq} = 45\Omega$

Z_{in} OF LINE 1 $= Z_1 \frac{Z_1}{Z_{eq}} = \frac{50 \times 50}{45} = 55.6\Omega$