SOLUTIONS

University of California, Berkeley
EECS 117

Spring 2007
Prof. A. Niknejad

Midterm Exam (closed book)
Tuesday, February 17, 2004

Guidelines: Closed book and notes. You may use a calculator. Do not unstaple the exam. *Warning:* Illustrations not to scale.

You may find the following formulas useful:
For a distributed circuit with impedance per unit length Z' and admittance per unit length Y', the voltage and current on a transmission line take the following form in harmonic steady-state

$$v(z) = v^+ e^{-\gamma z} + v^- e^{\gamma z}$$

$$i(z) = \frac{v^+}{Z_0} e^{-\gamma z} - \frac{v^-}{Z_0} e^{\gamma z}$$

where $\gamma = \sqrt{Y'Z'} = \alpha + j\beta$ is the propagation constant and $Z_0 = \sqrt{Z'/Y'}$ is the characteristic impedance of the line. The reflection coefficient from a termination Z_L is given by

$$\rho = \frac{Z_L - Z_0}{Z_L + Z_0}$$

The impedance on the transmission line at an arbitrary distance from the load is given by

$$Z_{in}(-\ell) = Z_0 \frac{Z_L + Z_0 \tanh(\gamma \ell)}{Z_0 + Z_L \tanh(\gamma \ell)}$$

For a lossless line, the equation take on the form

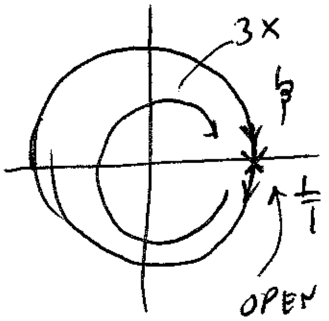
$$Z_{in}(-\ell) = Z_0 \frac{Z_L + jZ_0 \tan(\beta \ell)}{Z_0 + jZ_L \tan(\beta \ell)}$$

The standing-wave on the transmission line is given by

$$SWR = \frac{|v_{max}|}{|v_{min}|} = \frac{1 + |\rho|}{1 - |\rho|}$$

1. (*25 points*) Answer each question briefly.

   (a) Draw an equivalent circuit for a transmission line of length ℓ terminated by an open circuit for a small offset around the frequency $f = 1.5c/\ell$. You *do not* need to specify the component values.



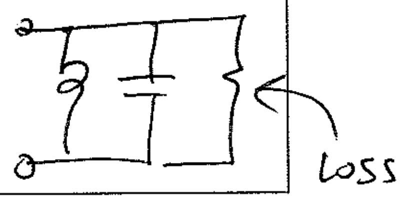
$\beta\ell = \frac{2\pi}{\lambda}\ell = \frac{2\pi}{c} f\ell = \frac{2\pi}{c}\,\frac{3c}{2\ell}\cdot\ell = 3\pi$

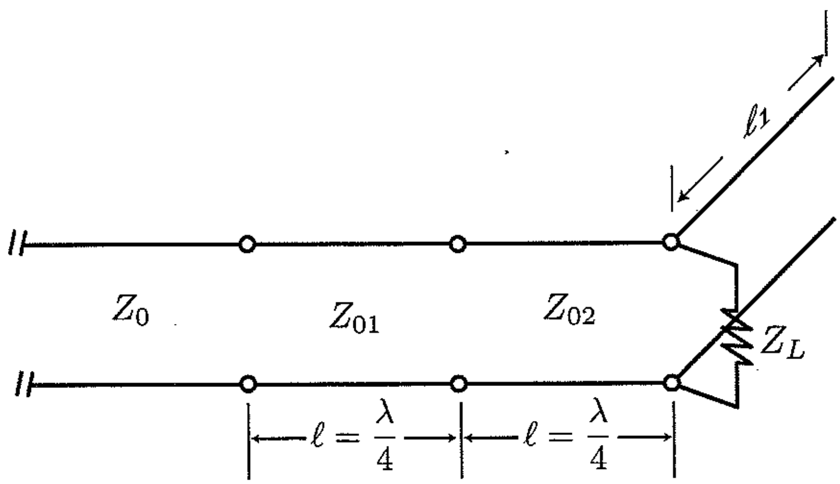
$2\beta\ell = 6\pi \Rightarrow$ 3X AROUND SMITH CHART

$f < f_0 \Rightarrow$

$f > f_0 \Rightarrow$

$\Rightarrow$





   (b) Consider the matching circuit shown above where the load Z_L is transformed into $Z_0 = 50\ \Omega$. Specify the component values Z_{02} and ℓ_1 to obtain an impedance match. ($Z_{01} = 62\ \Omega$ and $Z_L = 27.455 + j50\ \Omega$). The open stub of length ℓ_1 has an impedance of Z_0.

$Y_L = \frac{1}{Z_L} = \frac{\bar{Z}_L}{|Z_L|^2} = \frac{27 - j50}{3254} \simeq 8.4\text{mS} - j15\text{mS}$

$\rightarrow$ DESIGN OPEN STUB TO TUNE OUT $j\,15\text{mS}$

$j Y_0 \tan \beta \ell_1 = j 15 \text{ mS}$

$\Rightarrow$ SOLVE FOR ℓ_1

$$R_{Left} = \frac{1}{8.4\text{mS}} = 118.5\,\Omega$$

$$Z_S = \frac{Z_{01}^2}{Z_0} = \frac{62^2}{50} = 76.88\,\Omega$$

WANT $\frac{Z_{02}^2}{R_{Left}} = Z_S \Rightarrow Z_2 = \sqrt{Z_S R_{Left}} = 95.45\,\Omega$

(c) Draw the impedance $R + j\omega L$ on the Smith Chart as $\omega_0 = 0 \to \infty$. Assume $R < Z_0$. Label the point at DC, frequency $\omega_0 = R/L$, and ∞.

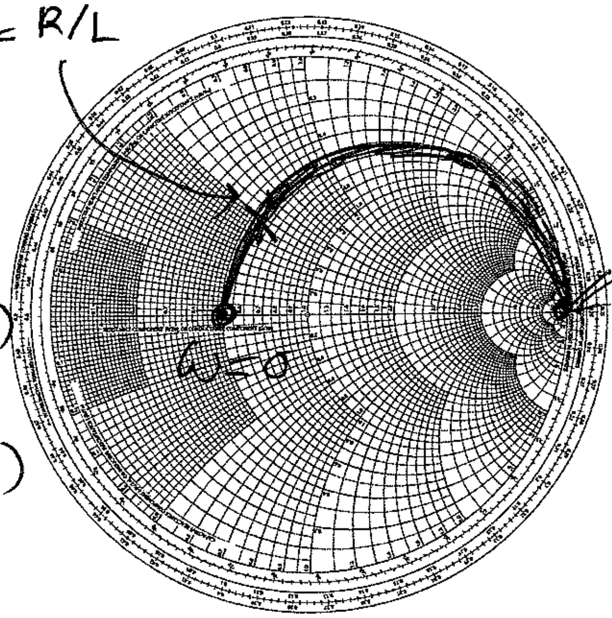
$W_0 = R/L$

$R < Z_0$ LEFT HAND AXIS

$\omega_0 = 0 \quad Z = R$

$\omega_0 \to \infty \quad Z \to \infty j$ (OPEN)

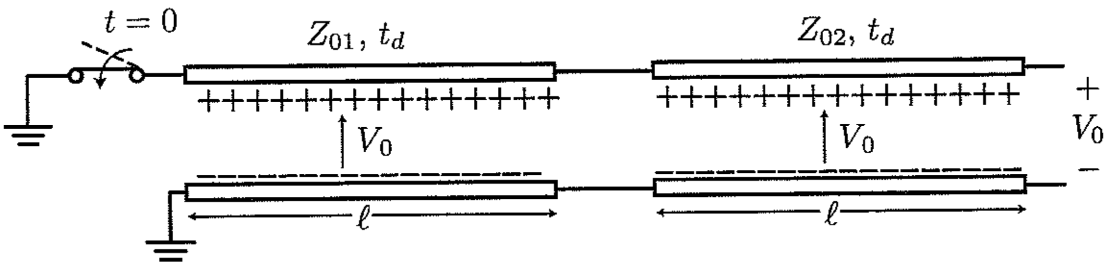
$\omega_0 \to \frac{R}{L} \quad Z \to R + jR$
$Re(Z) = Im(Z)$



(d) Calculate the ionization energy of the Hydrogen atom. The Bohr radius is approximately 0.53×10^{-10} m.

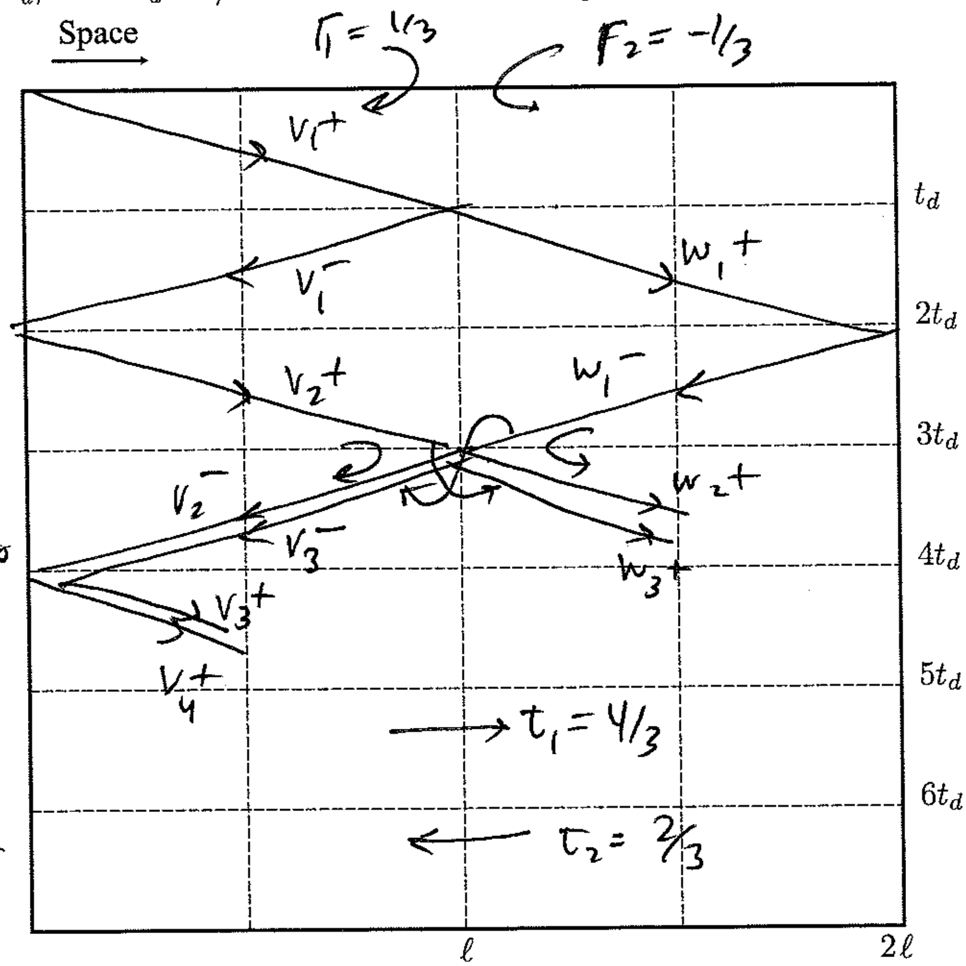
$$\phi = \frac{1}{4\pi\varepsilon_0} \frac{q_{nucleus}}{R_0}$$

$$E = e\phi = 27\text{ eV}$$



2. (*25 points*) Consider the following circuit consisting of two transmission lines of characteristic impedance $Z_{02} = 2Z_{01}$, equal length ℓ and equal propagation velocity. The lines are initially charged to 1V and then at time $t = 0$, one end of the line is discharged through an ideal switch.

   (a) Using the provided graph, draw the bounce diagram for the circuit from $t = 0$ to $t = 4t_d$, where $t_d = \ell/c$ is the time for a wave to propagation along one line.



$\Gamma_1 = \frac{100-50}{150} = \frac{1}{3}$

$\Gamma_2 = -\frac{1}{3}$

$t_1 = 1 + \Gamma_1 = 4/3$

$\tau_2 = 1 + \Gamma_2 = 2/3$

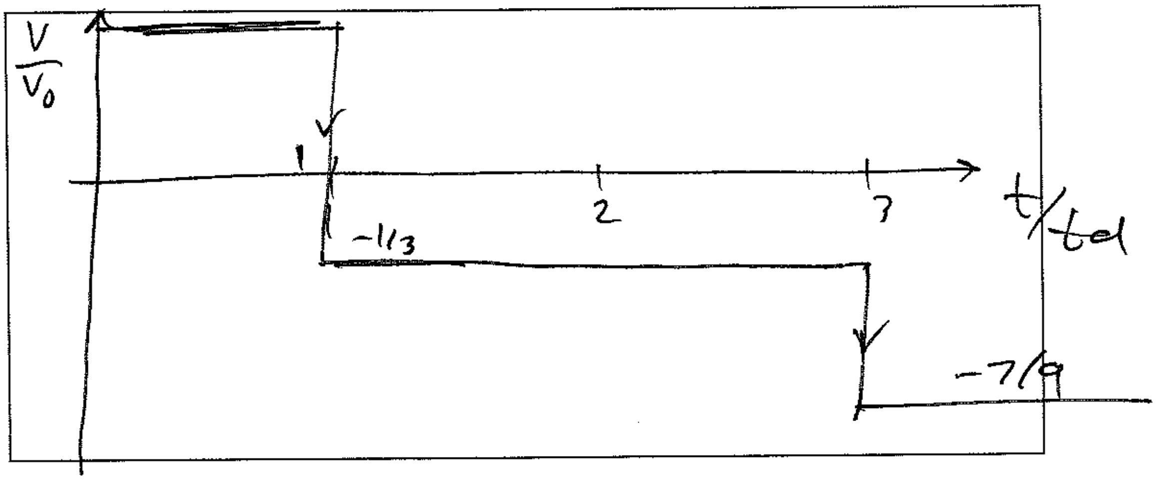
$V_1^+ = -V_0$	$W_1^+ = -\frac{4}{3} V_0$
$V_2^- = -\frac{V_0}{3}$	$W_1^- = -\frac{4}{3} V_0$
$V_2^+ = \frac{V_0}{3}$	$W_2^+ = \frac{4}{9} V_0$
$V_2^- = \frac{1}{9} V_0$	$W_3^+ = V_0 \frac{4}{9}$
$V_3^- = -\frac{8}{9} V_0$	
$V_3^+ = \frac{8}{9} V_0$	
$V_4^+ = -\frac{V_0}{9}$	

$\Gamma_S = -1$

$\Gamma_L = +1$

(b) Sketch the voltage waveform at the interface (versus time).

t	$\Delta V/V_0$
t_d	$V_1^+ + V_1^- = -\frac{4}{3}$
$3t_d$	$V_2^+ + V_2^- + V_3^- = -\frac{4}{9}$



(c) How much energy is stored on the line at time $t < 0$ and $t \to \infty$?

AT $t < 0$ THE ENERGY IS STORED IN THE LINE CAPACITANCE:

$$E = \frac{1}{2} C_1 V_0^2 + \frac{1}{2} C_2 V_0^2 = \frac{1}{2} \ell (C_1 + C_2) V_0^2$$

$$Z_{01} = \sqrt{\frac{L_1}{C_1}} = \sqrt{\frac{1}{C_1^2 v^2}} = \frac{1}{C_1 v} \qquad C_1 = \frac{1}{v Z_{01}}$$

$$E = \frac{1}{2} \ell \frac{1}{c Z_0} \cdot 3 \cdot V_0^2$$

(d) What is the final value of the voltage at the load? Explain.

IT OSCILLATES! SINCE THE LINES ARE LOSSLESS, THE CIRCUIT CANNOT RETURN TO A STABLE "DC" OPERATING POINT.

3. (*25 points*) In this problem you will design a two-wire communication link between two points separated by 100 km. In your calculations below, consider the losses due to resistivity of the wires but ignore the dielectric losses. Invoke a low loss approximation, e.g. Z_0 is approximately real and α is small.

For a two-wire line, the inductance per unit length is given by

$$L' = \frac{\mu}{\pi} \cosh^{-1} \frac{d}{2a}$$

where d is the separation between the wires and a is the wire radius.

(a) The two wires are embedded in a thick plastic casing for mechanical support. The dielectric constant of the plastic is 5. How long does it take to send information from origin to destination?

$$c = c_0/\sqrt{\varepsilon} \qquad t = \ell/c = \frac{100 \times 10^3}{c_0}\sqrt{\varepsilon} = 0.7\,\text{ms}$$

(b) Derive an expression for the propagation constant of the line. Assume $R' \ll \omega_0 L'$, where ω_0 is the operating frequency.

$$\gamma = \sqrt{(j\omega L' + R')(j\omega C')}$$

$$= \sqrt{j\omega L' \cdot j\omega C'}\sqrt{1 + \frac{R'}{j\omega L'}}$$

$$= j\omega\sqrt{L'C'}\left(1 - \frac{j}{2}\frac{R'}{\omega L'}\right)$$

$$\alpha = \cancel{\omega}\sqrt{L'C'} \cdot \frac{R'}{2\cancel{\omega} L'} = \frac{R'}{2\sqrt{L'/C'}} \approx \frac{R'}{2Z_0}$$

(c) In order to communicate over this distance, specify the maximum tolerable resistance per unit length if the minimum detectable signal is 6 μV while the wires melt if the current exceeds 5 A. Assume the characteristic impedance of the line is 600Ω.

$$i^+ = \frac{V^+}{Z_0} = 5A \Rightarrow V^+ = 600\Omega \cdot 5A = 3kV \text{ (MAX)}$$

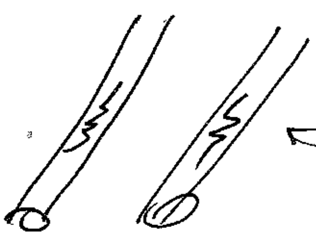
$$V^+(\ell) = V^+(0)\, e^{-\alpha \ell} = 6\mu$$

$$\Rightarrow \alpha \ell = -\log \frac{6\mu}{3K} = \log \frac{3}{6} \times 10^9$$

$$\alpha < 8.7 \times 10^{-5}$$

$$R' < 2 Z_0 \alpha = 0.104\, \Omega/m$$

(d) Specify the wire radius and separation to satisfy the previous conditions. If you were not able to solve the previous problem, assume that $\alpha = 1 \times 10^{-6}$. Assume that the conductivity of the wire $\sigma = 10^7$ S/m.



$$R' = \underbrace{2}_{\text{TWO WIRES}} \times \frac{\rho}{A} = \frac{2}{\sigma \pi^2 a} \Rightarrow a = \sqrt{\frac{2}{\sigma \pi R'}}$$

$$= 0.25 \text{ mm}$$

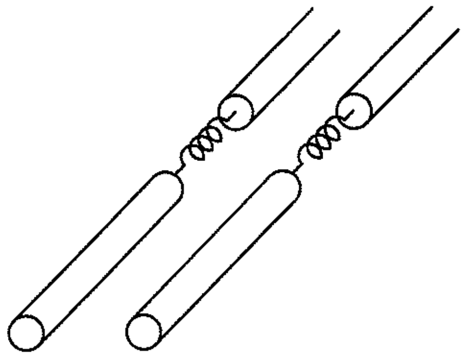
$$L' = \frac{\mu}{\pi} \cosh^{-1} \frac{d}{2a}$$

SOLVE FOR d

$$\frac{L'}{C'} = Z_0^2 \qquad L'C' = \frac{1}{v^2}$$

$$L' = C' Z_0^2 = \frac{1}{L' v^2} Z_0^2 \Rightarrow$$

$$L' = \frac{Z_0}{v} = \frac{600\Omega}{3 \times 10^8} = 2\mu H/m$$



(e) In order to increase the communication distance further by a factor of 10, lumped inductors are periodically loaded onto the line as shown above. Specify the number and value of inductance needed to do this if the maximum allowed spacing between the inductors is $\lambda/10$ and the communication frequency is 1 MHz. Explain why this works.

NEED $e^{-\alpha' l \cdot 10} = e^{-\alpha l}$

$\Rightarrow \alpha'$ SHOULD BE 10X SMALLER

$\Rightarrow Z_0$ 10X LARGER

$\Rightarrow L'$ 10X LARGER $20\,\mu H/m$

NEED ADDITIONAL $18\,\mu H/m$

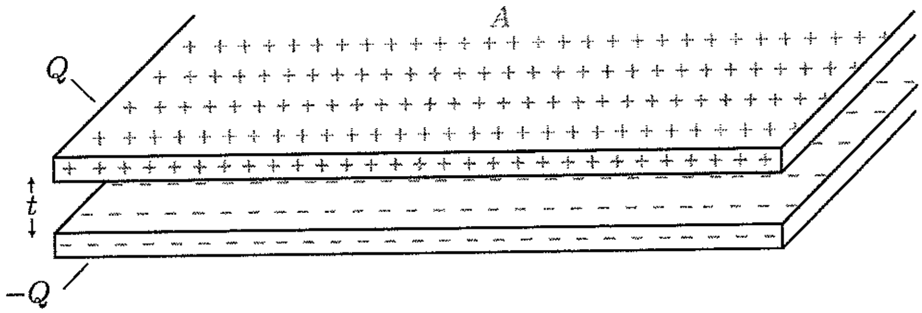
OR $L = \frac{18\,\mu H}{m} \times 100 \times 10^3 \times 10 = 18 H$

AT $1\,MHz$ $\lambda = c/f = 300\,m$ $\lambda/10 = 30\,m$

$N = \frac{10^5}{30} = 3333$ INDUCTORS

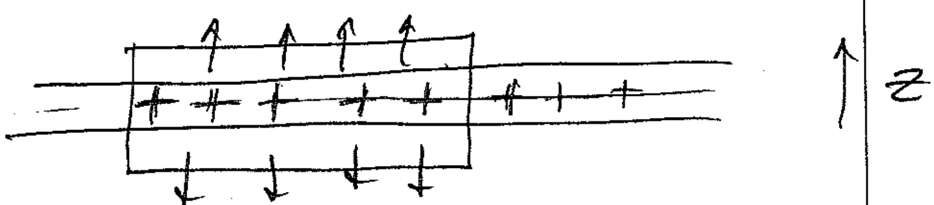
$L_0 = \frac{18 H}{N} = .54\,mH$

4. (*25 points*) Consider a simple parallel plate structure consisting of a two large conducting plates of area A separated by a small distance t. Note that $t \ll \sqrt{A}$. Suppose that a charge $+|Q|$ is placed on the top conductor and $-|Q|$ is placed on the bottom conductor as shown below.



(a) Calculate the electric field inside and outside of the plates.

USE GAUSS' LAW & SUPERPOSITION



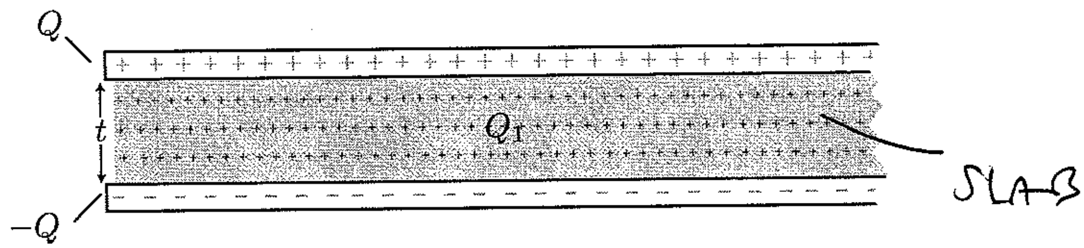
$2 \cdot E_0^+ \cdot A = -\frac{\rho^+}{\varepsilon_0} A$ $\qquad E_0^+ = -\frac{Q}{2\varepsilon_0 A}$ $\qquad$ NO Z-DEP

INSIDE:

$$E = E_0^+ + E_0^- = -\frac{Q}{\varepsilon_0 A}$$

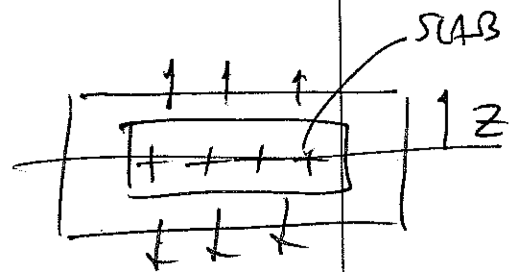
OUTSIDE:

$$E = E_0^+ - E_0^- = 0$$



(b) Calculate the electric field inside and outside of the plates when the region between the plates is filled with a uniformly charge. In other words, a positive charge $|Q_1| < |Q|$ is distributed evenly between the plates. The cross section of the setup is shown above.

FIRST CALC FIELD OF Q_1 ALONE
(ASSUME CHARGE DIST DOES NOT CHANGE)



$$2E_1(z) \cdot A \cdot \varepsilon_0 = \rho_1 A \cdot z$$
$$= \frac{Q_1}{A t} \cdot A \cdot z$$

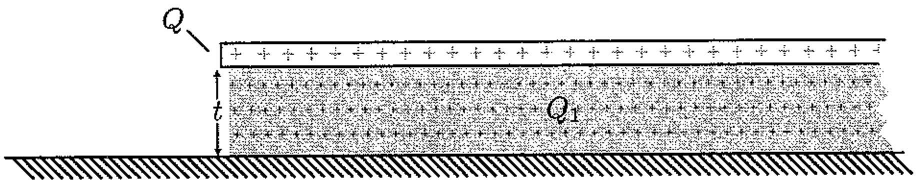
$$E_1(z) = \frac{Q_1}{2\varepsilon_0} \frac{\left(\frac{z}{t}\right)}{A}$$ INSIDE

INSIDE:

$$E(z) = E_0^+ + E_0^- + E_1(z)$$
$$= -\frac{Q}{\varepsilon_0 A} + \frac{Q_1}{2\varepsilon_0} \frac{\left(\frac{z}{t}\right)}{A}$$

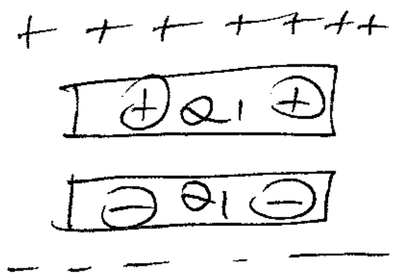
OUTSIDE: JUST THE FIELD OF THE SLAB REMAINS

$$E_1(z > t/2) = \frac{Q_1}{2\varepsilon_0 A}$$
$$E_1(z < -t/2)^0 = \frac{-Q_1}{2\varepsilon_0 A}$$



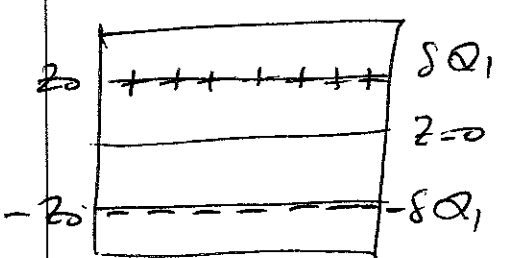
(c) Now assume the bottom plate is replaced by an infinitely large conducting plane (ground). Recalculate the fields inside and outside for this case. Note the charge on the top plate and the charge between the regions remains the same.

USE GAUSS' LAW BUT TAKE ~~ONE~~ EQ IMAGE SYSTEM



INSIDE: $= E_0^+ + E_0^- + \delta E_1^+ + \delta E_1^-$

$= 2E_0^+ + 2\delta E_1^+$



$\delta E(z<z_0) = \dfrac{Q_1/t}{\varepsilon_0 A}$

$\delta E(z>z_0) = 0$

$$E(z_0) = \int \delta E = \int_0^{z_0} + \cancel{\int_{z_0}^{t}}^{\,0} = \int_0^{z_0} = \left(\frac{z_0}{t}\right)\frac{Q_1}{\varepsilon_0 A}$$

$z_0 < t$: $E_T = -\left(\frac{z_0}{t}\right)\frac{Q_1}{\varepsilon_0 A} - \frac{Q}{\varepsilon_0 A}$

$z_0 > t$: $E_T = 0$