EECS 117

Lecture 5: Transmission Line Impedance Matching

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Open Line I/V

- The open transmission line has infinite VSWR and $\rho_L = 1$. At any given point along the transmission line

$$v(z) = V^+(e^{-j\beta z} + e^{j\beta z}) = 2V^+ \cos(\beta z)$$

whereas the current is given by

$$i(z) = \frac{V^+}{Z_0}(e^{-j\beta z} - e^{j\beta z})$$

or

$$i(z) = \frac{-2jV^+}{Z_0} \sin(\beta z)$$

Open Line Impedance (I)

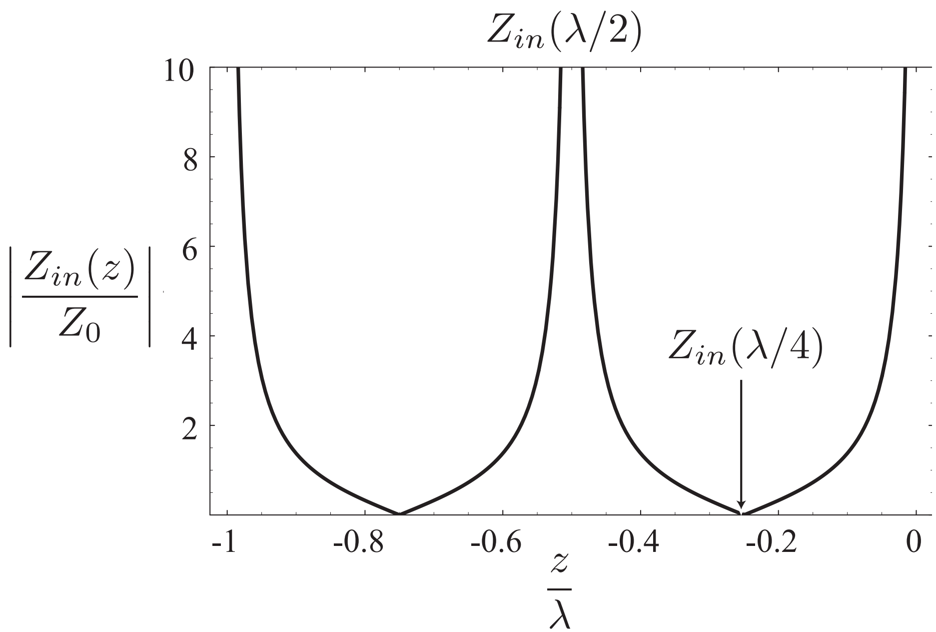
- The impedance at any point along the line takes on a simple form

$$Z_{in}(-\ell) = \frac{v(-\ell)}{i(-\ell)} = -jZ_0 \cot(\beta\ell)$$

- This is a special case of the more general transmission line equation with $Z_L = \infty$.
- Note that the impedance is purely imaginary since an open lossless transmission line cannot dissipate any power.
- We have learned, though, that the line stores reactive energy in a distributed fashion.

Open Line Impedance (II)

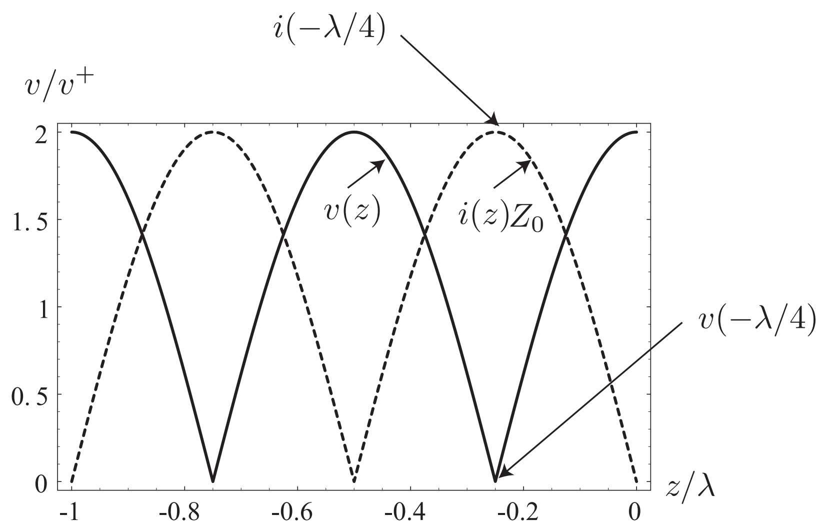
- A plot of the input impedance as a function of z is shown below



- The cotangent function takes on zero values when $\beta\ell$ approaches $\pi/2$ modulo 2π

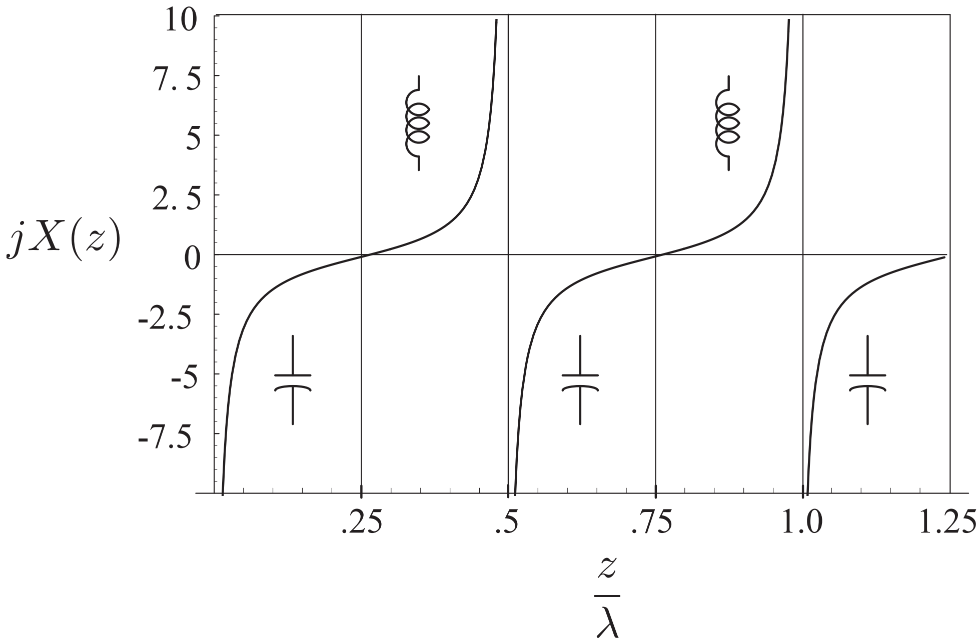
Open Line Impedance (III)

- Open transmission line can have zero input impedance!
- This is particularly surprising since the open load is in effect transformed from an open
- A plot of the voltage/current as a function of z is shown below



Open Line Reactance

- $\ell \ll \lambda/4 \rightarrow$ capacitor
- $\ell < \lambda/4 \rightarrow$ capacitive reactance
- $\ell = \lambda/4 \rightarrow$ short (acts like resonant series LC circuit)
- $\ell > \lambda/4$ but $\ell < \lambda/2 \rightarrow$ inductive reactance
- And the process repeats ...



$\lambda/2$ Transmission Line

- Plug into the general T-line equation for any multiple of $\lambda/2$

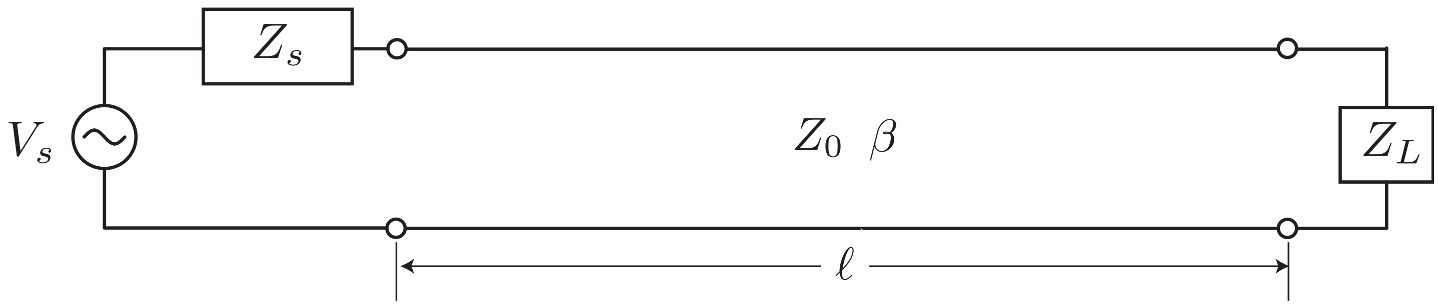
$$Z_{in}(-m\lambda/2) = Z_0 \frac{Z_L + jZ_0 \tan(-\beta\lambda/2)}{Z_0 + jZ_L \tan(-\beta\lambda/2)}$$

- $\beta\lambda m/2 = \frac{2\pi}{\lambda}\frac{\lambda m}{2} = \pi m$
- $\tan m\pi = 0$ if $m \in \mathcal{Z}$
- $Z_{in}(-\lambda m/2) = Z_0 \frac{Z_L}{Z_0} = Z_L$
- Impedance does not change ... it's periodic about $\lambda/2$ (not λ)

$\lambda/4$ Transmission Line

- Plug into the general T-line equation for any multiple of $\lambda/4$
- $\beta\lambda m/4 = \frac{2\pi}{\lambda}\frac{\lambda m}{4} = \frac{\pi}{2}m$
- $\tan m\frac{\pi}{2} = \infty$ if m is an odd integer
- $Z_{in}(-\lambda m/4) = \frac{Z_0^2}{Z_L}$
- $\lambda/4$ line transforms or “inverts” the impedance of the load

Effect of Source Impedance



- Up to now we have considered only a terminated semi-infinite line (or matched source)
- Consider the effect of the source impedance Z_s
- The voltage at the input of the line is given by

$$v_i n = v(-\ell) = v^+ e^{j\beta\ell}(1 + \rho_L e^{-2j\beta\ell})$$

Effect of Source Impedance

- By voltage division, the voltage can also be expressed as

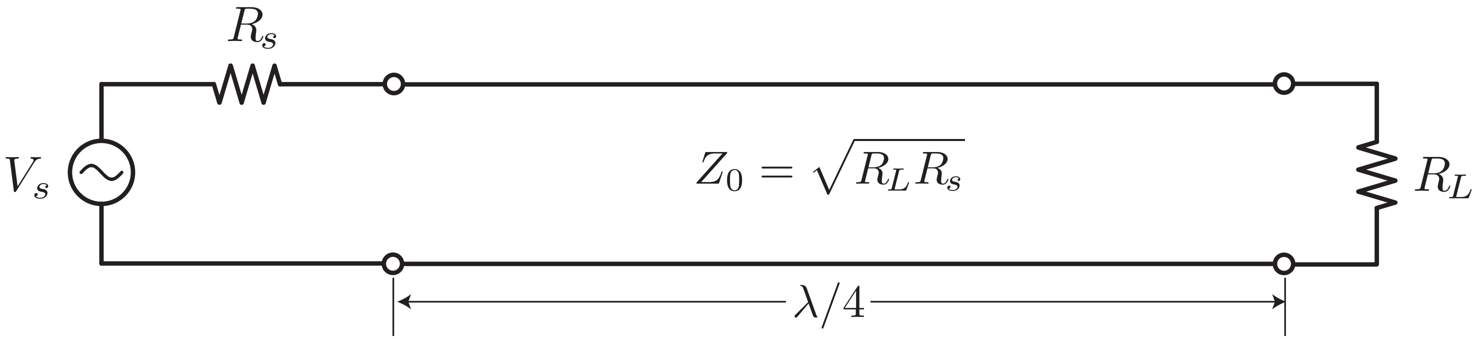
$$v_{in} = \frac{Z_{in}}{Z_{in} + Z_s} V_s$$

- Equating the two forms we arrive at

$$v^+ = \frac{Z_{in} V_s}{(Z_{in} + Z_s) e^{j\beta\ell} (1 + \rho_L e^{-2j\beta\ell})}$$

- In a matched system, we desire the input impedance seen into the T-line to be the conjugate of the source impedance (maximum power transfer)
- Impedance matching is required to acheive this goal

$\lambda/4$ Impedance Match



- If the source and load are real resistors, then a quarter-wave line can be used to match the source and load impedances
- Recall that the impedance looking into the quarter-wave line is the “inverse” of the load impedance

$$Z_{in}(z = -\lambda/4) = \frac{Z_0^2}{Z_L}$$

SWR on $\lambda/4$ Line

- In this case, therefore, we equate this to the desired source impedance $Z_{in} = \frac{Z_0^2}{R_L} = R_s$
- The quarter-wave line should therefore have a characteristic impedance that is the geometric mean $Z_0 = \sqrt{R_s R_L}$
- Since $Z_0 \neq R_L$, the line has a non-zero reflection coefficient

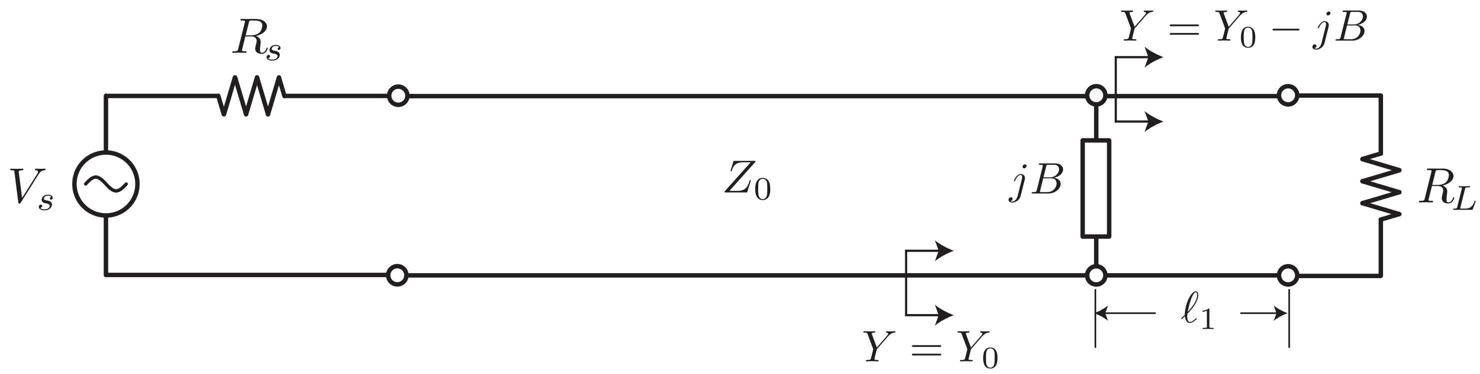
$$SWR = \frac{R_L - \sqrt{R_L R_s}}{R_L + \sqrt{R_L R_s}}$$

- It also therefore has standing waves on the T-line
- The non-unity SWR is given by $\frac{1+|\rho_L|}{1-|\rho_L|}$

Interpretation of SWR on $\lambda/4$ Line

- Consider a generic lossless transformer ($R_L > R_s$)
- Thus to make the load look smaller to match to the source, the voltage of the source should be increased in magnitude
- But since the transformer is lossless, the current will likewise decrease in magnitude by the same factor
- With the $\lambda/4$ transformer, the location of the voltage minimum to maximum is $\lambda/4$ from load (since the load is real)
- Voltage/current is thus increased/decreased by a factor of $1 + |\rho_L|$ at the load
- Hence the impedance decreased by a factor of $(1 + |\rho_L|)^2$

Matching with Lumped Elements (I)



- Recall the input impedance looking into a T-line varies periodically

$$Z_{in}(-\ell) = Z_0 \frac{Z_L + jZ_0 \tan(\beta\ell)}{Z_0 + jZ_L \tan(\beta\ell)}$$

- Move a distance ℓ_1 away from the load such that the real part of Z_{in} has the desired value

Matching with Lumped Elements (II)

- Then place a shunt or series impedance on the T-line to obtain desired reactive part of the input impedance (e.g. zero reactance for a real match)
- For instance, for a shunt match, the input admittance looking into the line is

$$y(z) = Y(z)/Y_0 = \frac{1 - \rho_L e^{j2\beta z}}{1 + \rho_L e^{j2\beta z}}$$

- At a distance ℓ_1 we desire the normalized admittance to be $y_1 = 1 - jb$
- Substitute $\rho_L = \rho e^{j\theta}$ and solve for ℓ_1 and let $\psi = 2\beta z + \theta$

$$\frac{1 - \rho e^{j\psi}}{1 + \rho e^{j\psi}} = \frac{1 - \rho^2 - j2\rho \sin\psi}{1 + 2\rho\cos\psi + \rho^2}$$

Matching with Lumped Elements (III)

- Solve for ψ (and then ℓ_1) from $\Re(y) = 1$

$$\psi = \theta - 2\beta\ell = \cos^{-1}(-\rho)$$

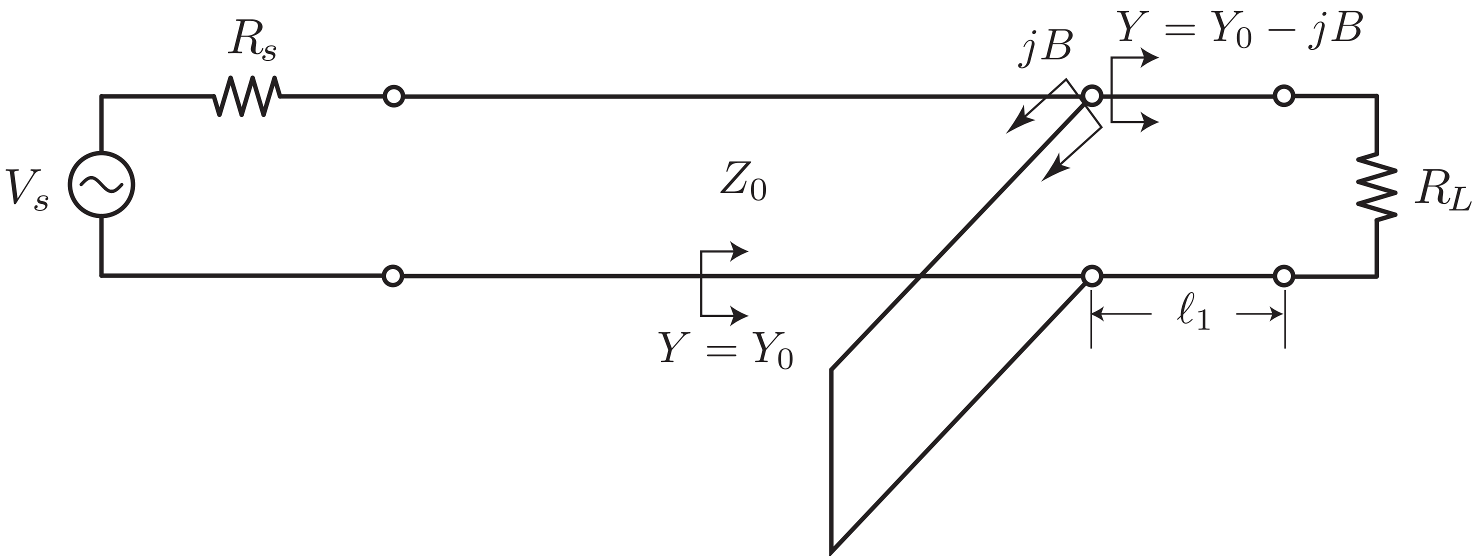
$$\ell_1 = \frac{\theta - \psi}{2\beta} = \frac{\lambda}{4\pi}\left(\theta - \cos^{-1}(-\rho)\right)$$

- At ℓ_1, the imaginary part of the input admittance is

$$b = \Im(y_1) = \pm\frac{2\rho}{\sqrt{1-\rho^2}}$$

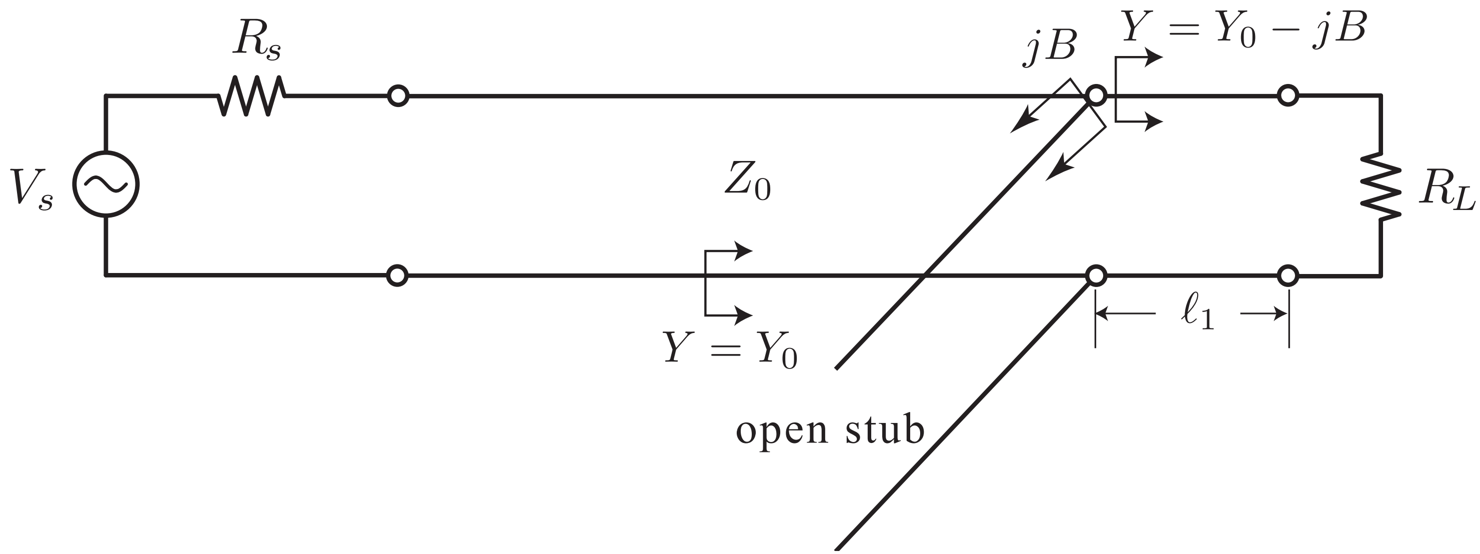
- Placing a reactance of value $-b$ in shunt provided impedance match at this particular frequency
- If the location of ℓ_1 is not convenient, we can achieve the same result by move back a multiple of $\lambda/2$

Matching with Stubs (I)



- At high frequencies the matching technique discussed above is difficult due to the lack of lumped passive elements (inductors and capacitors)
- But short/open pieces of transmission lines simulate fixed reactance over a narrow band
- A shorted stub with $\ell < \lambda/4$ looks like an inductor

Matching with Stubs (II)



- An open stub with $\ell < \lambda/4$ looks like a capacitor
- The procedure is identical to the case with lumped elements but instead of using a capacitor or inductor, we use shorted or open transmission lines
- Shunt stubs are easier to fabricate than series stubs

Lossy Transmission Lines

- Lossy lines are analyzed in the same way as lossless lines
- Low-loss lines are often approximated as lossless lines
- Recall the general voltage and current on the line

$$v(z) = v^+ e^{-\gamma z} + v^- e^{\gamma z} \qquad i(z) = \frac{v^+}{Z_0} e^{-\gamma z} - \frac{v^-}{Z_0} e^{\gamma z}$$

- Where $\gamma = \alpha + j\beta$ is the complex propagation constant. On an infinite line, α represents an exponential decay in the wave amplitude

$$v(z) = e^{-\alpha z} \times \left(v^+ e^{-j\beta z} \right)$$

Transmission Line Dispersion

- What about dispersion? Is the amplitude attenuation a function of frequency? If so, the wave will distort. Moreover, how does the speed of propagation vary with frequency?
- For a dispersionless line, the output should be a linearly scaled delayed version of the input $v_{out}(t) = K v_{in}(t - \tau)$, or in the frequency domain

$$V_{out}(j\omega) = K V_{in}(j\omega) e^{-j\omega\tau}$$

- The transfer function has constant magnitude $|H(j\omega)|$ and linear phase $\angle H(j\omega) = -\omega\tau$
- The propagation constant $j\beta$ should therefore be a linear function of frequency and α should be a constant
- In general, a lossy transmission line has dispersion