EECS 117

Lecture 4: Transmission Lines with Time Harmonic Excitation

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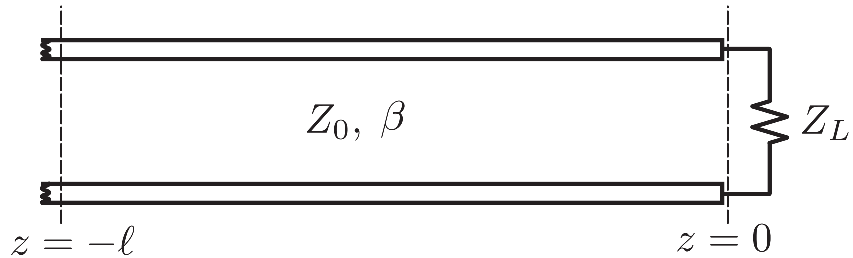
Lossless T-Line Termination



- Okay, lossless line means $\gamma = j\beta$ ($\alpha = 0$), and $\Im(Z_0) = 0$ (real characteristic impedance independent of frequency)
- The voltage/current phasors take the standard form

$$v(z) = V^+ e^{-\gamma z} + V^- e^{\gamma z}$$

$$i(z) = \frac{V^+}{Z_0} e^{-\gamma z} - \frac{V^-}{Z_0} e^{\gamma z}$$

Lossless T-Line Termination (cont)

- At load $Z_L = \frac{v(0)}{i(0)} = \frac{V^+ + V^-}{V^+ - V^-} Z_0$
- The reflection coefficient has the same form

$$\rho_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

- Can therefore write

$$v(z) = V^+ \left(e^{-j\beta z} + \rho_L e^{j\beta z} \right)$$

$$i(z) = \frac{V^+}{Z_0} \left(e^{-j\beta z} - \rho_L e^{j\beta z} \right)$$

Power on T-Line (I)

- Let's calculate the average power dissipation on the line at point z

$$P_{av}(z) = \frac{1}{2}\Re\left[v(z)i(z)^*\right]$$

- Or using the general solution

$$P_{av}(z) = \frac{1}{2}\frac{|V^+|^2}{Z_0}\Re\left(\left(e^{-j\beta z} + \rho_L e^{j\beta z}\right)\left(e^{j\beta z} - \rho_L^* e^{-j\beta z}\right)\right)$$

- The product in the $\Re$ terms can be expanded into four terms

$$1 + \underbrace{\rho_L e^{2j\beta z} - \rho_L^* e^{2j\beta z}}_{a-a^*} - |\rho_L|^2$$

- Notice that $a - a^* = 2j\Im(a)$

Power on T-Line (II)

- The average power dissipated at z is therefore

$$P_{av} = \frac{|V^+|^2}{2Z_0}\left(1 - |\rho_L|^2\right)$$

- Power flow is constant (independent of z) along line (lossless)
- No power flows if $|\rho_L| = 1$ (open or short)
- Even though power is constant, voltage and current are not!

Voltage along T-Line

- When the termination is matched to the line impedance $Z_L = Z_0$, $\rho_L = 0$ and thus the voltage along the line $|v(z)| = |V^+|$ is constant. Otherwise

$$|v(z)| = |V^+||1 + \rho_L e^{2j\beta z}| = |V^+||1 + \rho_L e^{-2j\beta\ell}|$$

- The voltage magnitude along the line can be written as

$$|v(-\ell)| = |V^+||1 + |\rho_L|e^{j(\theta - 2\beta\ell)}|$$

- The voltage is maximum when the $2\beta\ell$ is a equal to $\theta + 2k\pi$, for any integer k; in other words, the reflection coefficient phase modulo 2π

$$V_{max} = |V^+|(1 + |\rho_L|)$$

Voltage Standing Wave Ratio (SWR)

- Similarly, minimum when $\theta + k\pi$, where k is an integer $k \neq 0$

$$V_{min} = |V^+|(1 - |\rho_L|)$$

- The ratio of the maximum voltage to minimum voltage is an important metric and commonly known as the voltage standing wave ratio, VSWR (Sometimes pronounced viswar), or simply the standing wave ratio SWR

$$VSWR = \frac{V_{max}}{V_{min}} = \frac{1 + |\rho_L|}{1 - |\rho_L|}$$

- It follows that for a shorted or open transmission line the VSWR is infinite, since $|\rho_L| = 1$.

SWR Location

- Physically the maxima occur when the reflected wave adds in phase with the incoming wave, and minima occur when destructive interference takes place. The distance between maxima and minima is π in phase, or $2\beta\delta x = \pi$, or

$$\delta x = \frac{\pi}{2\beta} = \frac{\lambda}{4}$$

- VSWR is important because it can be deduced with a *relative* measurement. Absolute measurements are difficult at microwave frequencies. By measuring VSWR, we can readily calculate $|\rho_L|$.

VSWR $\rightarrow$ Impedance Measurement

- By measuring the location of the voltage minima from an unknown load, we can solve for the load reflection coefficent phase θ

$$\psi_{min} = \theta - 2\beta\ell_{min} = \pm\pi$$

- Note that

$$|v(-\ell_{min})| = |V^+||1 + |\rho_L|e^{j\psi_{min}}|$$

- Thus an unknown impedance can be characterized at microwave frequencies by measuring VSWR and ℓ_{min} and computing the load reflection coefficient. This was an important measurement technique that has been largely supplanted by a modern network analyzer with built-in digital calibration and correction.

VSWR Example

- Consider a transmission line terminated in a load impedance $Z_L = 2Z_0$. The reflection coefficient at the load is purely real

$$\rho_L = \frac{z_L - 1}{z_L + 1} = \frac{2-1}{2+1} = \frac{1}{3}$$

- Since $1 + |\rho_L| = 4/3$ and $1 - |\rho_L| = 2/3$, the VSWR is equal to 2.
- Since the load is real, the voltage minima will occur at a distance of $\lambda/4$ from the load

Impedance of T-Line (I)

- We have seen that the voltage and current along a transmission line are altered by the presence of a load termination. At an arbitrary point z, wish to calculate the input impedadnce, or the ratio of the voltage to current relative to the load impdance Z_L

$$Z_{in}(-\ell) = \frac{v(-\ell)}{i(-\ell)}$$

- It shall be convenient to define an analogous reflection coefficient at an arbitrary position along the line

$$\rho(-\ell) = \frac{V^- e^{-j\beta\ell}}{V^+ e^{j\beta\ell}} = \rho_L e^{-2j\beta\ell}$$

Impedance of T-Line (II)

- $\rho(z)$ has a constant magnitude but a periodic phase. From this we may infer that the input impedance of a transmission line is also periodic (relation btwn ρ and Z is one-to-one)

$$Z_{in}(-\ell) = Z_0 \frac{1 + \rho_L e^{-2j\beta\ell}}{1 - \rho_L e^{-2j\beta\ell}}$$

- The above equation is of paramount important as it expresses the input impedance of a transmission line as a function of position ℓ away from the termination.

Impedance of T-Line (III)

- This equation can be transformed into another more useful form by substituting the value of ρ_L

$$\rho_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$Z_{in}(-\ell) = Z_0 \frac{Z_L(1 + e^{-2j\beta\ell}) + Z_0(1 - e^{-2j\beta\ell})}{Z_0(1 + e^{-2j\beta\ell}) + Z_L(1 - e^{-2j\beta\ell})}$$

Using the common complex expansions for sine and cosine, we have

$$\tan(x) = \frac{\sin(x)}{\cos(x)} = \frac{(e^{jx} - e^{-jx})/2j}{(e^{jx} + e^{-jx})/2}$$

Impedance of T-Line (IV)

- The expression for the input impedance is now written in the following form

$$Z_{in}(-\ell) = Z_0 \frac{Z_L + jZ_0 \tan(\beta\ell)}{Z_0 + jZ_L \tan(\beta\ell)}$$

- This is the most important equation of the lecture, known sometimes as the "transmission line equation"

Shorted Line I/V

- The shorted transmission line has infinite VSWR and $\rho_L = -1$. Thus the minimum voltage $V_{min} = |V^+|(1 - |\rho_L|) = 0$, as expected. At any given point along the transmission line

$$v(z) = V^+(e^{-j\beta z} - e^{j\beta z}) = -2jV^+ \sin(\beta z)$$

whereas the current is given by

$$i(z) = \frac{V^+}{Z_0}(e^{-j\beta z} + e^{j\beta z})$$

or

$$i(z) = \frac{2V^+}{Z_0} \cos(\beta z)$$

Shorted Line Impedance (I)

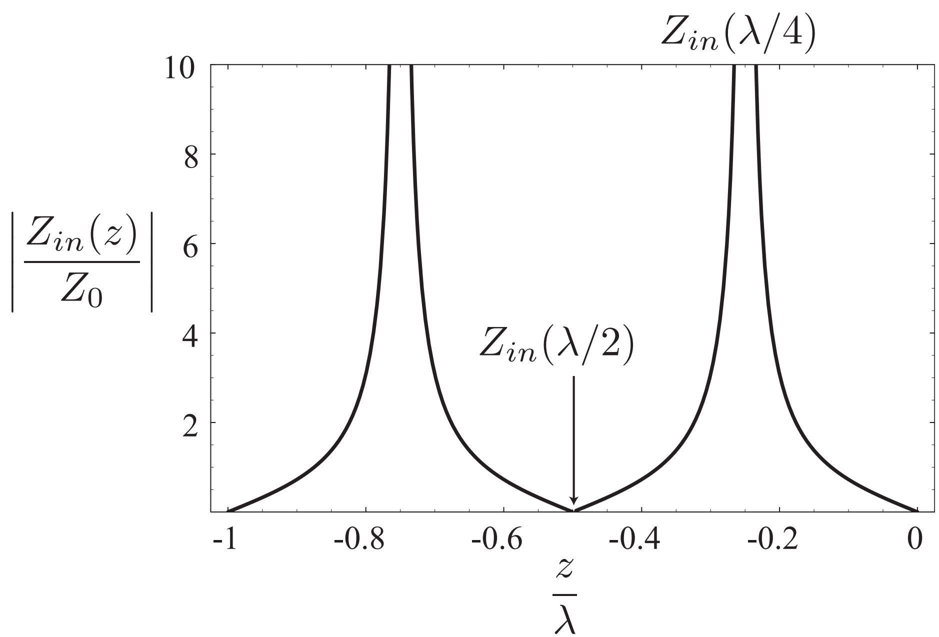
- The impedance at any point along the line takes on a simple form

$$Z_{in}(-\ell) = \frac{v(-\ell)}{i(-\ell)} = jZ_0 \tan(\beta\ell)$$

- This is a special case of the more general transmision line equation with $Z_L = 0$.
- Note that the impedance is purely imaginary since a shorted lossless transmission line cannot dissipate any power.
- We have learned, though, that the line stores reactive energy in a distributed fashion.

Shorted Line Impedance (II)

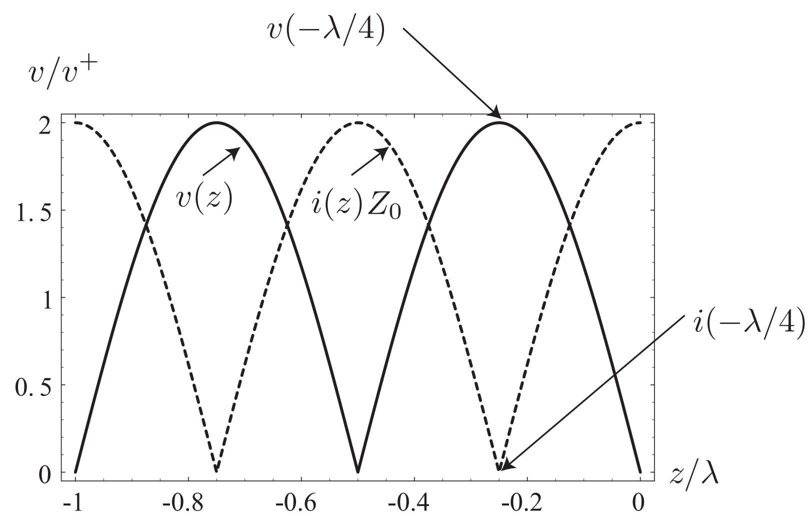
- A plot of the input impedance as a function of z is shown below



- The tangent function takes on infinite values when $\beta\ell$ approaches $\pi/2$ modulo 2π

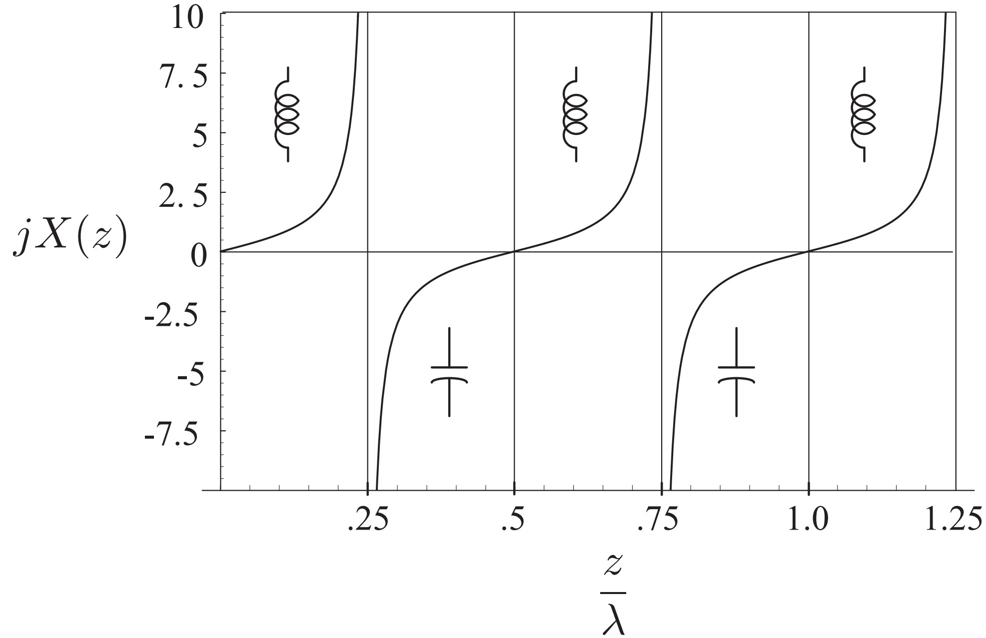
Shorted Line Impedance (III)

- Shorted transmission line can have infinite input impedance!
- This is particularly surprising since the load is in effect transformed from a short of $Z_L = 0$ to an infinite impedance.
- A plot of the voltage/current as a function of z is shown below



Shorted Line Reactance

- $\ell \ll \lambda/4 \rightarrow$ inductor
- $\ell < \lambda/4 \rightarrow$ inductive reactance
- $\ell = \lambda/4 \rightarrow$ open (acts like resonant parallel LC circuit)
- $\ell > \lambda/4$ but $\ell < \lambda/2 \rightarrow$ capacitive reactance
- And the process repeats ...



Open Line I/V

- The open transmission line has infinite VSWR and $\rho_L = 1$. At any given point along the transmission line

$$v(z) = V^+(e^{-j\beta z} + e^{j\beta z}) = 2V^+\cos(\beta z)$$

whereas the current is given by

$$i(z) = \frac{V^+}{Z_0}(e^{-j\beta z} - e^{j\beta z})$$

or

$$i(z) = \frac{-2jV^+}{Z_0}\sin(\beta z)$$

Open Line Impedance (I)

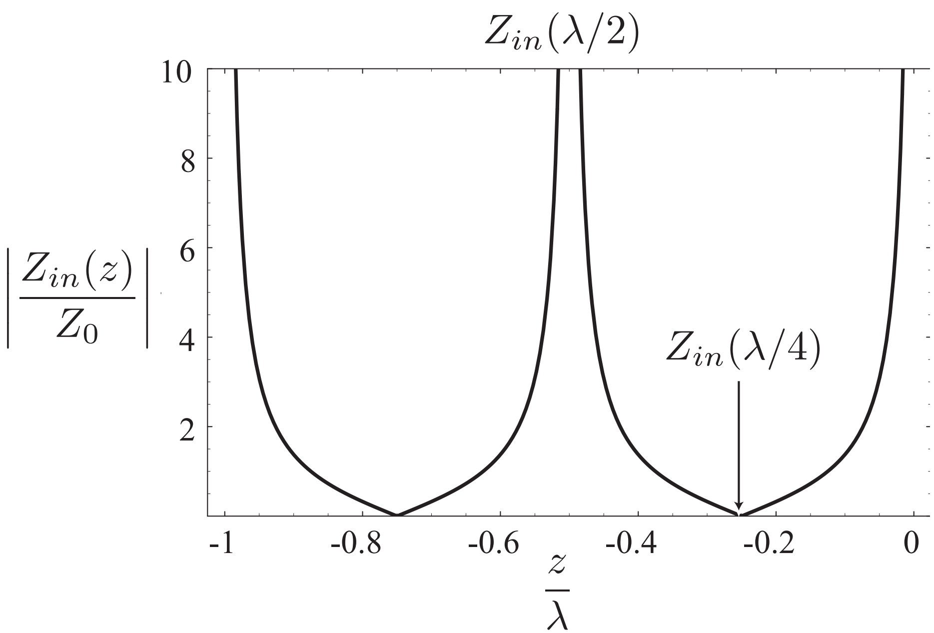
- The impedance at any point along the line takes on a simple form

$$Z_{in}(-\ell) = \frac{v(-\ell)}{i(-\ell)} = -jZ_0 \cot(\beta\ell)$$

- This is a special case of the more general transmision line equation with $Z_L = \infty$.
- Note that the impedance is purely imaginary since an open lossless transmission line cannot dissipate any power.
- We have learned, though, that the line stores reactive energy in a distributed fashion.

Open Line Impedance (II)

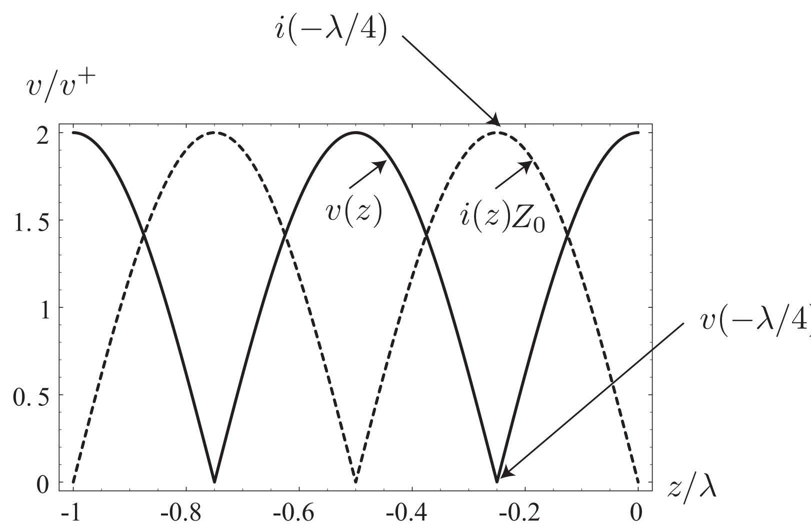
- A plot of the input impedance as a function of z is shown below



- The cotangent function takes on zero values when $\beta\ell$ approaches $\pi/2$ modulo 2π

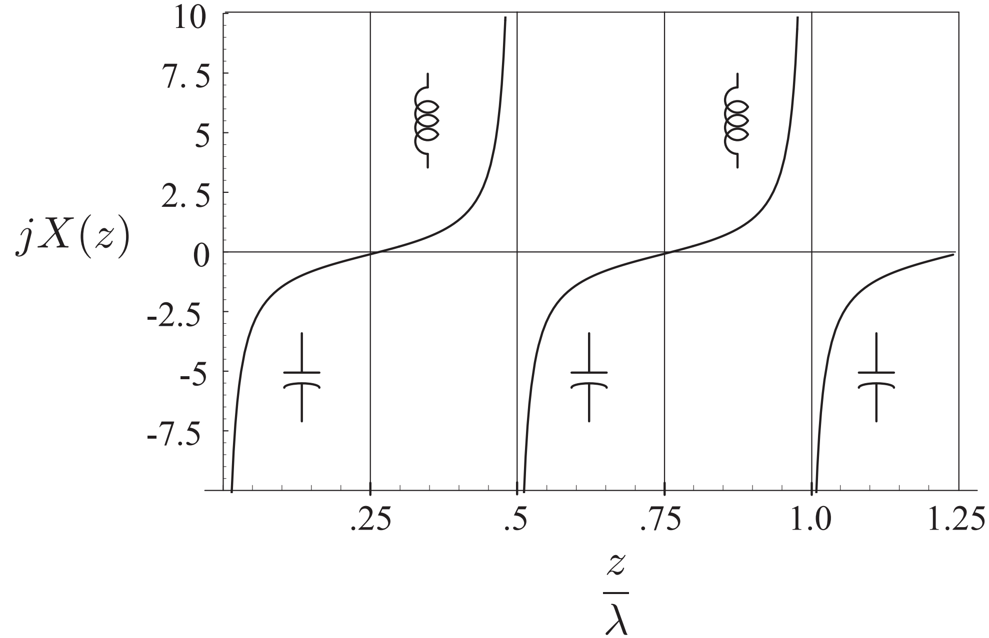
Open Line Impedance (III)

- Open transmission line can have zero input impedance!
- This is particularly surprising since the open load is in effect transformed from an open
- A plot of the voltage/current as a function of z is shown below



Open Line Reactance

- $\ell \ll \lambda/4 \rightarrow$ capacitor
- $\ell < \lambda/4 \rightarrow$ capacitive reactance
- $\ell = \lambda/4 \rightarrow$ short (acts like resonant series LC circuit)
- $\ell > \lambda/4$ but $\ell < \lambda/2 \rightarrow$ inductive reactance
- And the process repeats ...



$\lambda/2$ Transmission Line

- Plug into the general T-line equaiton for any multiple of $\lambda/2$

$$Z_{in}(-m\lambda/2) = Z_0 \frac{Z_L + jZ_0 \tan(-\beta\lambda/2)}{Z_0 + jZ_L \tan(-\beta\lambda/2)}$$

- $\beta\lambda m/2 = \frac{2\pi}{\lambda}\frac{\lambda m}{2} = \pi m$
- $\tan m\pi = 0$ if $m \in \mathcal{Z}$
- $Z_{in}(-\lambda m/2) = Z_0 \frac{Z_L}{Z_0} = Z_L$
- Impedance does not change ... it's periodic about $\lambda/2$ (not λ)

$\lambda/4$ Transmission Line

- Plug into the general T-line equaiton for any multiple of $\lambda/4$
- $\beta\lambda m/4 = \frac{2\pi}{\lambda}\frac{\lambda m}{4} = \frac{\pi}{2}m$
- $\tan m\frac{\pi}{2} = \infty$ if m is an odd integer
- $Z_{in}(-\lambda m/4) = \frac{Z_0^2}{Z_L}$
- $\lambda/4$ line transforms or “inverts” the impedance of the load