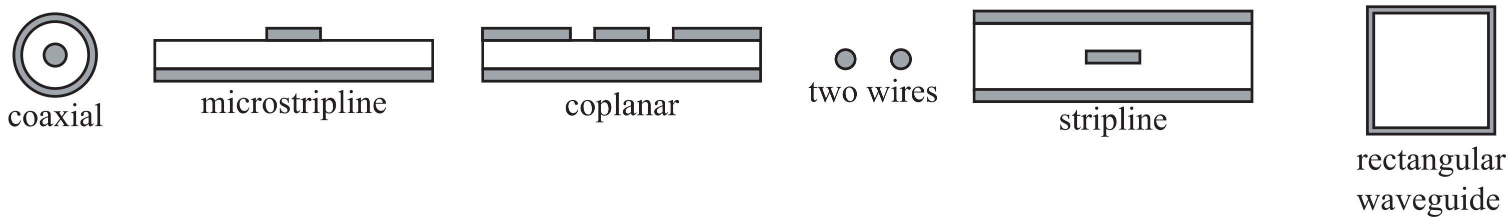
EECS 117

Lecture 3: Transmission Line Junctions / Time Harmonic Excitation

Prof. Niknejad

University of California, Berkeley

Transmission Line Menagerie

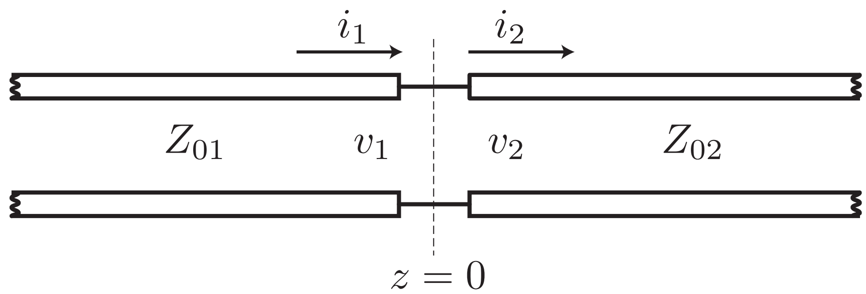


- T-Lines come in many shapes and sizes
- Coaxial usually 75Ω or 50Ω (cable TV, Internet)
- Microstrip lines are common on printed circuit boards (PCB) and integrated circuit (ICs)
- Coplanar also common on PCB and ICs
- Twisted pairs is almost a T-line, ubiquitous for phones/Ethernet

Waveguides and Transmission Lines

- The transmission lines we've been considering have been propagating the "TEM" mode or Transverse Electro-Magnetic. Later we'll see that they can also propagation other modes
- Waveguides cannot propagate TEM but propagation TM (Transverse Magnetic) and TE (Transverse Electric)
- In general, *any* set of more than one lossless conductors with uniform cross-section can transmit TEM waves. Low loss conductors are commonly approximated as lossless.

Cascade of T-Lines (I)



- Consider the junction between two transmission lines Z_{01} and Z_{02}
- At the interface $z = 0$, the boundary conditions are that the voltage/current has to be continuous

$$v_1^+ + v_1^- = v_2^+$$

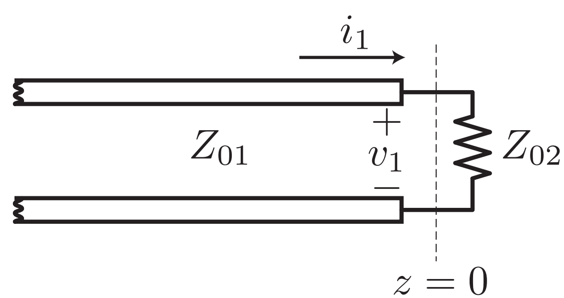
$$(v_1^+ - v_1^-)/Z_{01} = v_2^+/Z_{02}$$

Cascade of T-Lines (II)

- Solve these equations in terms of v_1^+
- The reflection coefficient has the same form (easy to remember)

$$\Gamma = \frac{v_1^-}{v_1^+} = \frac{Z_{02} - Z_{01}}{Z_{01} + Z_{02}}$$

- The second line looks like a load impedance of value Z_{02}



Transmission Coefficient

- The wave launched on the new transmission line at the interface is given by

$$v_2^+ = v_1^+ + v_1^- = v_1^+(1+\Gamma) = \tau v_1^+$$

- This "transmitted" wave has a coefficient

$$\tau = 1 + \Gamma = \frac{2Z_{02}}{Z_{01} + Z_{02}}$$

- Note the incoming wave carries a power

$$P_{in} = \frac{|v_1^+|^2}{2Z_{01}}$$

Conservation of Energy

- The reflected and transmitted waves likewise carry a power of

$$P_{ref} = \frac{|v_1^-|^2}{2Z_{01}} = |\Gamma|^2 \frac{|v_1^+|^2}{2Z_{01}} \qquad P_{tran} = \frac{|v_2^+|^2}{2Z_{02}} = |\tau|^2 \frac{|v_1^+|^2}{2Z_{02}}$$

- By conservation of energy, it follows that

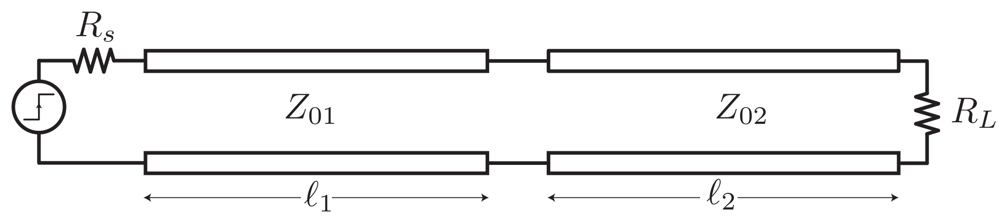
$$P_{in} = P_{ref} + P_{tran}$$

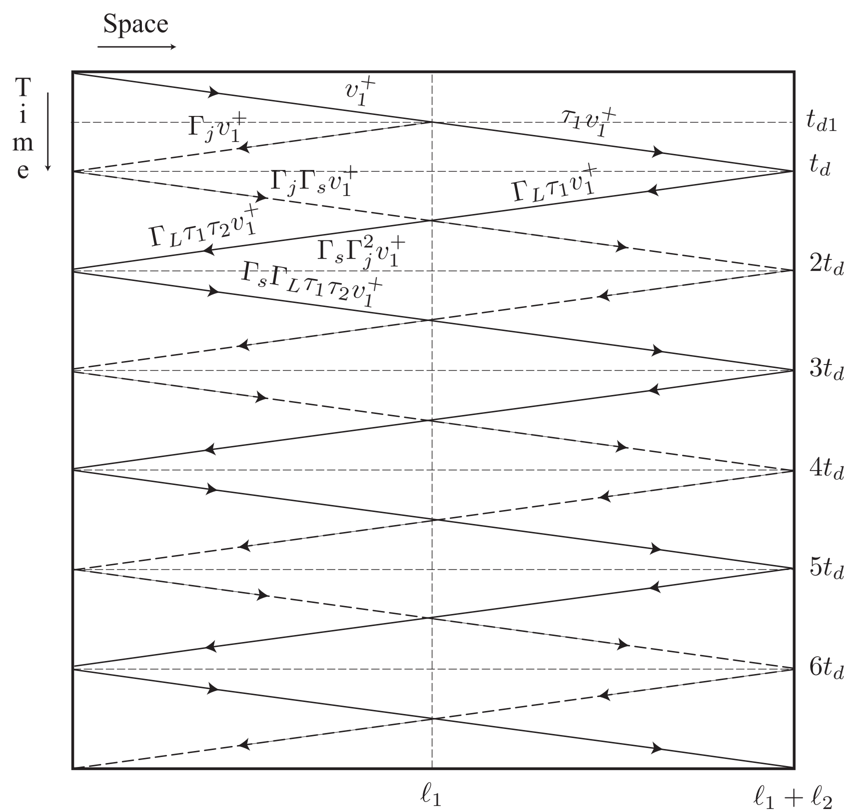
$$\frac{1}{Z_{02}}\tau^2 + \frac{1}{Z_{01}}\Gamma^2 = \frac{1}{Z_{01}}$$

- You can verify that this relation holds!

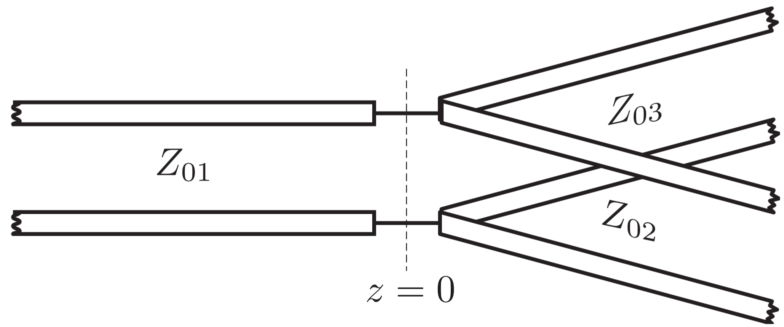
Bounce Diagram

- Consider the bounce diagram for the following arrangement





Junction of Parallel T-Lines

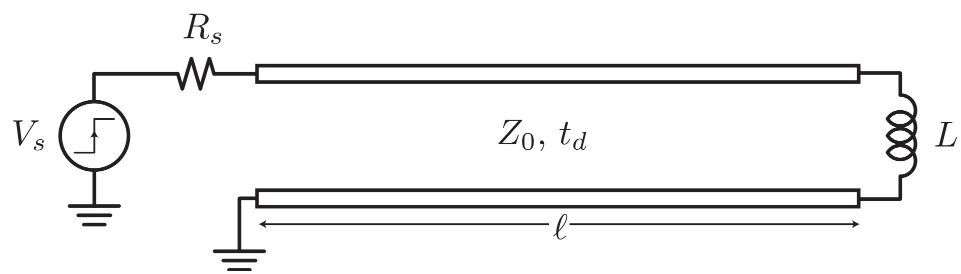


- Again invoke voltage/current continuity at the interface

$$v_1^+ + v_1^- = v_2^+ = v_3^+ \qquad \frac{v_1^+ - v_1^-}{Z_{01}} = \frac{v_2^+}{Z_{02}} + \frac{v_3^+}{Z_{02}}$$

- But $v_2^+ = v_3^+$, so the interface just looks like the case of two transmission lines Z_{01} and a new line with char. impedance $Z_{01}||Z_{02}$.

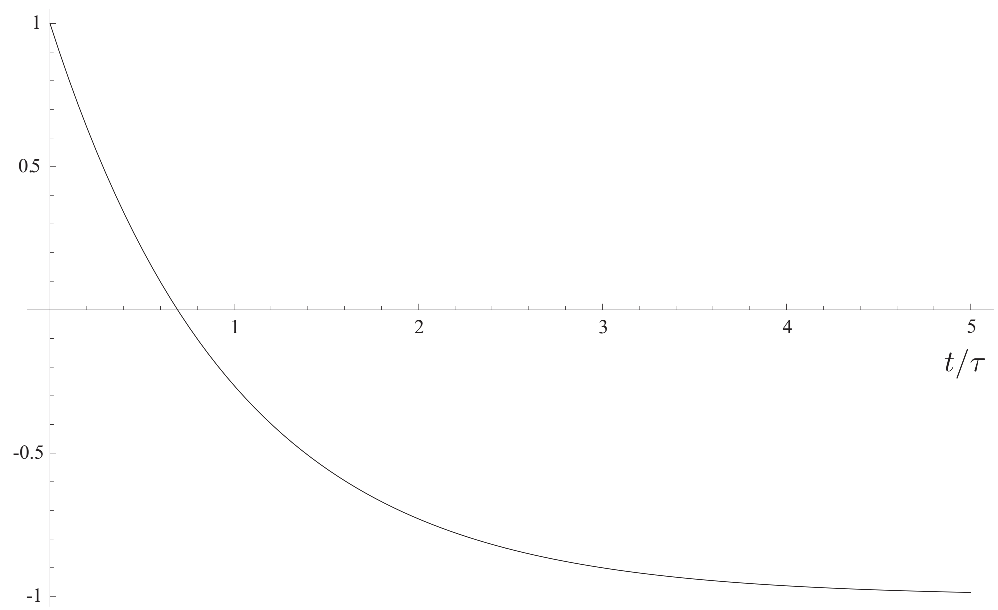
Reactive Terminations (I)



- Let’s analyze the problem intuitively first
- When a pulse first “sees” the inductance at the load, it looks like an open so $\Gamma_0 = +1$
- As time progresses, the inductor looks more and more like a short! So $\Gamma_\infty = -1$

Reactive Terminations (II)

- So intuitively we might expect the reflection coefficient to look like this:



- The graph starts at $+1$ and ends at -1. In between we'll see that it goes through exponential decay (1st order ODE)

Reactive Terminations (III)

- Do equations confirm our intuition?

$$v_L = L\frac{di}{dt} = L\frac{d}{dt}\left(\frac{v^+}{Z_0} - \frac{v^-}{Z_0}\right)$$

- And the voltage at the load is given by $v^+ + v^-$

$$v^- + \frac{L}{Z_0}\frac{dv^-}{dt} = \frac{L}{Z_0}\frac{dv^+}{dt} - v^+$$

- The right hand side is known, it's the incoming waveform

Solution for Reactive Term

- For the step response, the derivative term on the RHS is zero at the load

$$v^+ = \frac{Z_0}{Z_0 + R_s} V_s$$

- So we have a simpler case $\frac{dv^+}{dt} = 0$
- We must solve the following equation

$$v^- + \frac{L}{Z_0}\frac{dv^-}{dt} = -v^+$$

- For simplicity, assume at $t = 0$ the wave v^+ arrives at load

Laplace Domain Solution I

- In the Laplace domain

$$V^-(s) + \frac{sL}{Z_0}V^-(s) - \frac{L}{Z_0}v^-(0) = -v^+/s$$

- Solve for reflection $V^-(s)$

$$V^-(s) = \frac{v^-(0)L/Z_0}{1 + sL/Z_0} - \frac{v^+}{s(1 + sL/Z_0)}$$

- Break this into basic terms using partial fraction expansion

$$\frac{-1}{s(1 + sL/Z_0)} = \frac{-1}{1 + sL/Z_0} + \frac{L/Z_0}{1 + sL/Z_0}$$

Laplace Domain Solution (II)

- Invert the equations to get back to time domain $t > 0$

$$v^-(t) = (v^-(0) + v^+)e^{-t/\tau} - v^+$$

- Note that $v^-(0) = v^+$ since initially the inductor is an open
- So the reflection coefficient is

$$\Gamma(t) = 2e^{-t/\tau} - 1$$

- The reflection coefficient decays with time constant L/Z_0

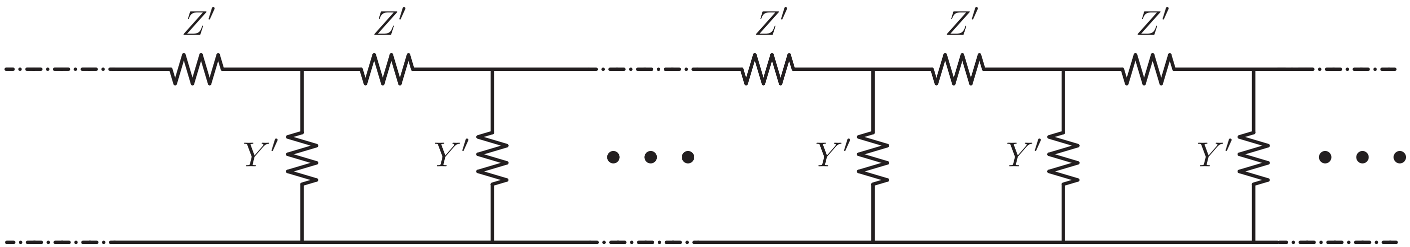
Time Harmonic Steady-State

- Compared with general transient case, sinusoidal case is very easy $\frac{\partial}{\partial t} \rightarrow j\omega$
- Sinusoidal steady state has many important applications for RF/microwave circuits
- At high frequency, T-lines are like interconnect for distances on the order of λ
- Shorted or open T-lines are good resonators
- T-lines are useful for impedance matching

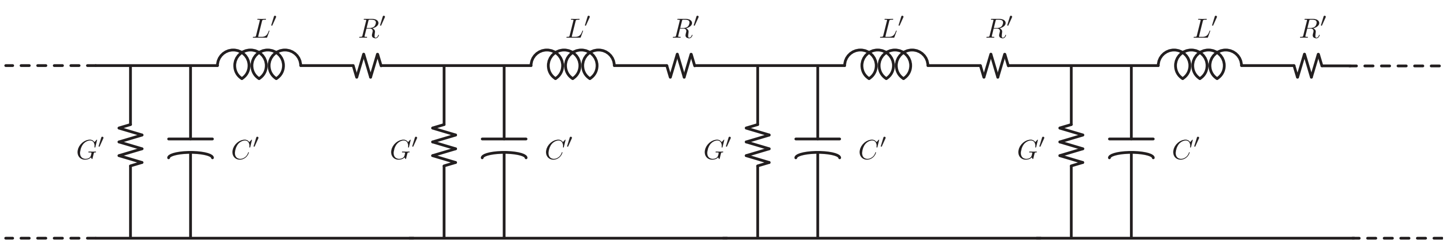
Why Sinusoidal Steady-State?

- Typical RF system modulates a sinusoidal carrier (either frequency or phase)
- If the modulation bandwidth is much smaller than the carrier, the system looks like it's excited by a pure sinusoid
- Cell phones are a good example. The carrier frequency is about $1\ \mathrm{GHz}$ and the voice digital modulation is about $200\ \mathrm{kHz}$(GSM) or $1.25\ \mathrm{MHz}$(CDMA), less than a 0.1% of the bandwidth/carrier

Generalized Distributed Circuit Model



- Z': impedance per unit length (e.g. $Z' = j\omega L' + R'$)
- Y': admittance per unit length (e.g. $Y' = j\omega C' + G'$)
- A lossy T-line might have the following form (but we'll analyze the general case)



Time Harmonic Telegrapher's Equations

- Applying KCL and KVL to a infinitesimal section

$$v(z+\delta z) - v(z) = -Z'\delta z i(z)$$

$$i(z+\delta z) - i(z) = -Y'\delta z v(z)$$

- Taking the limit as before ($\delta z \to 0$)

$$\frac{dv}{dz} = -Zi(z)$$

$$\frac{di}{dz} = -Yv(z)$$

Sin. Steady-State (SSS) Voltage/Current

- Taking derivatives (notice z is the only variable) we arrive at

$$\frac{d^2v}{dz^2} = -Z\frac{di}{dz} = YZv(z) = \gamma^2 v(z)$$

$$\frac{d^2i}{dz^2} = -Y\frac{dv}{dz} = YZi(z) = \gamma^2 i(z)$$

- Where the propagation constant γ is a complex function

$$\gamma = \alpha + j\beta = \sqrt{(R' + j\omega L')(G' + j\omega C')}$$

- The general solution to $D^2G - \gamma^2 G = 0$ is $e^{\pm\gamma z}$

Lossless Line for SSS

- The voltage and current are related (just as before, but now easier to derive)

$$v(z) = V^+ e^{-\gamma z} + V^- e^{\gamma z}$$

$$i(z) = \frac{V^+}{Z_0} e^{-\gamma z} - \frac{V^-}{Z_0} e^{\gamma z}$$

- Where $Z_0 = \sqrt{\frac{Z'}{Y'}}$ is the characteristic impedance of the line (function of frequency with loss)
- For a lossless line we discussed before, $Z' = j\omega L'$ and $Y' = j\omega C'$
- Propagation constant is imaginary

$$\gamma = \sqrt{j\omega L' j\omega C'} = j\sqrt{L'C'}\omega$$

Back to Time-Domain

- Recall that the *real* voltages and currents are the $\Re$ and $\Im$ parts of

$$v(z,t) = e^{\pm\gamma z} e^{j\omega t} = e^{j\omega t \pm \beta z}$$

- Thus the voltage/current waveforms are sinusoidal in space and time
- Sinusoidal source voltage is transmitted unaltered onto T-line (with delay)
- If there is loss, then γ has a real part α, and the wave decays or grows on the T-line

$$e^{\pm\gamma z} = e^{\pm\alpha z} e^{\pm j\beta z}$$

- The first term represents amplitude response of the T-line

Passive T-Line/Wave Speed

- For a passive line, we expect the amplitude to decay due to loss on the line
- The speed of the wave is derived as before. In order to follow a constant point on the wavefront, you have to move with velocity

$$\frac{d}{dt}\left(\omega t \pm \beta z = \text{constant}\right)$$

- Or, $v = \frac{dz}{dt} = \pm\frac{\omega}{\beta} = \pm\sqrt{\frac{1}{L'C'}}$