EECS 117

Lecture 1: Transmission Lines

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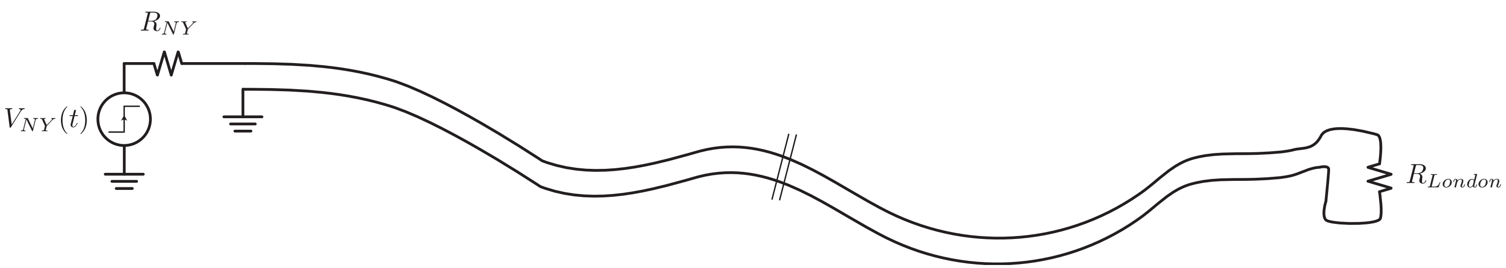
First Trans-Atlantic Cable

- Problem: A long cable – the trans-atlantic telephone cable – is laid out connecting NY to London. We would like analyze the electrical properties of this cable.
- For simplicity, assume the cable has a uniform cross-secitonal configuration (shown as two wires here)

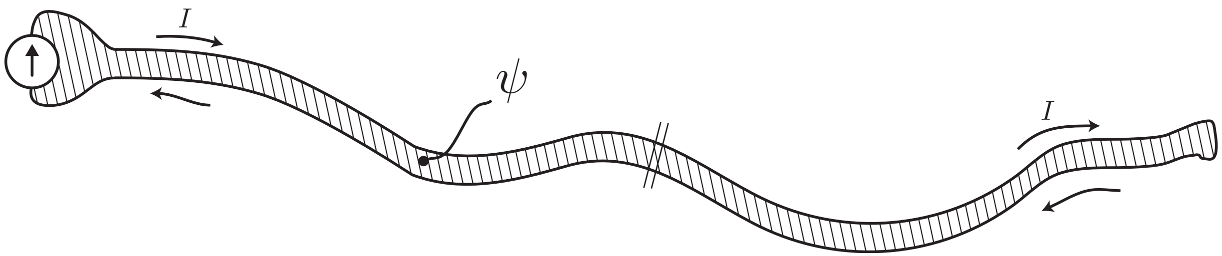


Trans-Atlantic Cable Analysis

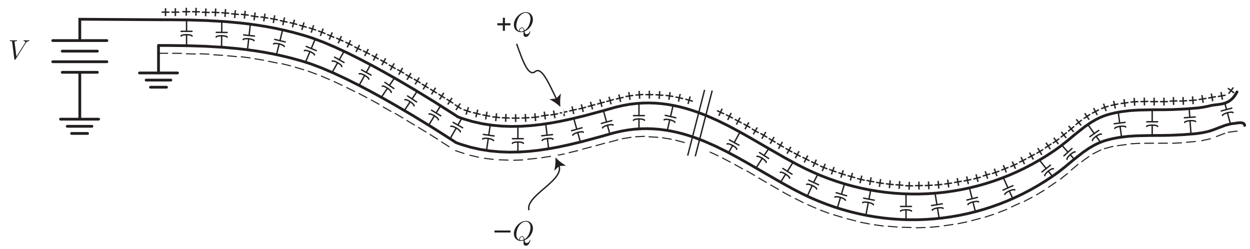
- Can we do it with circuit theory?
- Fundamental problem with circuit theory is that it assumes that the speed of light is infinite. So all signals are in phase: $V(z) = V(z + \ell)$
- Consequently, all variations in space are ignored: $\frac{\partial}{\partial z} \to 0$
- This allows the *lumped* circuit approximation.

Lumped Circuit Properties of Cable

- Shorted Line: The long loop has *inductance* since the magnetic flux ψ is not negligible (long cable) ($\psi = LI$)

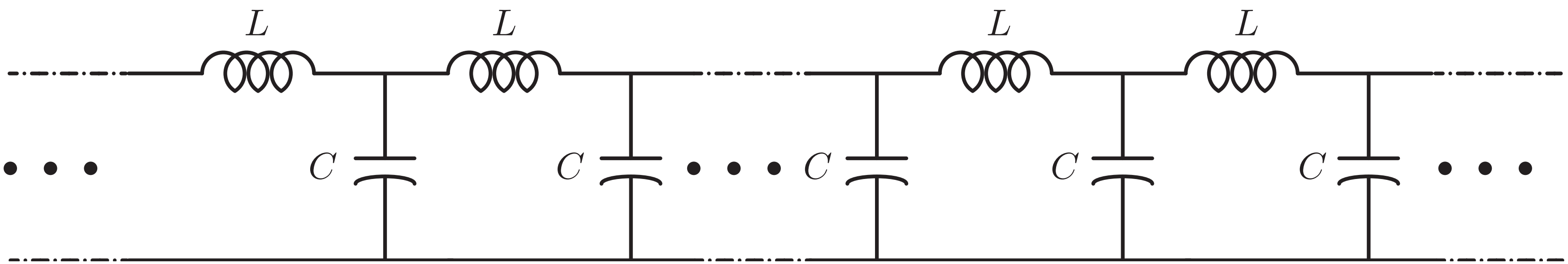


- Open Line: The cable also has substantial capacitance ($Q = CV$)



Sectional Model (I)

- So do we model the cable as an inductor or as a capacitor? Or both? How?
- Try a *distributed* model: Inductance and capacitance occur together. They are intermingled.

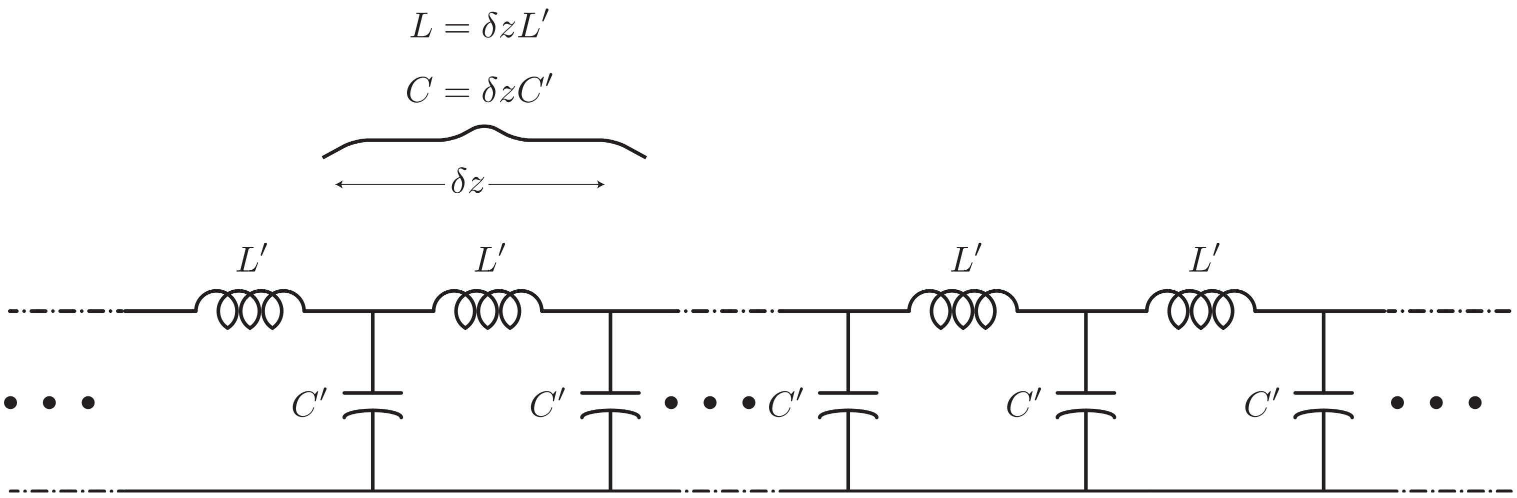


- Can add loss (series and shunt resistors) but let's keep it simple for now.
- Add more sections and solution should converge

Sectional Model (II)

- More sections $\rightarrow$ The equiv LC circuit represents a smaller and smaller section and therefore lumped circuit approximation is more valid
- This is an easy problem to solve with SPICE.
- But the people 1866 didn't have computers ... how did they analyze a problem with hundreds of inductors and capacitors?

Distributed Model



- Go to a fully distributed model by letting the number of sections go to infinity
- Define inductance and capacitance per unit length $L' = L/\ell$, $C' = C/\ell$
- For an infinitesimal section of the line, circuit theory applies since signals travel instantly over an infinitesimally small length

KCL and KVL for a small section

- KCL: $i(z) = \delta z C' \frac{\partial v(z)}{\partial t} + i(z + \delta z)$
- KVL: $v(z) = \delta z L' \frac{\partial i(z+\delta z)}{\partial t} + v(z + \delta z)$
- Take limit as $\delta z \to 0$

We arrive at “Telegrapher’s Equatins”

$$\lim_{\delta z \to 0} \frac{i(z) - i(z + \delta z)}{\delta z} = -\frac{\partial i}{\partial z} = C' \frac{\partial v}{\partial t}$$

$$\lim_{\delta z \to 0} \frac{v(z) - v(z + \delta z)}{\delta z} = -\frac{\partial v}{\partial z} = L' \frac{\partial i}{\partial t}$$

Derivation of Wave Equations

- We have two coupled equations and two unkowns (i and v) ... can reduce it to two de-coupled equations:

$$\frac{\partial^2 i}{\partial t \partial z} = -C' \frac{\partial^2 v}{\partial t^2} \qquad \frac{\partial^2 v}{\partial z^2} = -L' \frac{\partial^2 i}{\partial z \partial t}$$

- note order of partials can be changed (at least in EE)

$$\frac{\partial^2 v}{\partial z^2} = L'C' \frac{\partial^2 v}{\partial t^2}$$

- Same equation can be derived for current:

$$\frac{\partial^2 i}{\partial z^2} = L'C' \frac{\partial^2 i}{\partial t^2}$$

The Wave Equation

We see that the currents and voltages on the transmission line satisfy the one-dimensional wave equation. This is a partial differential equation. The solution depends on boundary conditions and the initial condition.

$$\frac{\partial^2 i}{\partial z^2} = L'C'\frac{\partial^2 i}{\partial t^2}$$

Wave Equation Solution

Consider the function $f(z,t) = f(z \pm vt) = f(u)$:

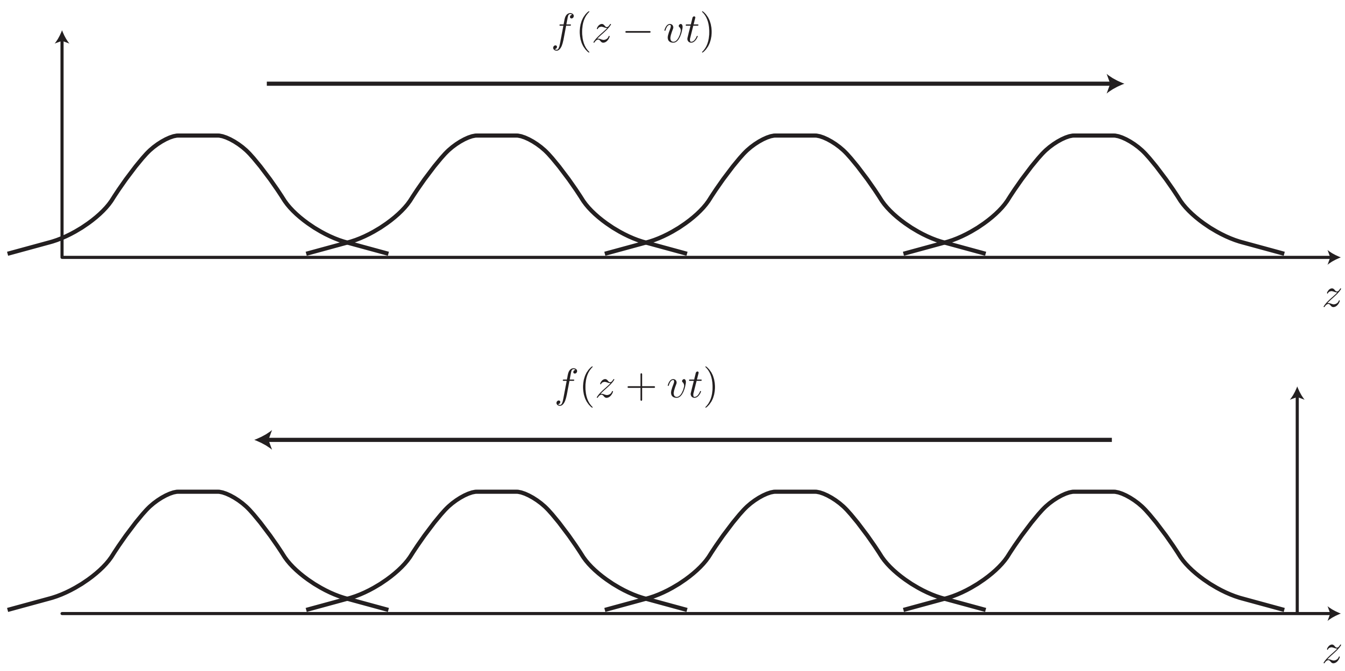
$$\frac{\partial f}{\partial z} = \frac{\partial f}{\partial u}\frac{\partial u}{\partial z} = \frac{\partial f}{\partial u} \qquad \frac{\partial f}{\partial t} = \frac{\partial f}{\partial u}\frac{\partial u}{\partial t} = \pm v \frac{\partial f}{\partial u}$$

$$\frac{\partial^2 f}{\partial^2 z} = \frac{\partial^2 f}{\partial u^2} \qquad \frac{\partial^2 f}{\partial t^2} = \pm v \frac{\partial}{\partial u}\left(\frac{\partial f}{\partial t}\right) = v^2 \frac{\partial^2 f}{\partial u^2}$$

$$\frac{\partial^2 f}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$$

It satisfies the wave equation!

Wave Motion



- General voltage solution: $v(z,t) = f^+(z-vt) + f^-(z+vt)$
- Where $v = \sqrt{\frac{1}{LC}}$

Wave Speed

- Speed of motion can be deduced if we observe the speed of a point on the aveform

$$z \pm vt = \text{constant}$$

- To follow this point as time elapses, we must move the z coordinate in step. This point moves with velocity

$$\frac{dz}{dt} \pm v = 0$$

- This is the speed at which we move with speed $\frac{dz}{dt} = \pm v$
- v is the velocity of wave propagation

Current / Voltage Relationship (I)

- Since the current also satisfies the wave equation

$$i(z,t) = g^+(z - vt) + g^-(z + vt)$$

- Recall that on a transmission line, current and voltage are related by

$$\frac{\partial i}{\partial z} = -C' \frac{\partial v}{\partial t}$$

- For the general function this gives

$$\frac{\partial g^+}{\partial u} + \frac{\partial g^-}{\partial u} = -C' \left(-v \frac{\partial f^+}{\partial u} + v \frac{\partial f^-}{\partial u} \right)$$

Current / Voltage Relationship (II)

- Since the forward waves are independent of the reverse waves

$$\frac{\partial g^+}{\partial u} = C' v \frac{\partial f^+}{\partial u} \qquad \frac{\partial g^-}{\partial u} = -C' v \frac{\partial f^-}{\partial u}$$

- Within a constant we have

$$g^+ = \frac{f^+}{Z_0} \qquad g^- = -\frac{f^-}{Z_0}$$

- Where $Z_0 = \sqrt{\frac{L'}{C'}}$ is the “Characteristic Impedance” of the line

Example: Step Into Infinite Line

- Excite a step function onto a transmission line
- The line is assumed uncharged: $Q(z,0)=0$, $\psi(z,0)=0$ or equivalently $v(z,0)=0$ and $i(z,0)=0$
- By physical intuiition, we would only expect a forward traveling wave since the line is infinite in extent
- The general form of current and voltage on the line is given by

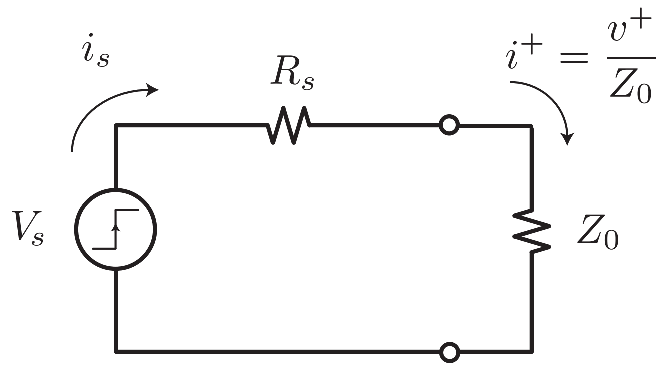
$$v(z,t)=v^+(z-vt)$$

$$i(z,t)=i^+(z-vt)=\frac{v^+(z-vt)}{Z_0}$$

- The T-line looks like a resistor of Z_0 ohms!

Example 1 (cont)

- We may therefore model the line with the following simple equivalent circuit



- Since $i_s = i^+$, the excited voltage wave has an amplitude of

$$v^+ = \frac{Z_0}{Z_0 + R_s} V_s$$

- It's surprising that the voltage on the line is not equal to the source voltage

Example 1 (cont)

- The voltage on the line is a delayed version of the source voltage