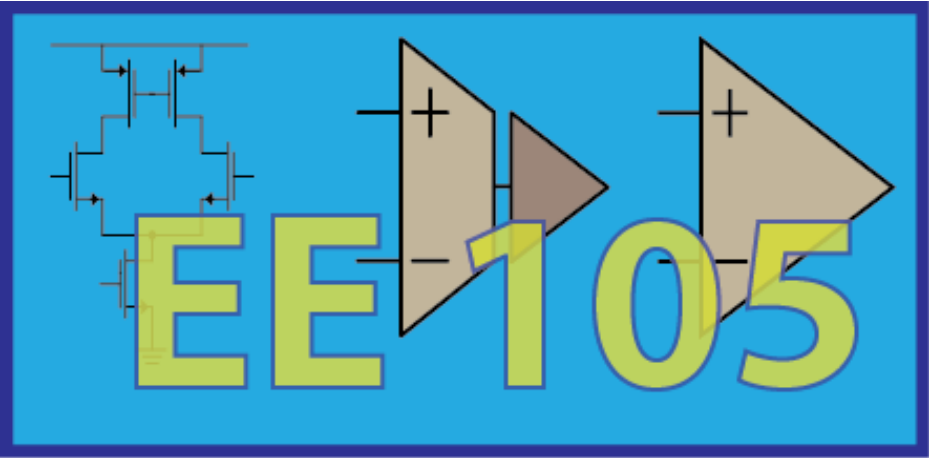




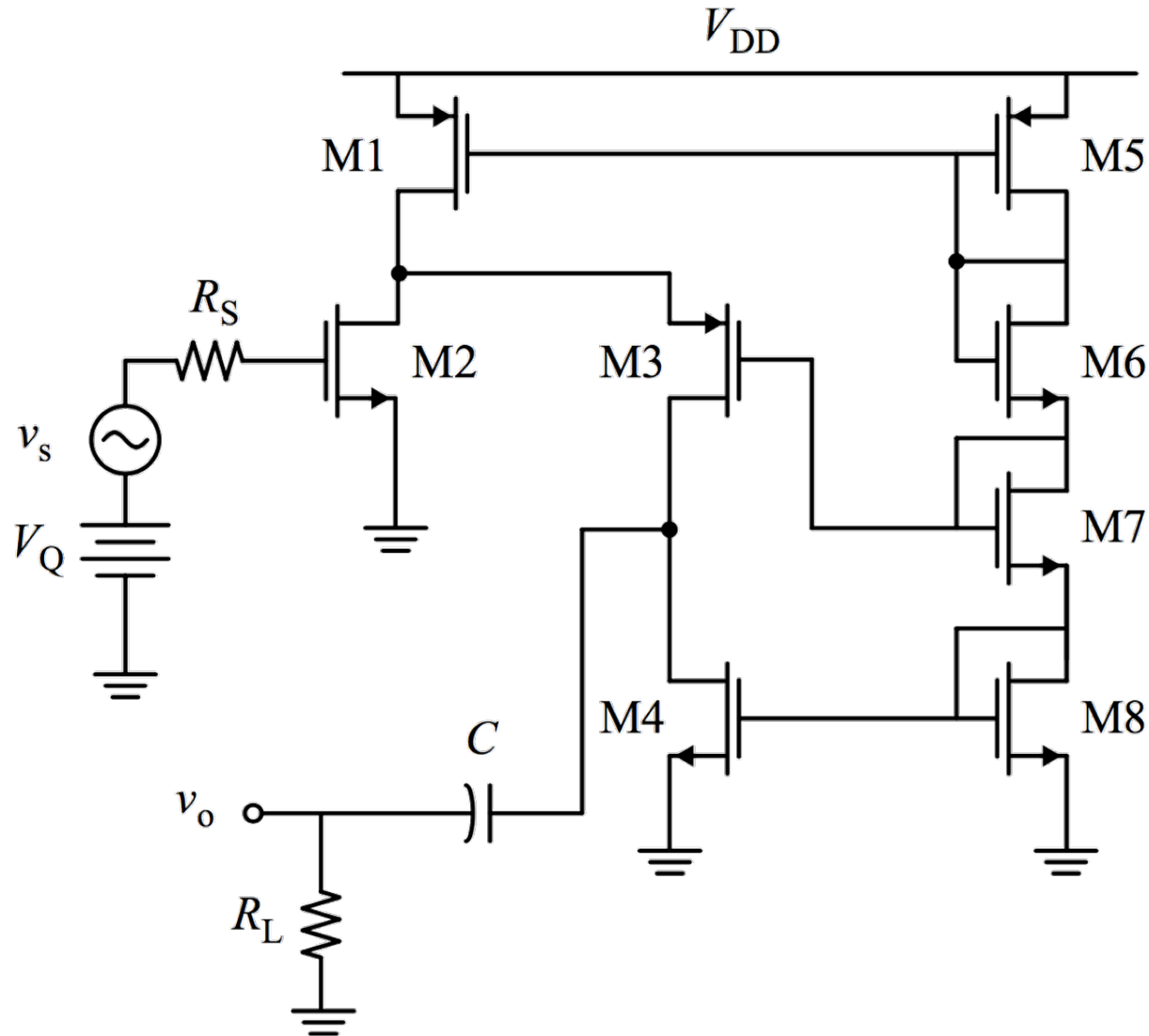
Differential Amplifiers

Prof. Ali M. Niknejad
Prof. Rikky Muller

Announcements

- HW11 due on Friday, last homework!
- Lab 6 due this week, last lab!
- Check web site for updates on RRR & finals week office hours and review sessions:
- <http://rfic.eecs.berkeley.edu/105/lectures.html>

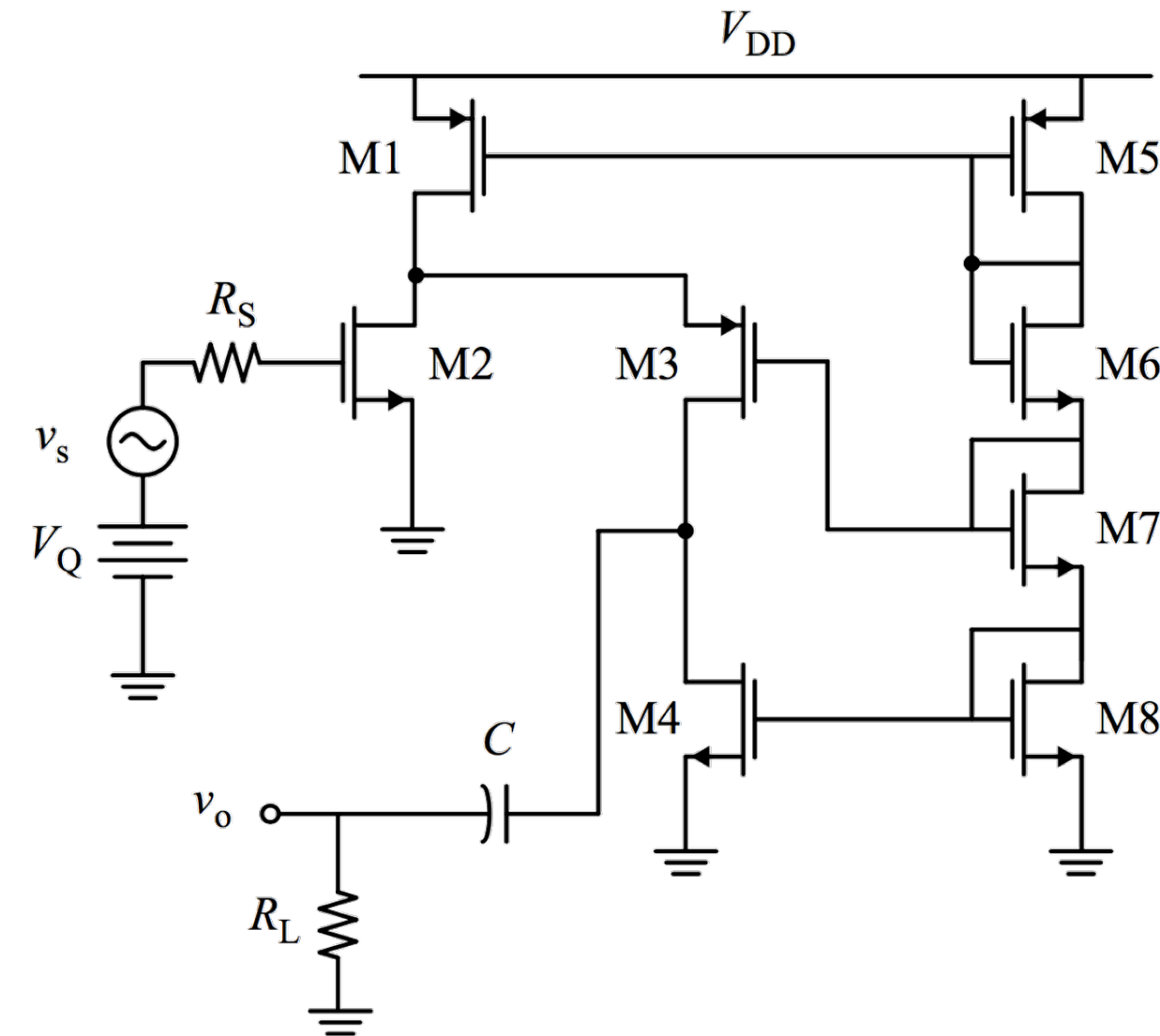
Multistage Analysis Example



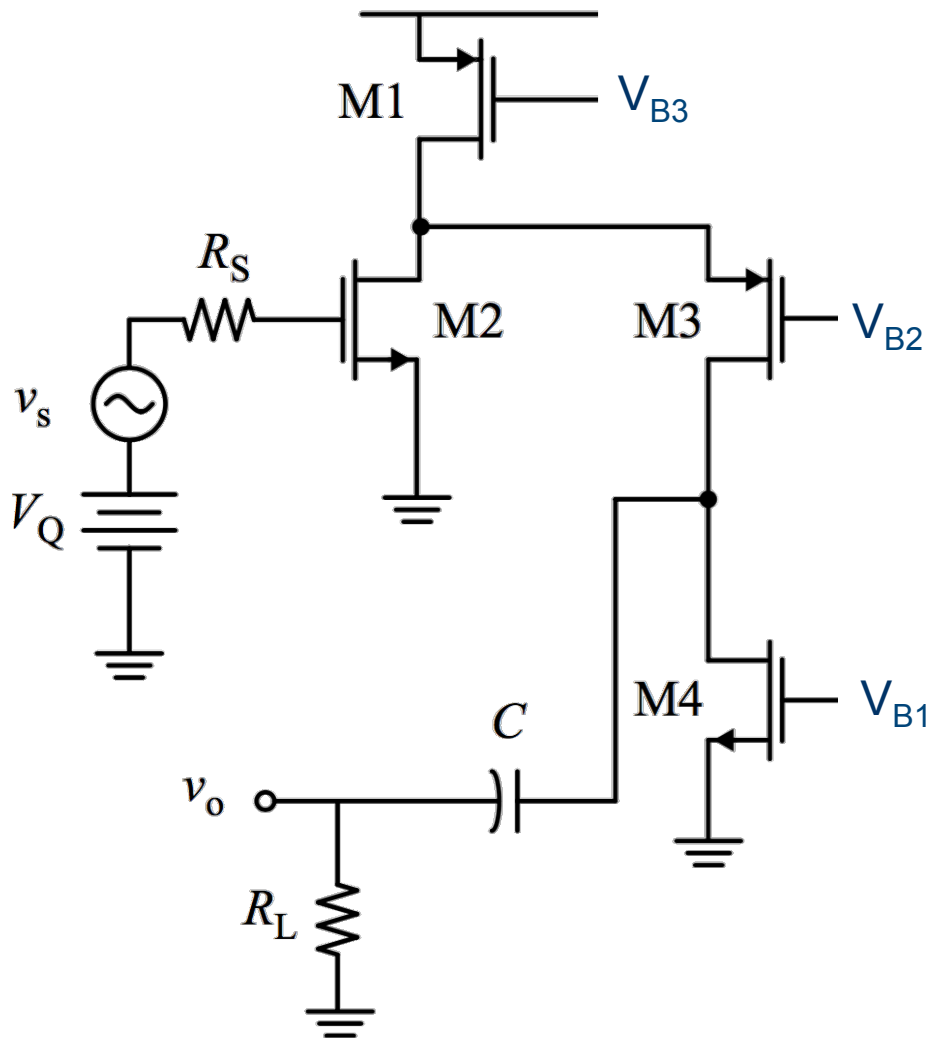
Cutting Through the Complexity

1. Identify the “signal path” between the input and output
2. Eliminate “background” transistors to reduce clutter
3. For “background transistors, understand their role (e.g. DC biasing)
4. For frequency response, identify “hi-Z” nodes.

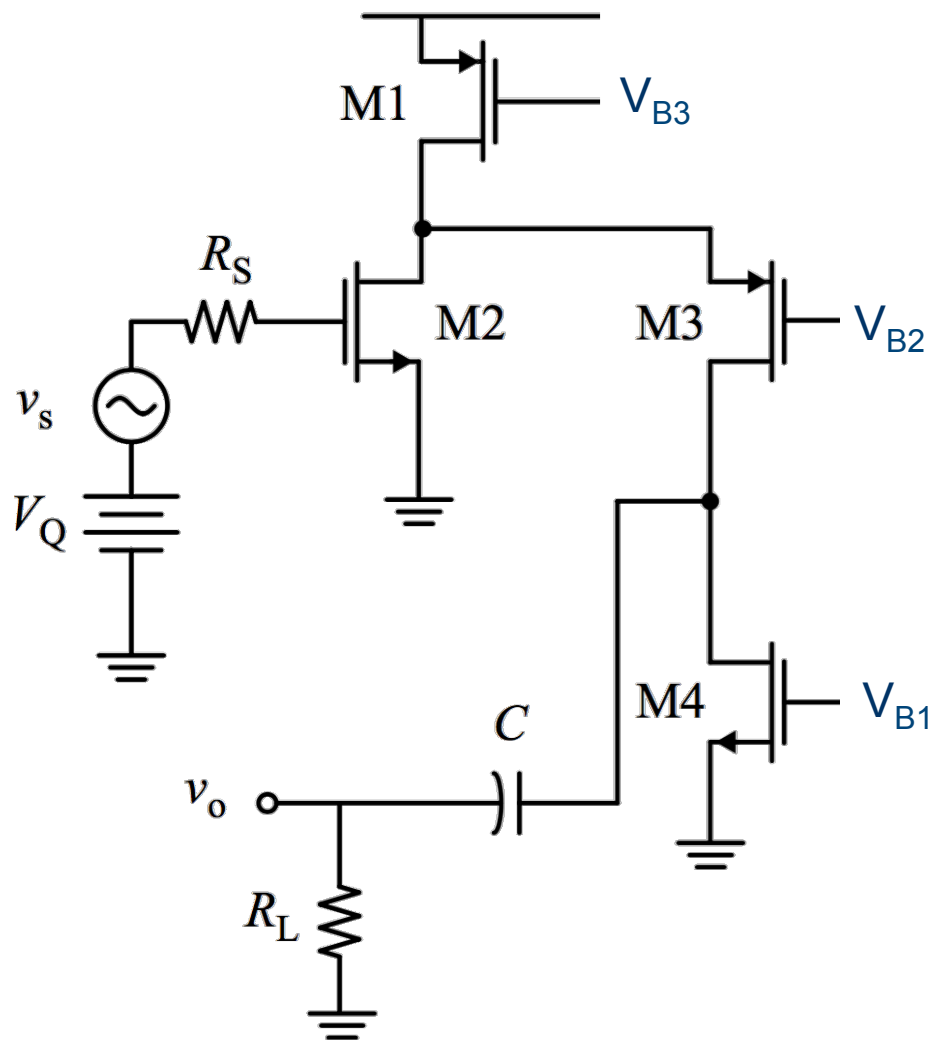
Eliminate Clutter



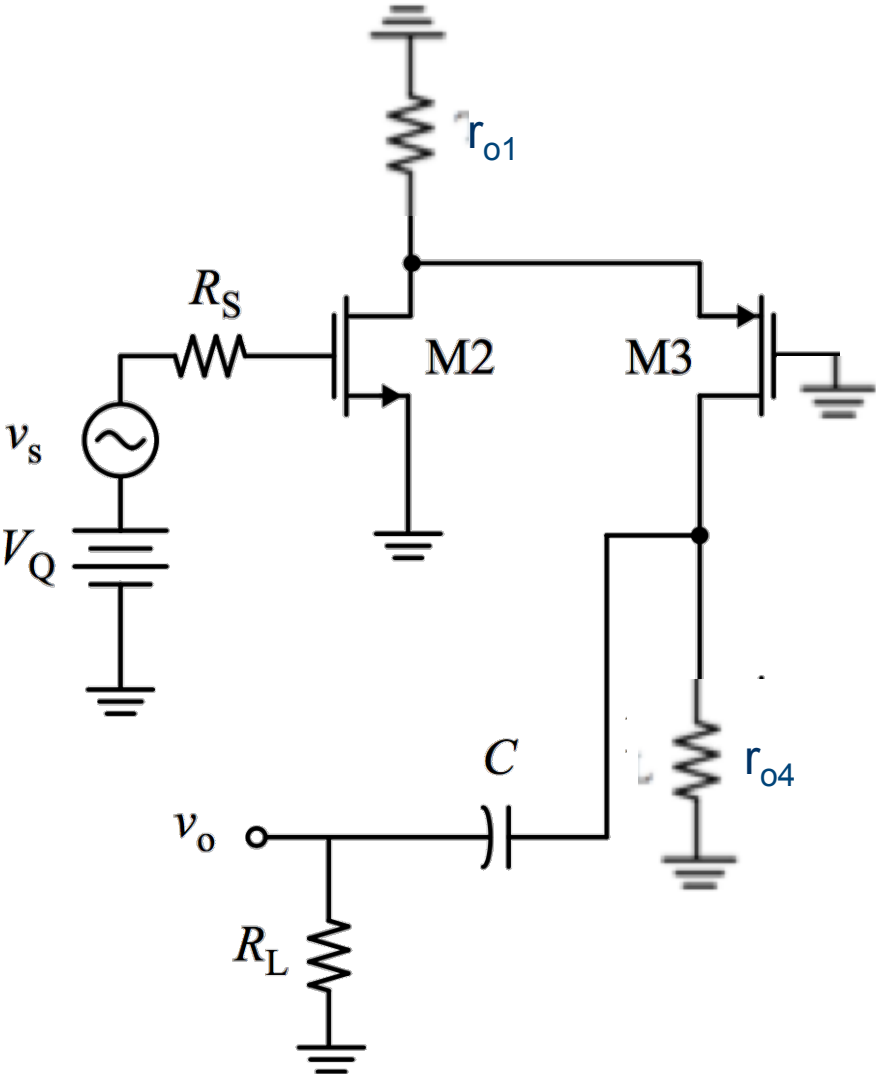
Identify Signal Path & Amplifier Stages



DC Biasing



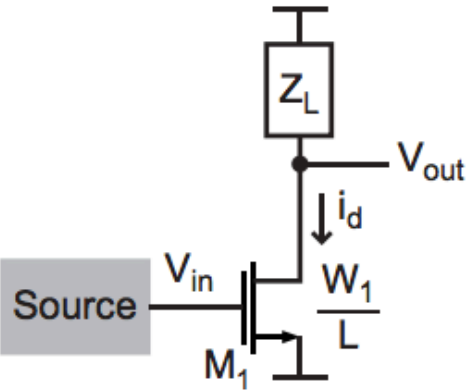
Small-Signal Models



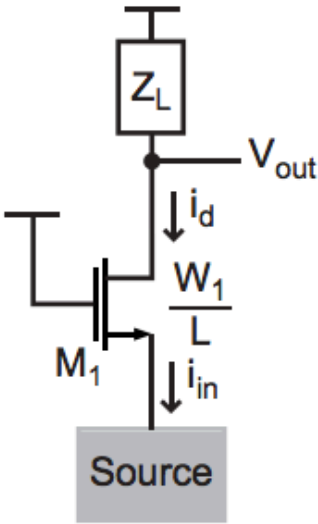
Two-Port Model

Basic Amplifier Types

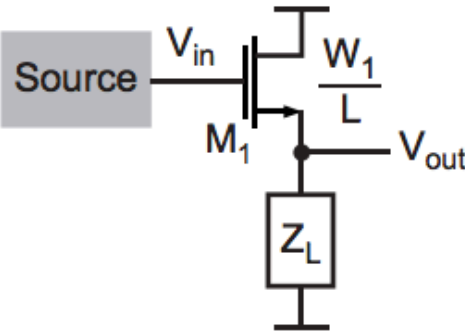
Common Source



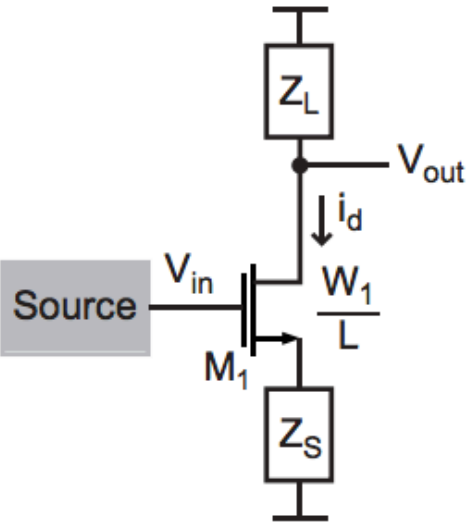
Common Gate



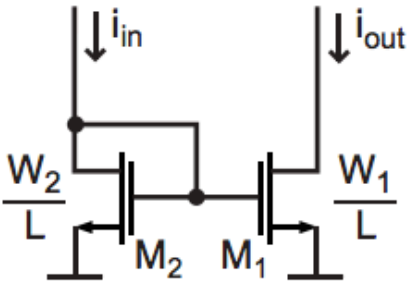
Source Follower



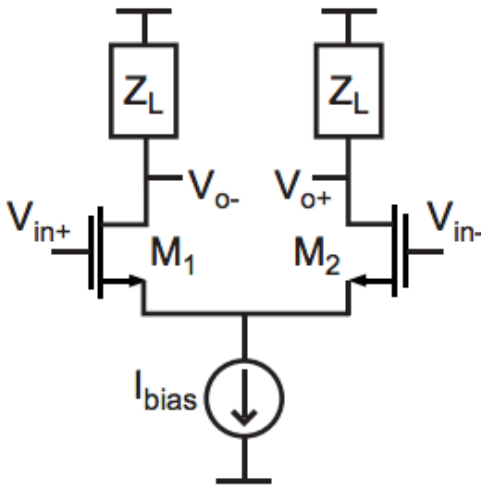
Common Source with Source Degeneration



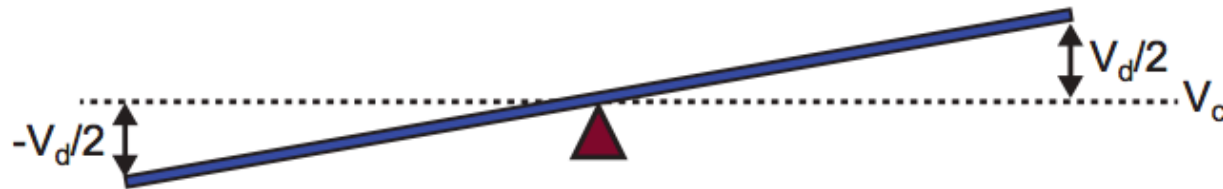
Current Mirror



Differential Amplifier



Differential & Common Mode Signals



- Consider positive and negative input terminal signals V_i^+ and V_i^-
- Define differential signal as: $V_{id} = V_{in}^+ - V_{in}^-$
- Define common mode signal as: $V_{ic} = (V_{in}^+ + V_{in}^-)/2$
- We can create arbitrary V_i^+ and V_i^- signals from differential and common mode components:

$$V_{in}^+ = V_{ic} + \frac{1}{2}V_{id} \qquad V_{in}^- = V_{ic} - \frac{1}{2}V_{id}$$

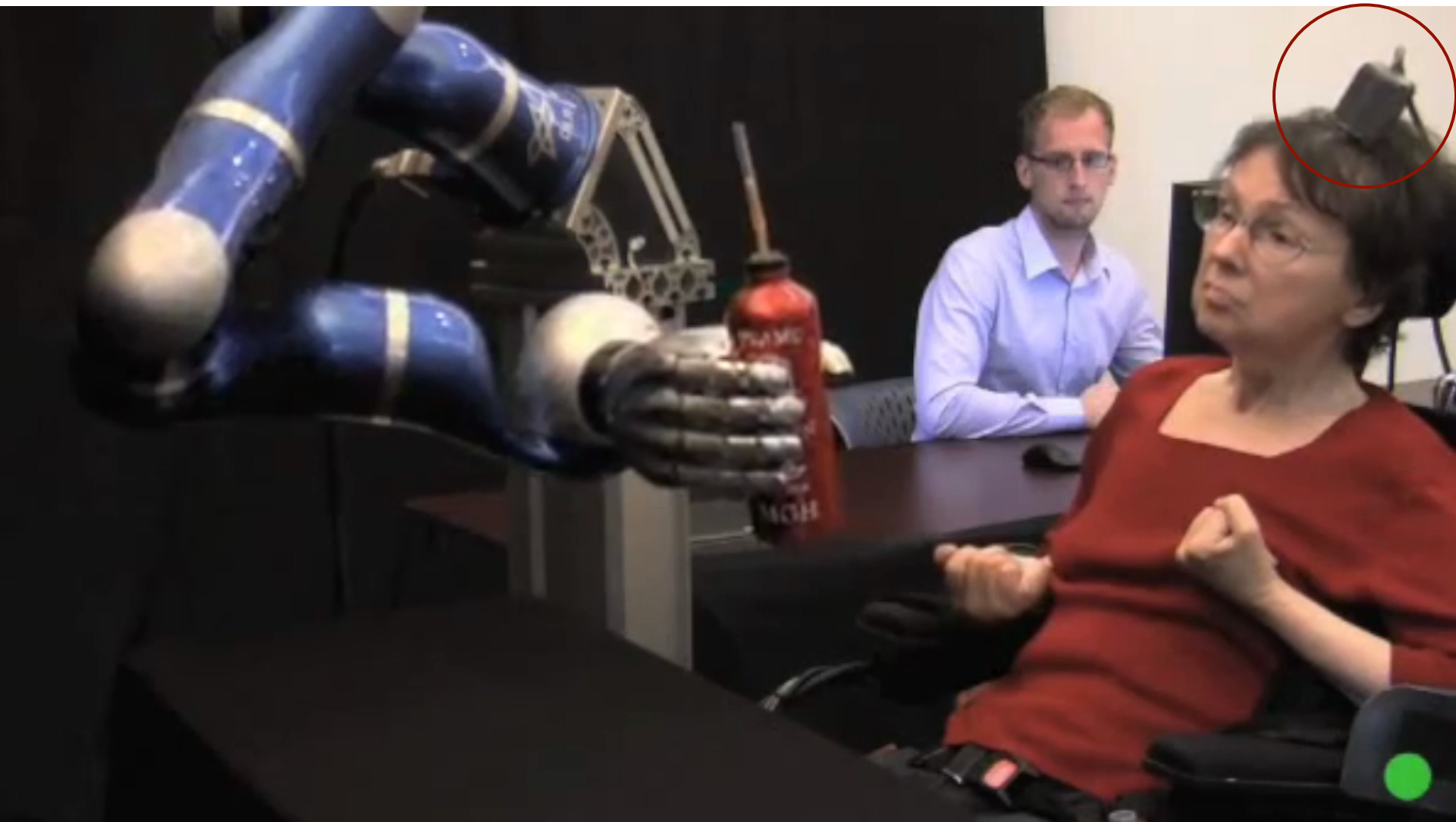
- This also applies to differential output signals:

$$V_o^+ = V_{oc} + \frac{1}{2}V_{od} \qquad V_o^- = V_{oc} - \frac{1}{2}V_{od}$$

Why Differential?

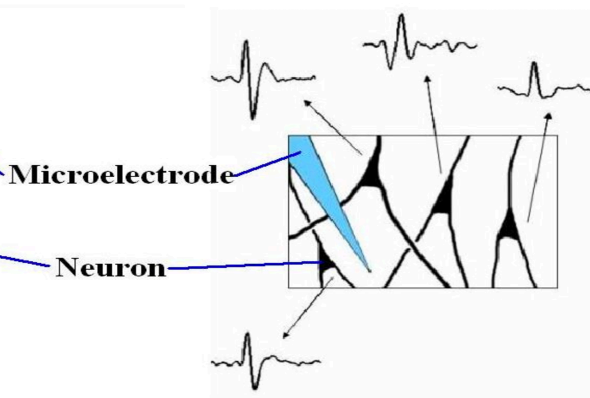
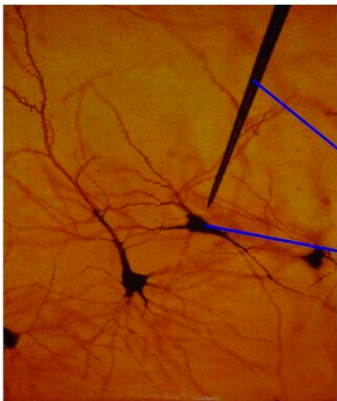
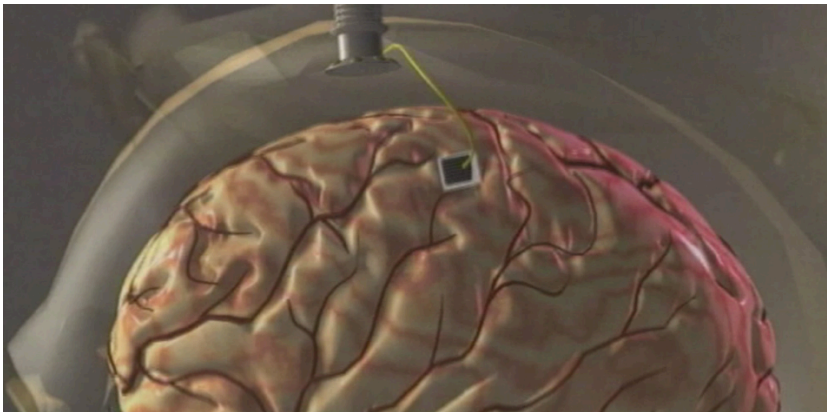
- Differential circuits are much less sensitive to noises and interferences
- Differential configuration enables us to bias amplifiers and connect multiple stages without using coupling or bypass capacitors
- Differential amplifiers are widely used in IC's
 - Excellent matching of transistors, which is critical for differential circuits
 - Differential circuits require more transistors → not an issue for IC

Brain-Machine Interfaces

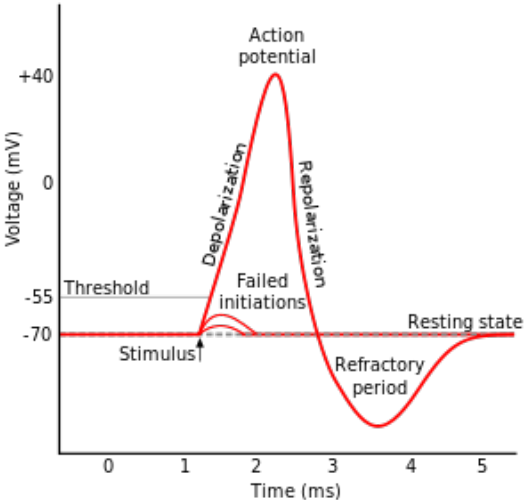
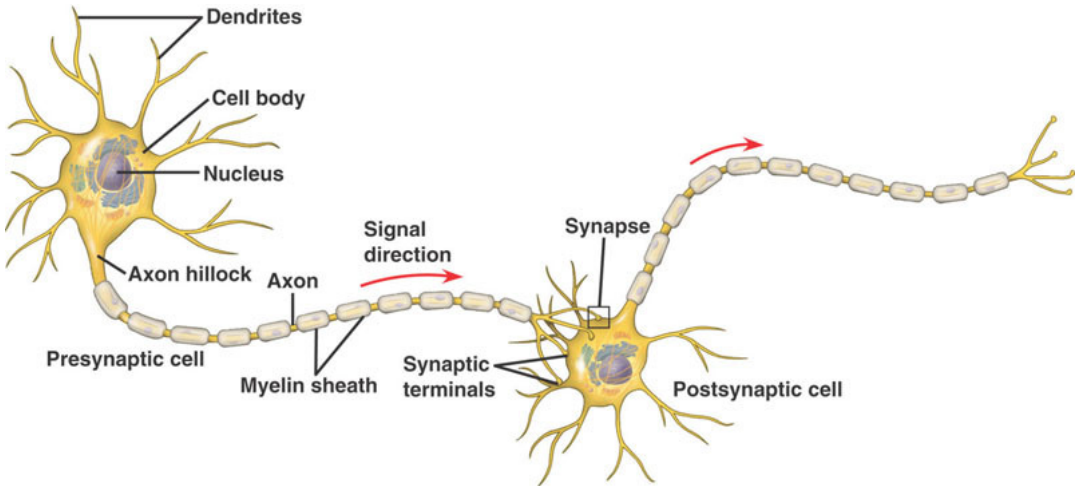


Source: Hochberg et al., *Nature* '12

Neural Recording

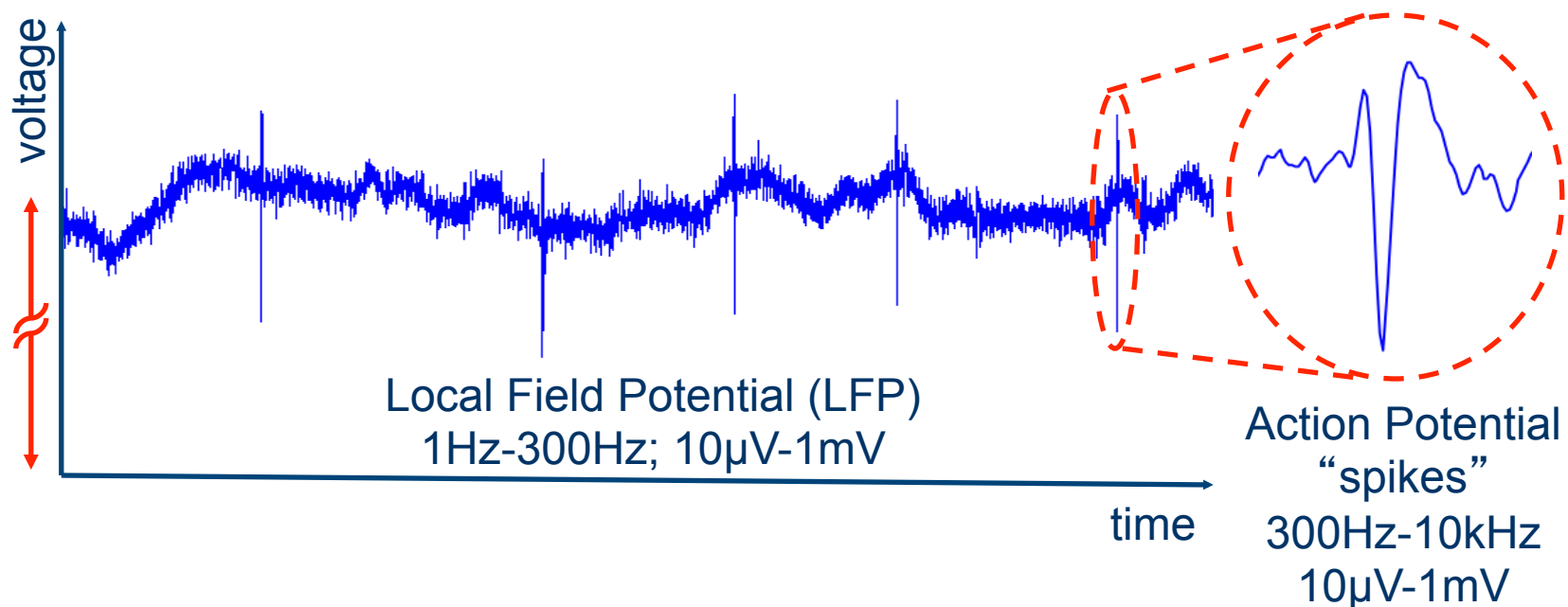


- An array of electrodes is implanted in the motor cortex and senses extracellular signals that include firing from nearby neurons



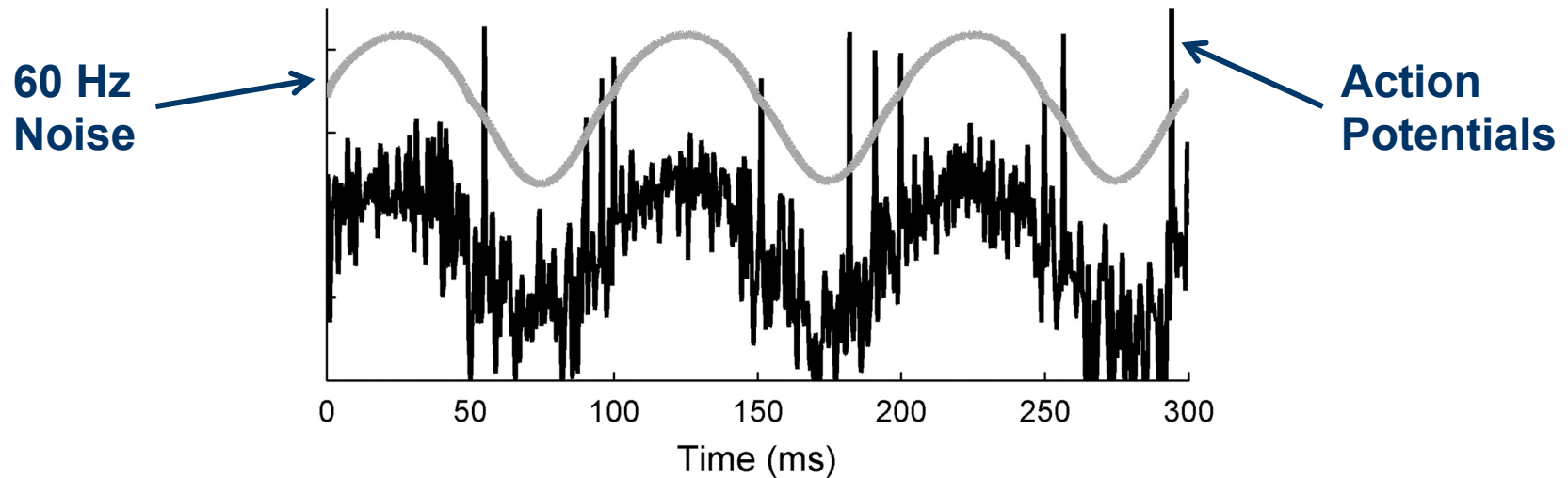
- The propagation of signals from neuron to neuron is called an Action Potential, which is analogous to a digital “pulse”

Extracellular Neuronal Signals



- The goal of a neural recording device is to record the small-amplitude neural signals and pick out the meaningful signals from the “noise”.
- These signals are then decoded to create trajectories, movements, and speeds for controlling prostheses, computers, etc.

60Hz and Other Interferers

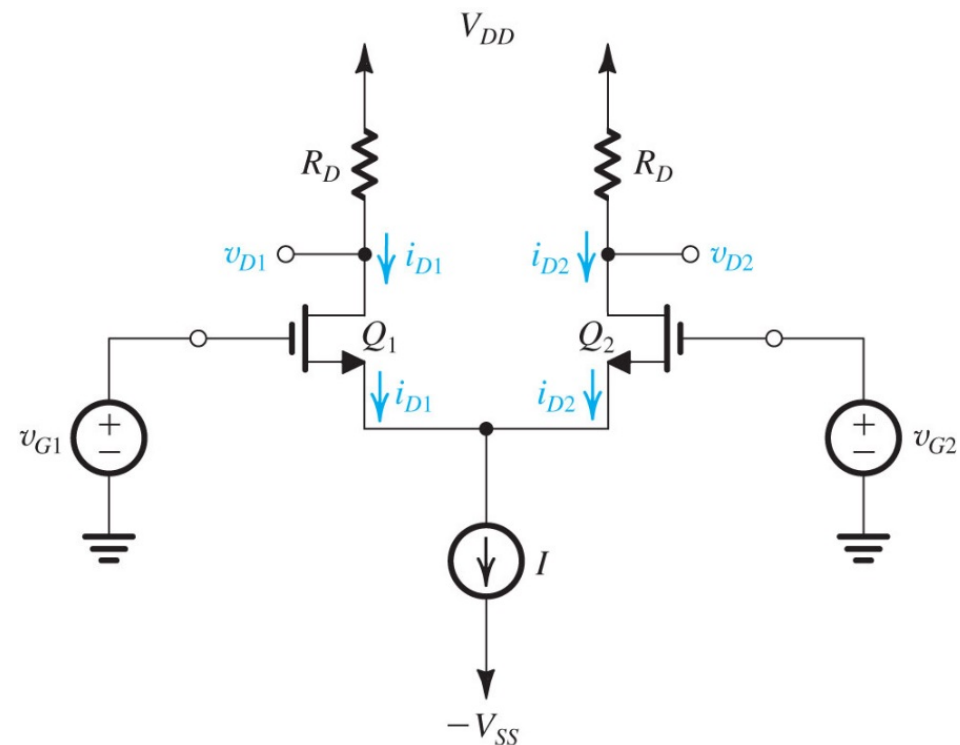


- In reality, the tiny signals recorded from the brain can get corrupted by numerous interferers.
- Ambient 60Hz noise couples into electrical signals in and on the body
- Motion can cause voltage artifacts from the movement of the electrodes relative to the neurons

Key Idea: Differential Recording

MOS Differential-Pair

Basic Configuration



Two matched MOS transistors

Common current bias

"Differential signals" applied to v_{G1} and v_{G2}
(equal amplitude but opposite sign)

"Differential outputs" are produced
at v_{D1} and v_{D2}

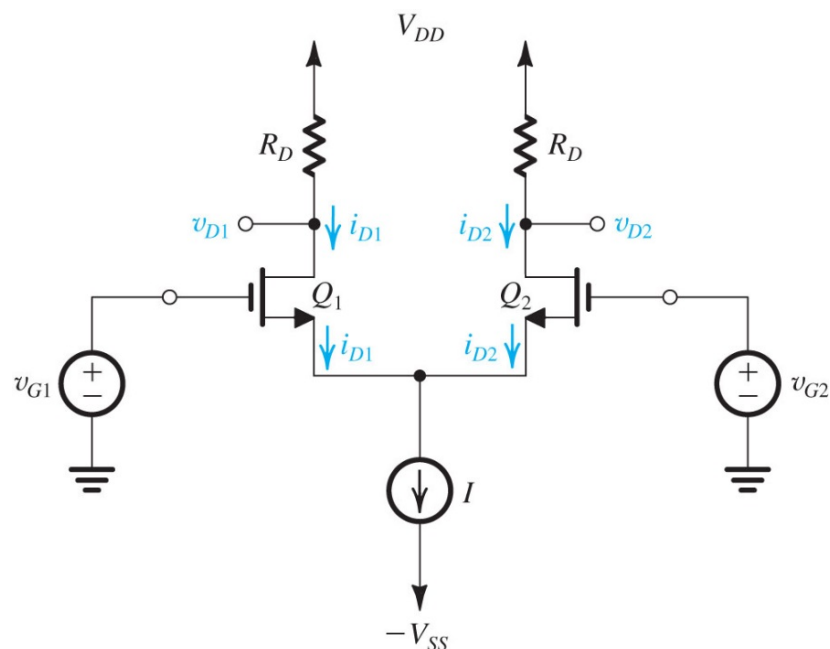
Note in differential configuration,
 V_{GS} is fixed for both Q_1 and Q_2

$$I_{D1} = I_{D2} = \frac{I}{2}$$

$$\frac{I}{2} = \frac{k_n}{2} (V_{GS} - V_{Tn})^2$$

$$V_{GS} = V_{Tn} + \sqrt{\frac{I}{k_n}}$$

Large Signal Behavior of Diff Mode Operation



$$v_{id} = v_{in+} - v_{in-} = \left(V_{Tn} + \sqrt{\frac{I_1}{k_n}} \right) - \left(V_{Tn} + \sqrt{\frac{I_2}{k_n}} \right)$$

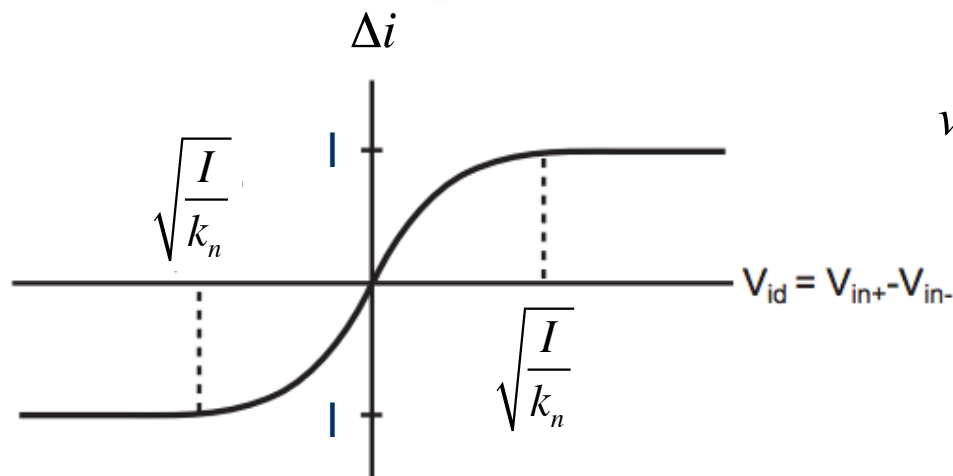
$$v_{id} = \sqrt{\frac{I_1}{k_n}} - \sqrt{\frac{I_2}{k_n}} = \sqrt{\frac{I + \Delta i/2}{k_n}} - \sqrt{\frac{I_2 - \Delta i/2}{k_n}}$$

$$\Delta i = i_{D1} - i_{D2}$$

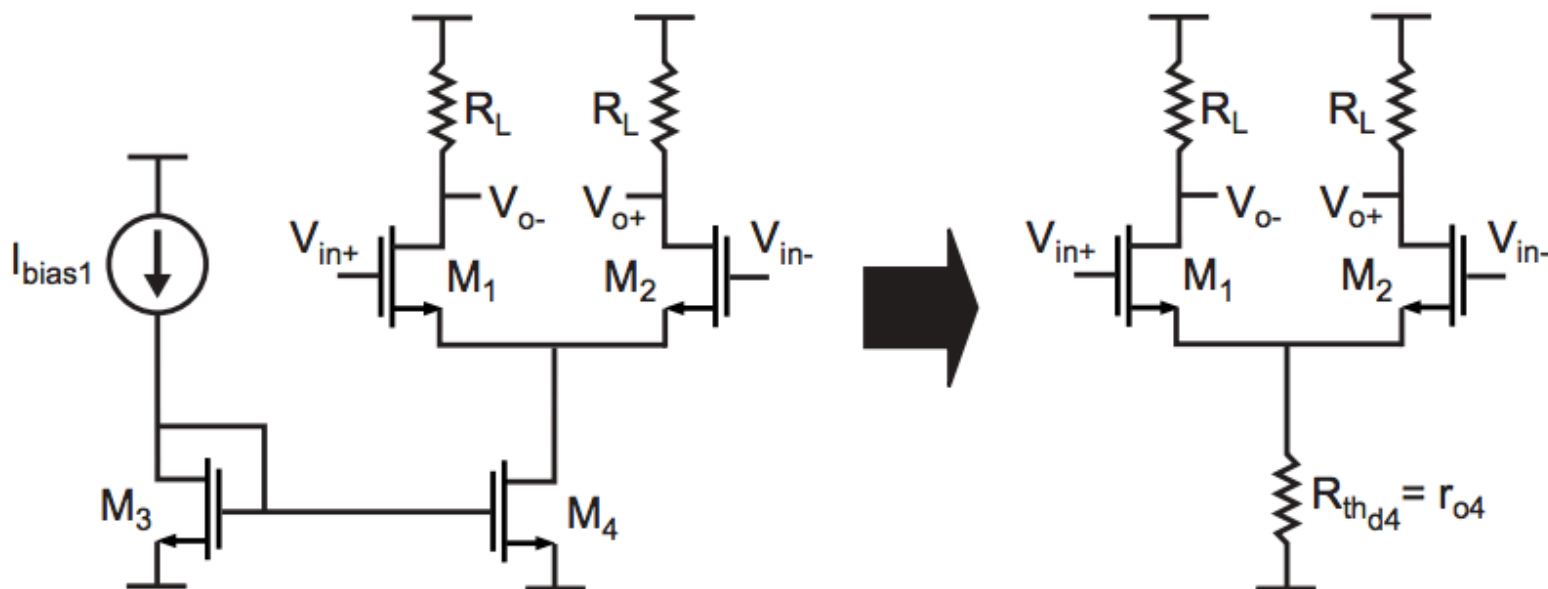
solve

$$\Delta i = k_n v_{id} \sqrt{\frac{2I}{k_n} - v_{id}^2}$$

$$v_{id} \leq \sqrt{\frac{I}{k_n}}$$



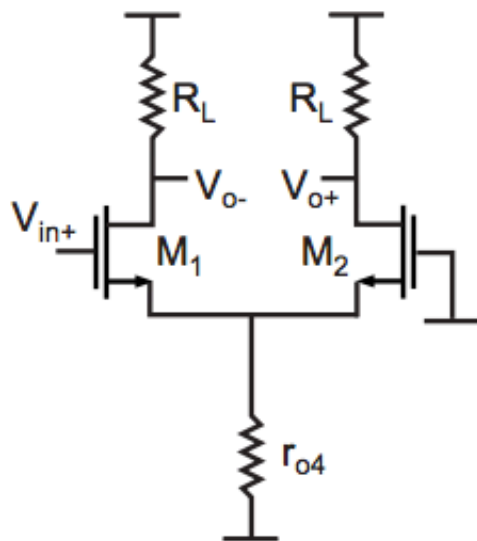
Small Signal Modeling: Step 1



■ Small signal analysis assumes linearity

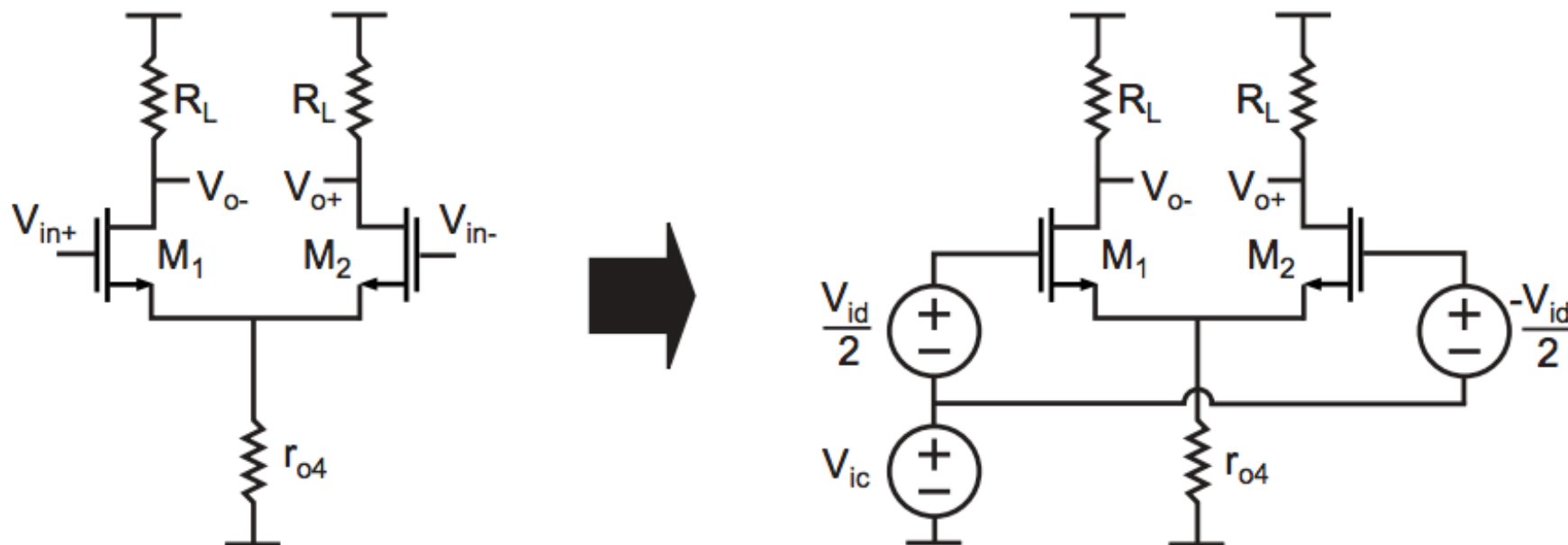
- Impact of M_4 on amplifier is to simply present its drain impedance to the diff pair transistors (M_1 and M_2)
- Impact of V_{in+} and V_{in-} can be evaluated separately and then added (i.e., superposition)
 - By symmetry, we need only determine impact of V_{in+}
 - Calculation of V_{in-} impact directly follows

Method 1: Use Small Signal Model



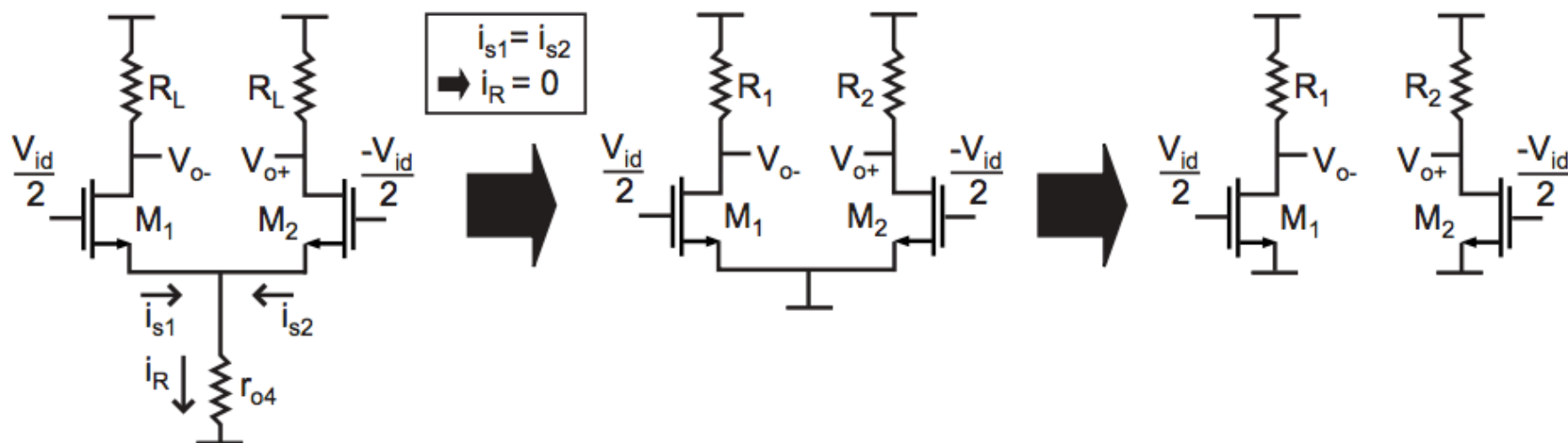
- Analysis follows easily, but there is a simpler way!

Method 2: Differential Amplifier Analysis



- Partition input signals into common-mode and differential components
- By superposition, we can add the results to determine the overall impact of the input signals

Differential Analysis

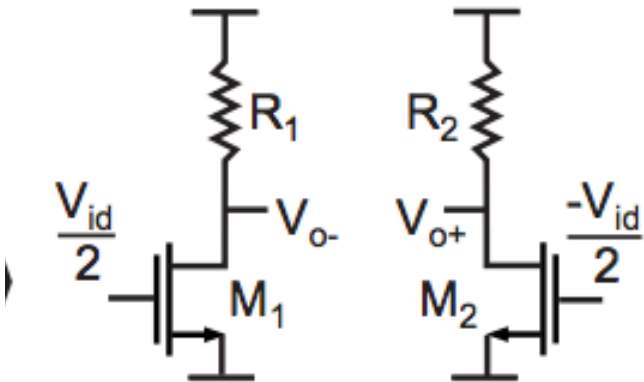


■ Key observations

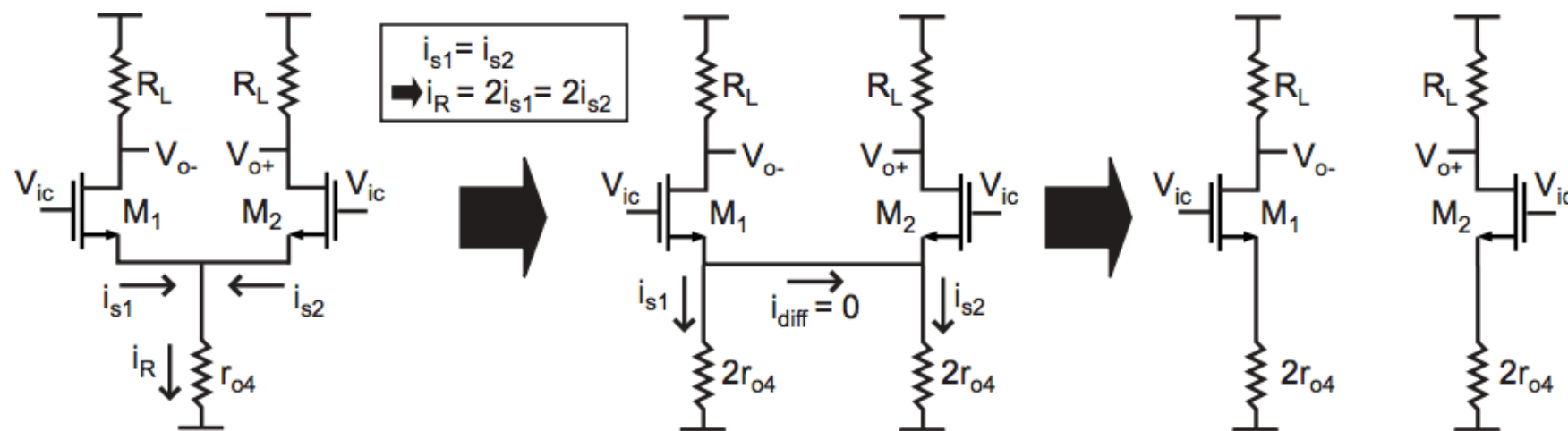
- Inputs are equal in magnitude but opposite in sign to each other
- By linearity and symmetry, i_{s1} must equal $-i_{s2}$
 - This implies i_R is zero, so that voltage drop across r_{o4} is zero
 - The sources of M_1 and M_2 are therefore at incremental ground and decoupled from each other!

■ Analysis can now be done on identical “half-circuits”

Find Differential Gain



Common Mode Analysis

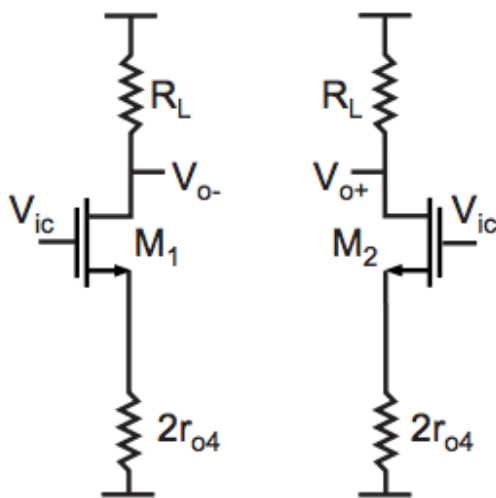


■ Key observations

- Inputs are equal to each other
- By linearity and symmetry, i_{s1} must equal i_{s2}
 - This implies $i_R = 2i_{s1} = 2i_{s2}$
- We can view r_{o4} as two parallel resistors that have equal current running through them

■ Analysis can also be done on two identical half-circuits

Common Mode Analysis



Common-Source
with degeneration

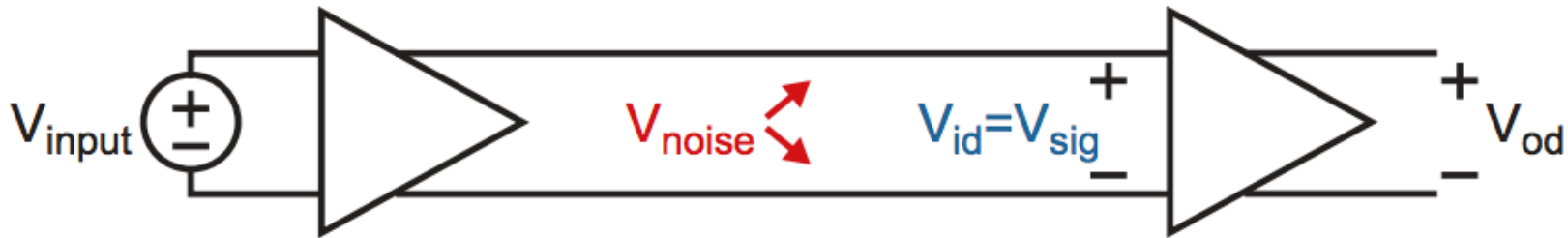
$$\frac{v_{o-}}{v_{ic}} = \frac{v_{o+}}{v_{ic}} = \frac{-R_L}{1/g_m + 2r_{o4}} = \frac{-g_m R_L}{1 + 2g_m r_{o4}}$$

Since $2r_{o4} \gg 1/g_m$,

$$\frac{v_{o-}}{v_{ic}} = \frac{v_{o+}}{v_{ic}} \approx \frac{-R_L}{2r_{o4}}$$

$$v_{od} = v_{o+} - v_{o-} = 0$$

Useful Metric for Diff Amps: CMRR



■ Common Mode Rejection Ratio (CMRR)

- Define: a_{vd} : differential gain, a_{vc} : common mode gain

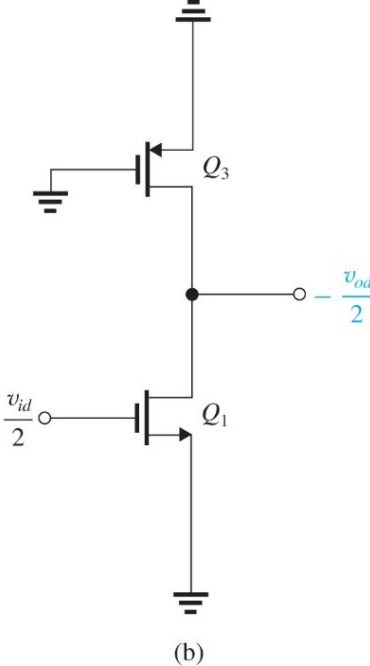
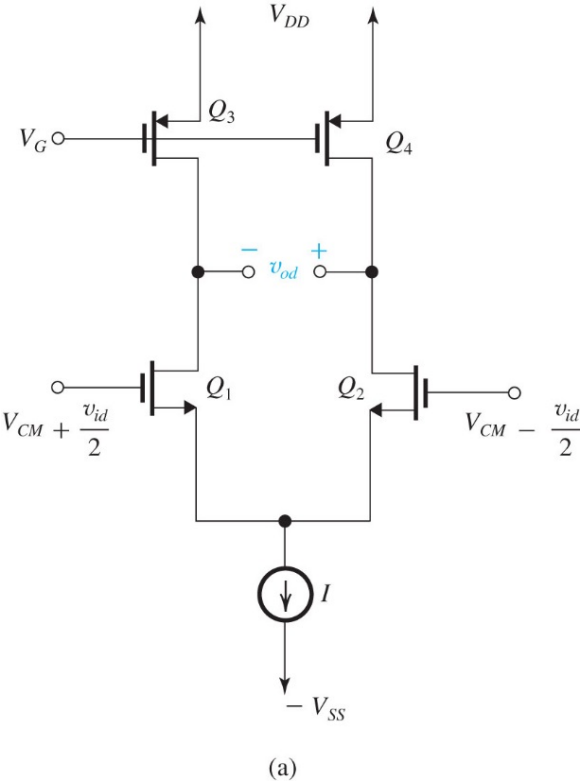
$$\text{CMRR} = \left(\frac{a_{vd}}{a_{vc}} \right)$$

- CMRR corresponds to ratio of differential to common mode gain and is related to received signal-to-noise ratio

$$V_{od} = a_{vd} V_{sig} + a_{vc} V_{noise}$$

$$\Rightarrow \frac{\text{Signal}}{\text{Noise}} = \left(\frac{a_{vd}}{a_{vc}} \right) \left(\frac{V_{sig}}{V_{noise}} \right) = \text{CMRR} \left(\frac{V_{sig}}{V_{noise}} \right)$$

Current-Source Loads

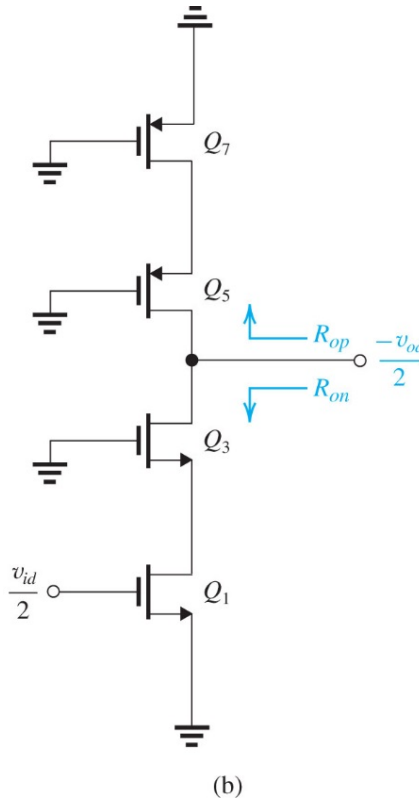
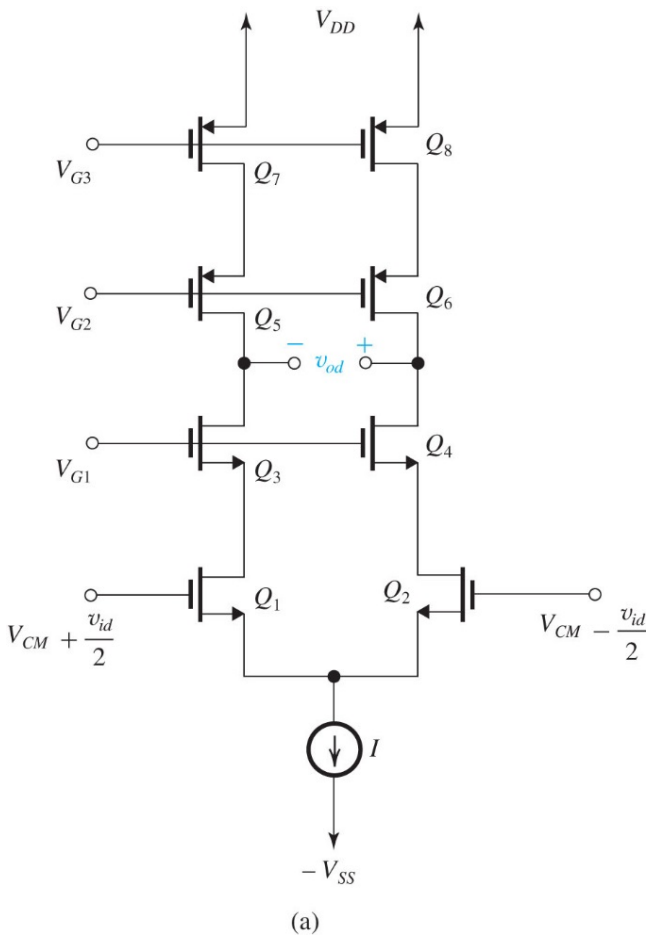


Q_3 and Q_4 are
PMOS current sources

From half-circuit:

$$A_d = \frac{v_{od}}{v_{id}} = -g_{m1} (r_{o1} \parallel r_{o3})$$

Cascode Differential Amplifier



Cascode configurations for both amplifying transistors and current source loads.

From half-circuit

$$A_d = \frac{v_{od}}{v_{id}} = g_{m1} (R_{on} \parallel R_{op})$$

$$R_{on} = (g_{m3} r_{o3}) r_{o1}$$

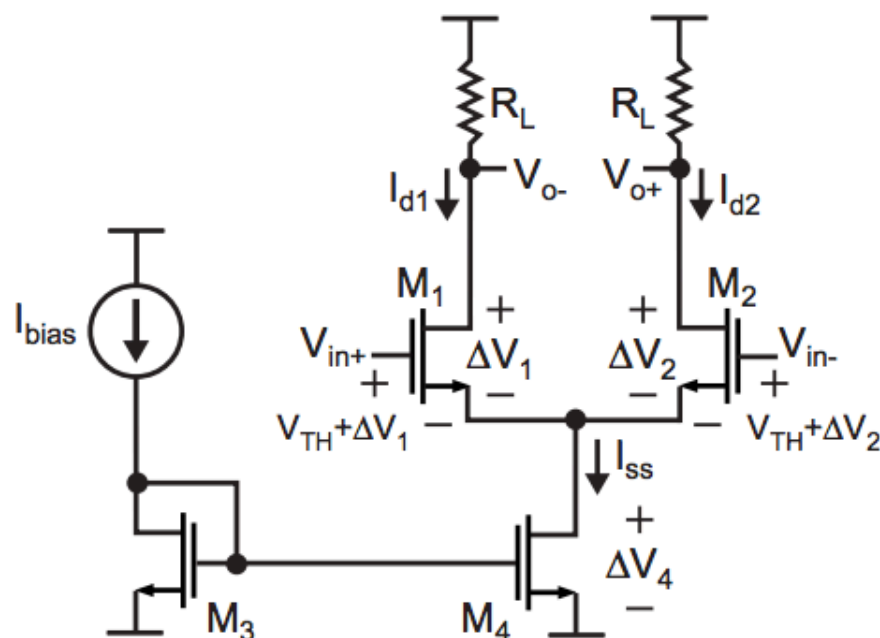
$$R_{op} = (g_{m5} r_{o5}) r_{o7}$$

If all transistors are identical,

$$R_{on} = R_{op} = g_m r_o^2$$

$$A_d = \frac{1}{2} g_m^2 r_o^2$$

Common Mode Voltage Range



- **While keeping all devices in saturation:**
 - **What is the maximum common mode output range?**
 - Assume $V_{id} = 0$
 - **What is the maximum common mode input range?**
 - Assume $V_{od} = 0$