Frequency Response

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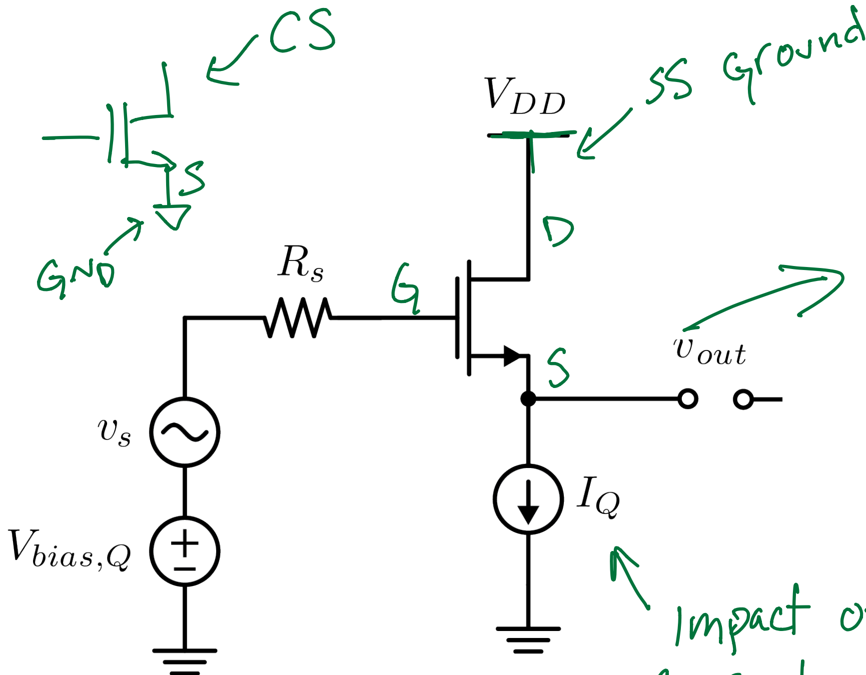
Prof. Rikky Muller

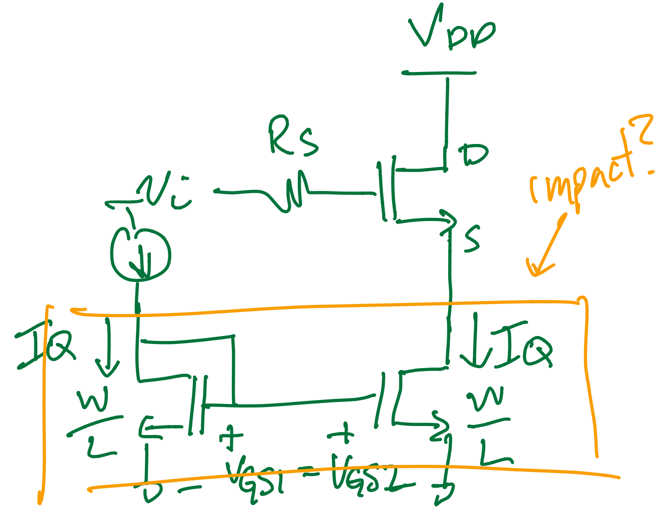
Announcements

- HW9 due on Friday
- LAB 5 EXTENDED

"SOURCE FOLLOWER"

Review: CD with Current Mirror

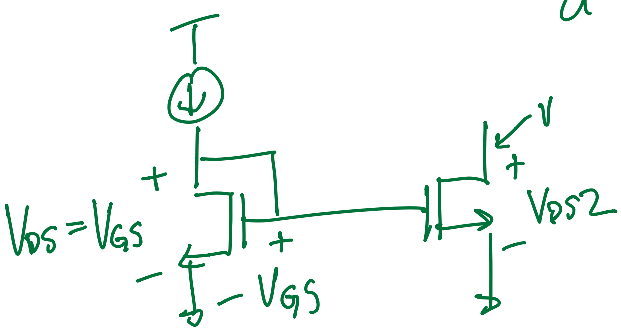




Impact of a real current source

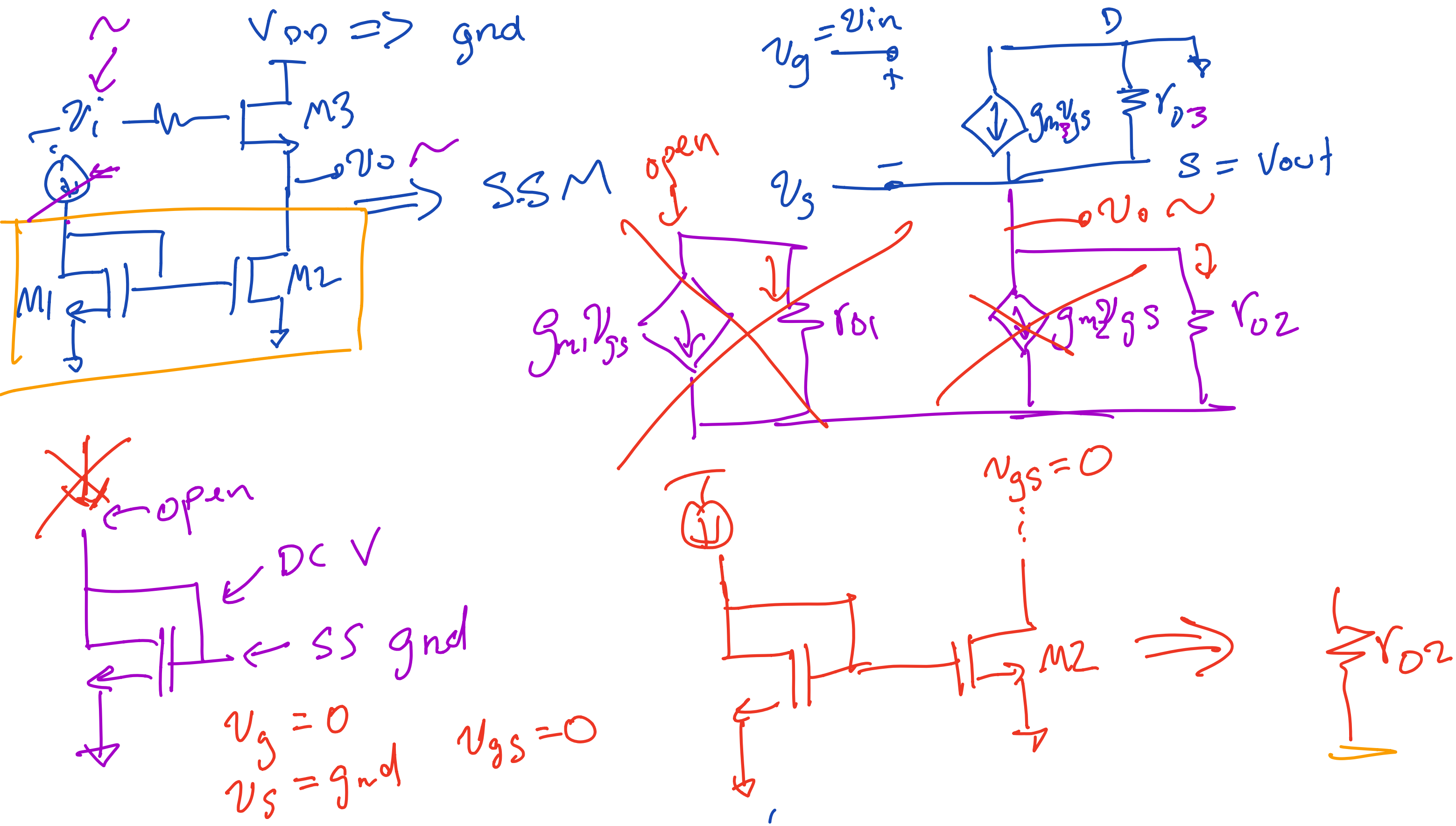
$$I_{DS} = \frac{1}{2}\mu C_{ox}\frac{W}{L}(V_{GS}-V_T)^2$$

$(\lambda \sim 0)$



$$I_{DS} = \frac{1}{2}\mu C_{ox}\frac{W}{L}(V_{gs}-V_T)^2(1+\lambda V_{DS})$$

Review: CD with Current Mirror



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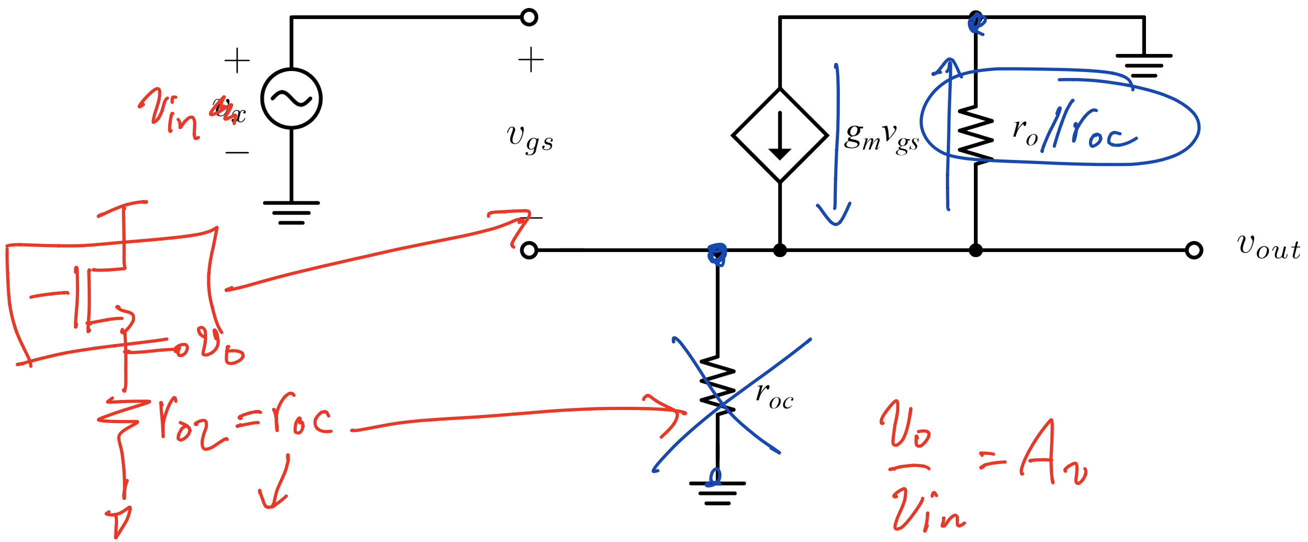
Review: CD with Current Mirror



$$\frac{v_o}{v_{in}} = A_v$$

KCL @ output node

$$\frac{v_o}{r_o // r_{oc}} = g_m v_{gs} = g_m (v_{in} - v_{out})$$

$$v_{out}\left(\frac{1}{r_o // r_{oc}} + g_m\right) = g_m v_{in}$$

$$\frac{v_{out}}{v_{in}} = \frac{g_m}{g_m + \frac{1}{r_o // r_{oc}}}$$

v_{out} $\quad g_m (r_o // r_{oc})$

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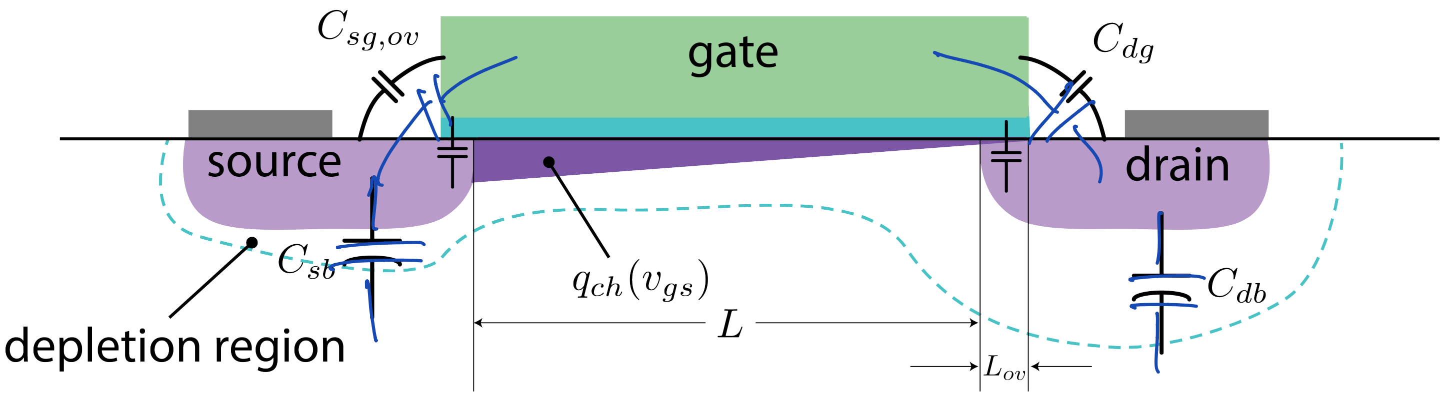
$\frac{}{v_{in}} = \frac{}{g_m(r_o \| r_{oc})+1}$

Capacitors in MOS Device



large cap

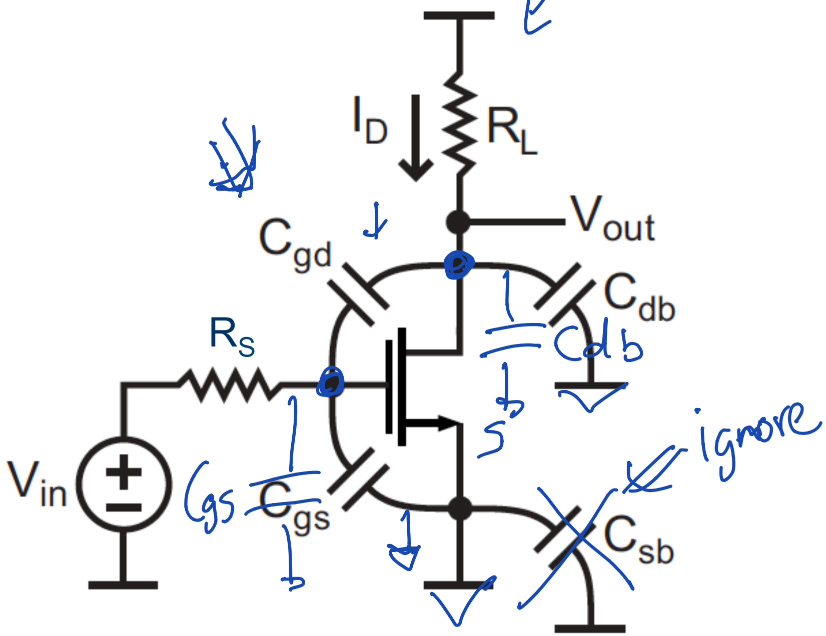
$$C_{gs} = (2/3)WLC_{ox} + C_{ov}$$ ← probably largest

$$C_{gd} = C_{ov}$$ ← small but problematic

$$C_{sb} = C_{jsb}(area + perimeter)\ junction$$

$$C_{db} = C_{jdb}(area + perimeter)\ junction$$

Common-Source Voltage Amplifier



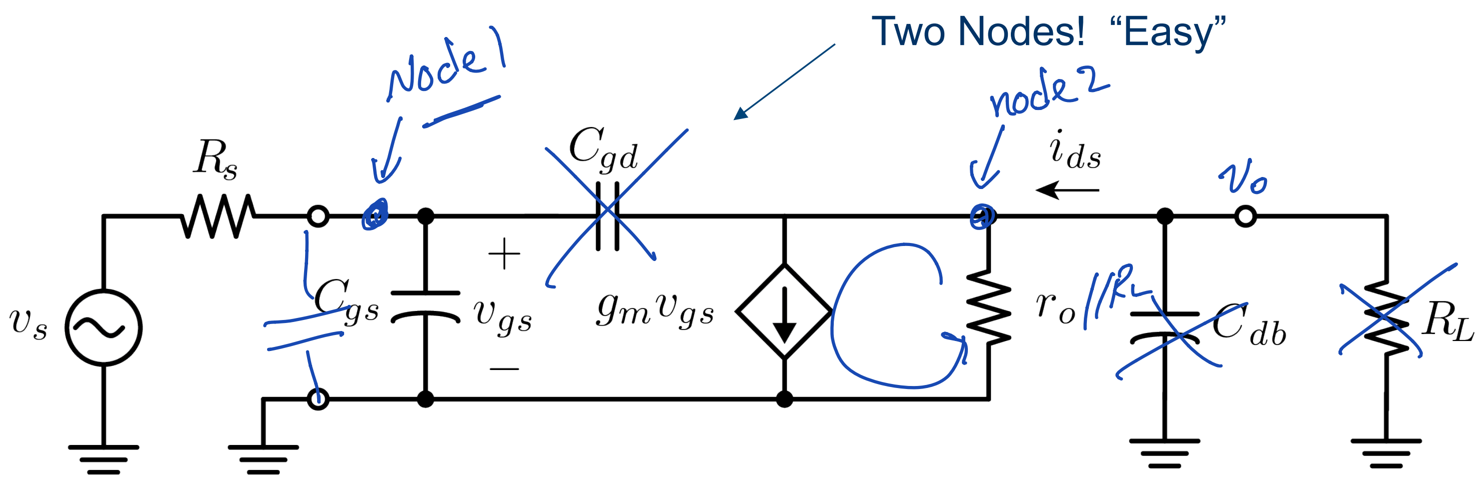
Small-signal model: C_{sb} is connected to gnd on both sides, therefore can be ignored

Can solve problem directly by nodal analysis or using 2-port models of transistor

OK if circuit is "small" (1-2 nodes)

We can find the complete transfer function of the circuit, but in many cases it's good enough to get an estimate of the -3dB bandwidth

CS Voltage Amp Small-Signal Model



For now we will ignore C_{db} to simplify the math

$$g_m v_{gs} = -\frac{v_o}{r_o \| R_L}$$

$$g_m v_{in} = -\frac{v_o}{r_o \| R_L} \qquad \frac{v_o}{v_g = v_i} = -g_m (r_o \| R_L)$$

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Frequency Response

KCL at input and output nodes; analysis is made complicated

DC gain

$$\frac{V_{out}}{V_{in}} = \frac{\underline{-g_m\left[r_o \parallel R_L\right]}\left(1 - j\omega/\omega_z\right)}{\left(1 + j\omega/\omega_{p1}\right)\left(1 + j\omega/\omega_{p2}\right)}$$

Zero

Two Poles

Low-frequency gain:

$$\frac{V_{out}}{V_{in}} = \frac{-g_m\left[r_o \parallel R_L\right]\left(1 - j0\right)}{\left(1 + j0\right)\left(1 + j0\right)} \rightarrow -g_m\left[r_o \parallel R_L\right]$$

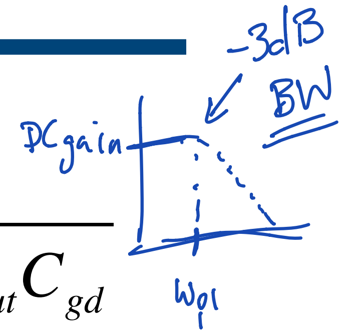
✓

Zero: $$\omega_z = \frac{g_m}{C_{gs} + C_{gd}}$$

Calculating the Poles

dominant pole

$$\omega_{p1} \approx \frac{1}{R_s\left\{C_{gs}+\left(1+g_m R'_{out}\right)C_{gd}\right\}+R'_{out}C_{gd}}$$



same denominator

Usually >> 1

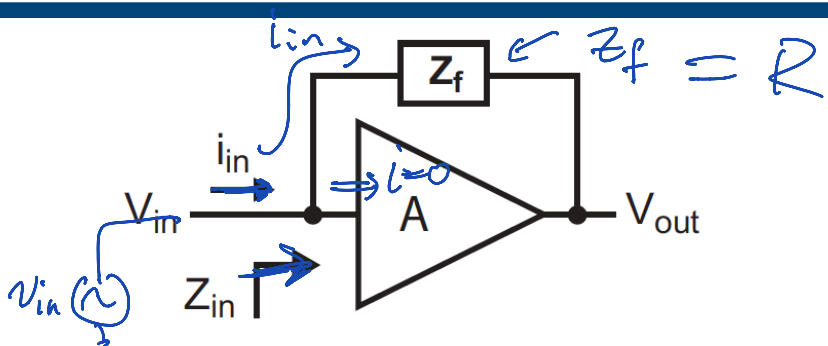
$$\omega_{p2} \approx \frac{R'_{out}/R_S}{R_S\left\{C_{gs}+\left(1+g_m R'_{out}\right)C_{gd}\right\}+R'_{out}C_{gd}}$$

much higher frequency

Results of complete analysis: not exact and little insight

These poles are calculated after doing some algebraic manipulations on the circuit. It's hard to get any intuition from the above expressions. There must be an easier way!

Method: The Miller Effect



- **Derive input impedance (assume gain of amplifier = *A*):**

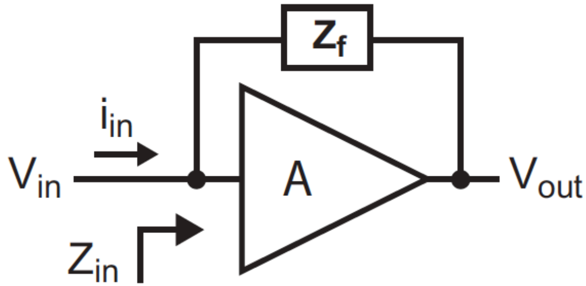
$$i_{in} = \frac{V_{in} - V_{out}}{Z_f} \qquad V_{out} = A v_{in}$$

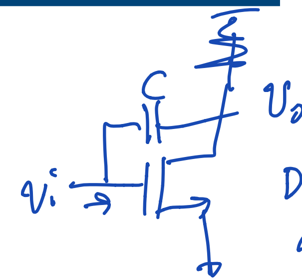
$$Z_{in} = \frac{V_{in}}{i_{in}}$$

$$i_{in} = \frac{V_{in} - A v_{in}}{Z_f} = \frac{V_{in}(1-A)}{Z_f}$$

$$\frac{V_{in}}{i_{in}} = \boxed{\frac{Z_f}{1-A}}$$

The Miller Effect





DC gain ~10

- **Derive input impedance (assume gain of amplifier = *A*):**

$$Z_{in} = \frac{V_{in}}{i_{in}} = \frac{V_{in}}{(V_{in} - V_{out})/Z_f} = \frac{V_{in} Z_f}{V_{in} - AV_{in}} = \frac{Z_f}{1 - A}$$ $= \frac{1}{sC}$

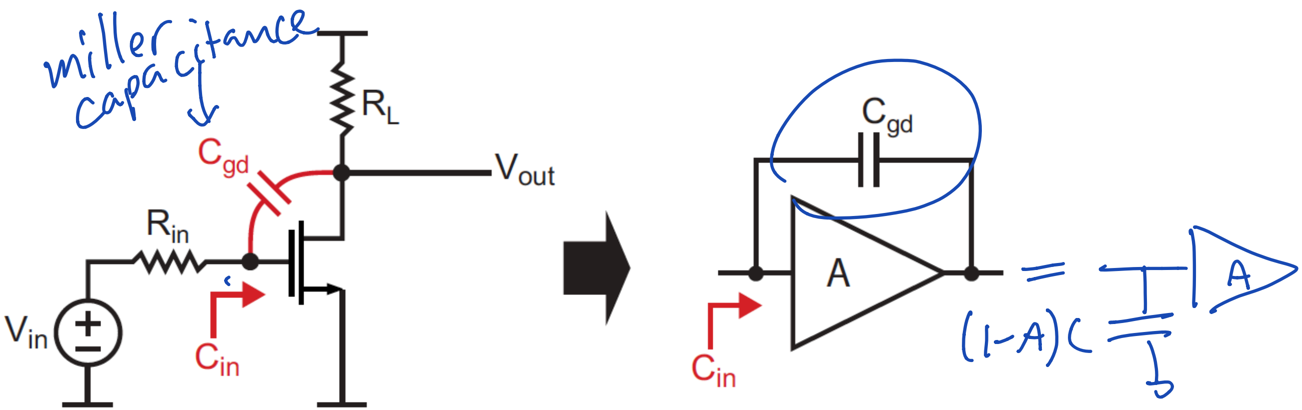
- **Consider the case where Z_f is a capacitor**

$$Z_f = \frac{1}{sC} \Rightarrow Z_{in} = \frac{1}{s(1-A)C}$$ ← very large capacitance

  - **For negative *A*, input impedance sees increased cap value**
  - **For *A* = 1, input impedance sees no influence from cap**
  - **For *A* > 1, input impedance sees negative capacitance!**

$Z_{in} = \frac{1}{s(11)C}$

Using The Miller Effect



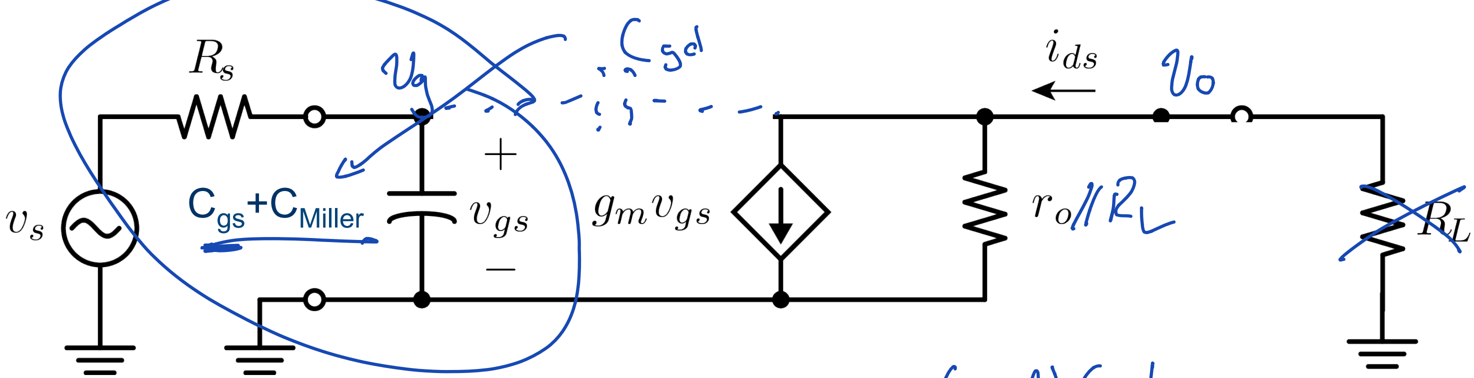
- **Notice that C_{gd} is in the feedback path of the common source amplifier**
  - **Recall Miller effect calculation:** $C_{in} = (1 - A)C_{gd}$

Effective input capacitance:

$$C_{in} = \frac{1}{j\omega C_{Miller}} = \left(\frac{1}{1 - A_{v,Cgd}}\right)\left(\frac{1}{j\omega C_{gd}}\right) = \frac{1}{j\omega\left[\left(1 - A_{vCgd}\right)C_{gd}\right]}$$

CS Voltage Amp Small-Signal Model

Modified Small-Signal Model with Miller Effect:



pole

$C_{miller} = (1-A)C_{gd}$

We can approximate the first pole by using Miller capacitance

This gives us a good approximation of the -3dB bandwidth approx. ω_{p1}

$$V_g = \frac{\frac{1}{(C_{gs}+C_{miller})s}}{\frac{1}{(C_{gs}+C_{miller})s} + R_s} = \frac{1}{1+R_s(C_{gs}+C_m)s}$$

$$\frac{V_o}{V_g} = -g_m(r_o // R_L)$$

$$\frac{V_o}{V_s} = \frac{-g_m(r_o // R_L)}{1+R_s(C_{gs}+C_m)s}$$

pole! $\omega_p = \frac{1}{R_s(C_{gs}+C_m)}$

Comparison with "Exact Analysis"

Miller result (calculate RC time constant of input pole):

$$\omega_{p1}^{-1} = R_S\left[C_{gs} + \underbrace{\overset{1-A}{\left(1 + g_m R'_{out}\right)} C_{gd}}_{C_{miller}}\right]$$

Exact result:

$$\omega_{p1}^{-1} = R_S\left[\underbrace{C_{gs} + \left(1 + g_m R'_{out}\right) C_{gd}}_{\text{large}}\right] + R'_{out} C_{gd}$$

As a result of the Miller effect there is a fundamental gain-bandwidth tradeoff

DC gain $= -g_m(r_o \| R_L) = -g_m R_{out}$

$\omega_{p1}^{-1} \approx R_S C_{gs} + R_S(1 + g_m R_{out}')C_{gd}$

tradeoff

if we multiply gain × BW ~ constant

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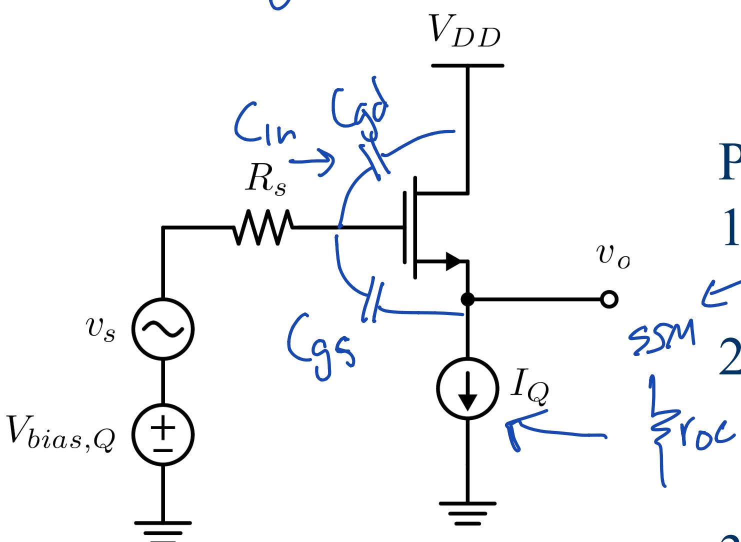
Common Drain Amplifier

"source follower"

DC result is different
gain ~ +1

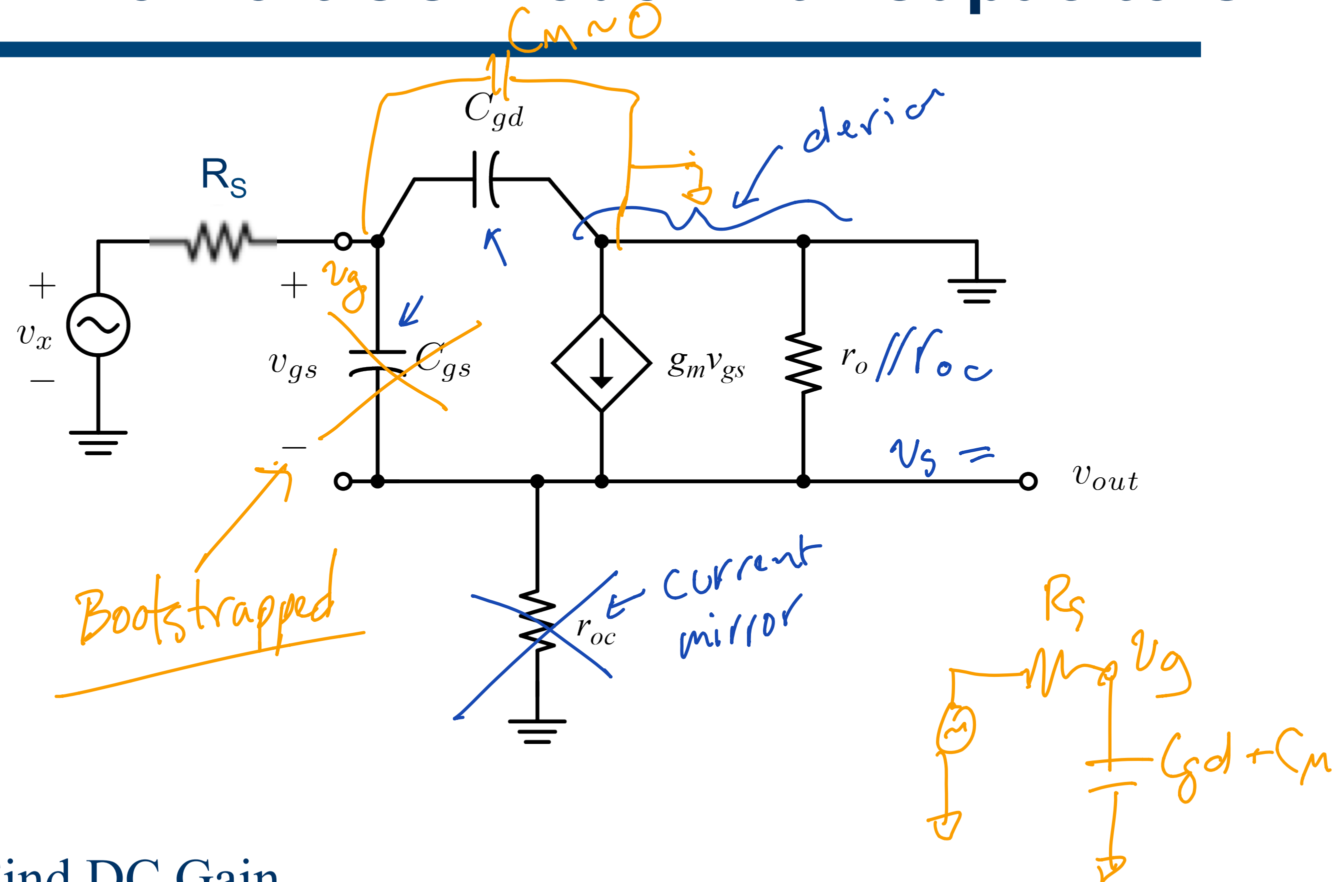


Calculate Bandwidth of the Common Drain (Source-Follower)

Procedure:

1. Replace current source with MOSFET-based current mirror
2. Draw small-signal model with capacitors (for simplicity, we will focus on C_{gd} and C_{gs})
3. Find the ~~DC~~ SF small-signal gain ~1
4. Use the Miller effect to calculate the input capacitance
5. Calculate the dominant pole

Two-Port CC Model with Capacitors



-Find DC Gain

-Find Miller capacitor for C_{gs} -- *note that the gate-source capacitor is between the input and output!*

Voltage Gain Across C_{gs}

Write KCL at output node:

$$\frac{v_{out}}{r_o \| r_{oc}} = g_m v_{gs} = g_m (v_{in} - v_{out})$$

$$v_{out}\left(\frac{1}{r_o \| r_{oc}} + g_m\right) = g_m v_{in}$$

$$\frac{v_{out}}{v_{in}} = \frac{g_m}{\left(\frac{1}{r_o \| r_{oc}} + g_m\right)} = \boxed{\frac{g_m (r_o \| r_{oc})}{1 + g_m (r_o \| r_{oc})}} = A_{vCgs}$$

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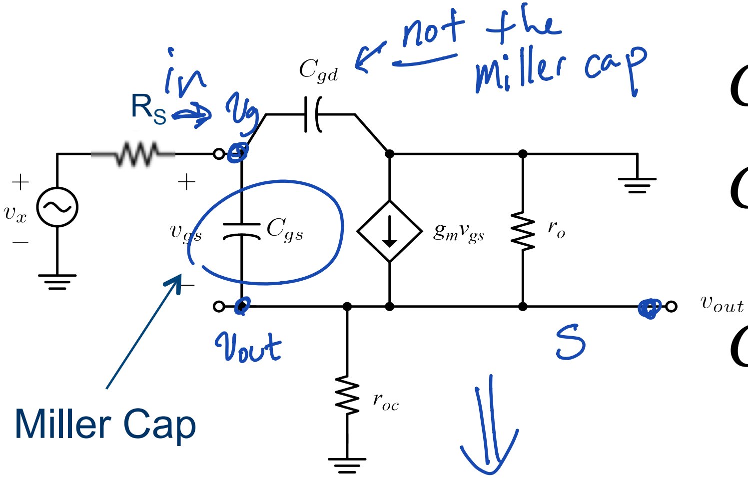
Compute Miller Effected Capacitance

Now use the Miller Effect to compute C_{in}:

Remember that C_{gs} is the capacitor from the input to the output



$$C_{in} = C_{gd} + C_M$$

$$C_{in} = C_{gd} + (1 - A_{vC_{gs}})C_{gs}$$

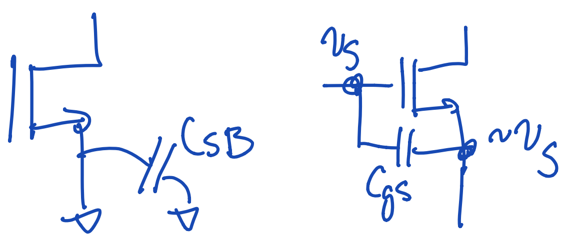
$$C_{in} = C_{gd} + \left(1 - \frac{g_m(r_o \| r_{oc})}{1 + g_m(r_o \| r_{oc})}\right)C_{gs}$$

$$C_{in} = C_{gd} + \left(\frac{1}{1 + g_m(r_o \| r_{oc})}\right)C_{gs}$$

heavily attenuated

$$C_{in} \approx C_{gd}$$

(for large $g_m(r_o // r_{oc})$)

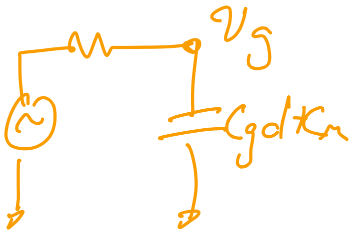


Bandwidth of Source Follower

Input low-pass filter's –3 dB frequency:

$$\omega_p^{-1} = R_S\left(C_{gd} + \frac{C_{gs}}{1 + g_m(r_o \| r_{oc})}\right)$$

Small



Substitute favorable values of R_S, r_o:

$$R_S \approx 1/g_m \qquad r_o >> 1/g_m$$

$$\omega_p^{-1} \approx (1/g_m)\left(C_{gd} + \frac{C_{gs}}{1 + BIG}\right) \approx C_{gd}/g_m$$

$$\omega_p \approx g_m / C_{gd}$$

Very high frequency! Model not valid at these high frequencies

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Some Examples

Common source amplifier:

$$A_{vCgd} = \text{ Negative, large number (-100)}$$

$$C_{Miller} = (1 - A_{V,C_{gd}})C_{gd} \approx 100C_{gd}$$

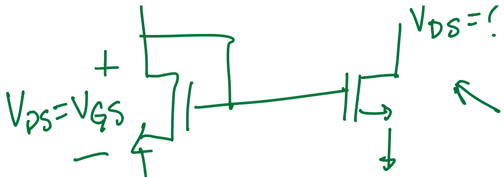
Miller Multiplied Cap has detrimental impact on bandwidth

Common drain amplifier: source follower

$$A_{vCgs} = \text{ Slightly less than 1}$$ & positive

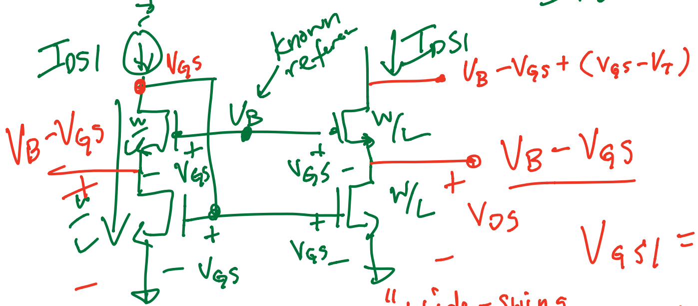
$$C_{Miller} = (1 - A_{V,Cgs})C_{gs} \simeq 0$$

"Bootstrapped" cap has negligible impact on bandwidth!



$V_{DS1} \neq V_{DS2} \Rightarrow r_o$

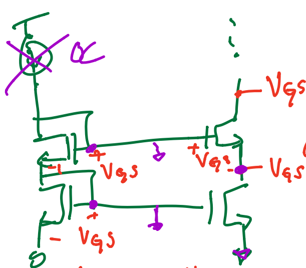
$I_{DS1} \neq I_{DS2}$



"version 2"

$V_{GS1} = V_{GS2}$ { $V_{DS1} = V_{DS2}$

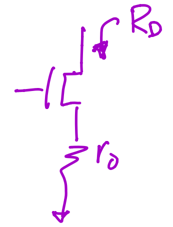
"wide-swing cascode current mirror"

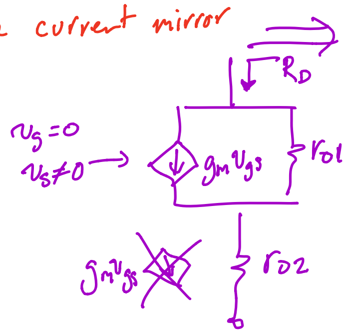


← BIGGER

cascode current mirror

"version 1"





$R_D \approx \underline{g_m r_o^2}$

complete soln'

$R_D = r_{o1} + r_{o2} + \underline{g_m r_{o1} r_{o2}}$