

# Metal Oxide Semiconductor Field Effect Transistors (MOSFETs)

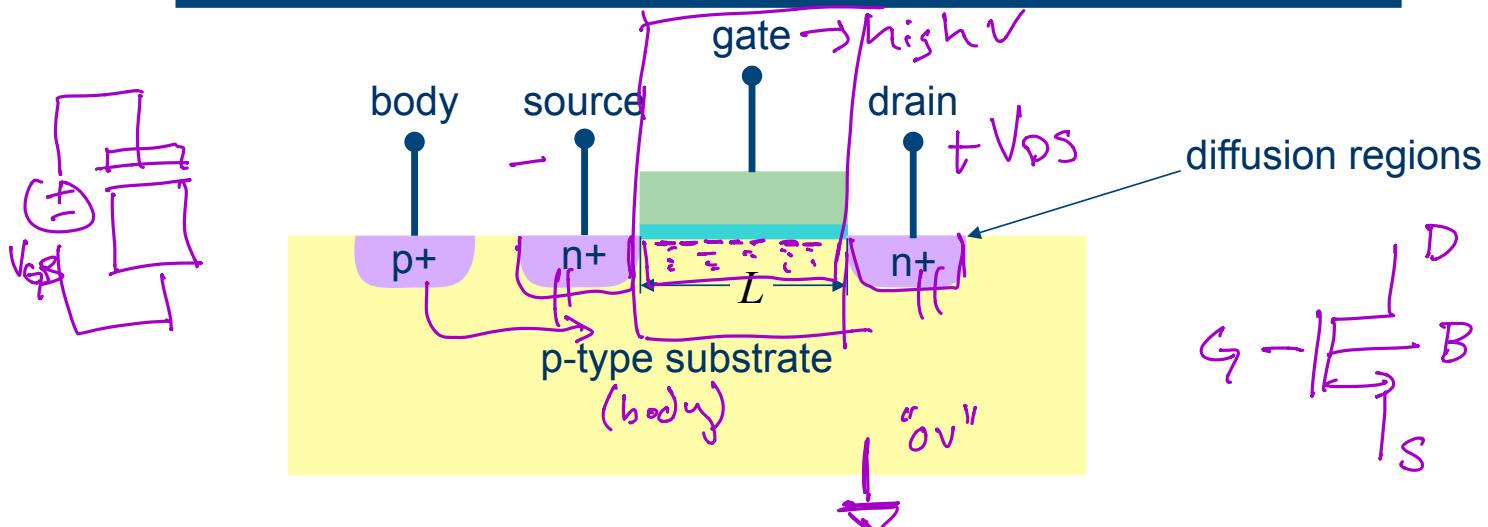
Prof. Ali M. Niknejad  
Prof. Rikky Muller

# Announcements

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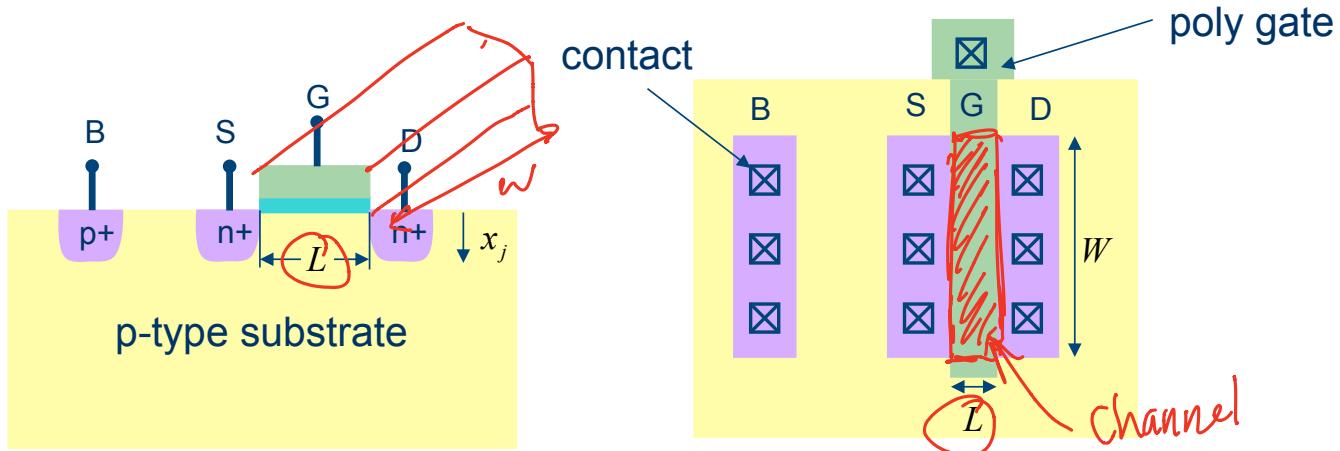
- Pick up Midterm 1 if you haven't already!
- Two options: (1) from my office hours, or (2) ~~from lecture today~~ AFTER LECTURE FROM MY OFFICE
- MIDTERM SOLUTIONS ON BCOURSES

# MOSFET Cross Section



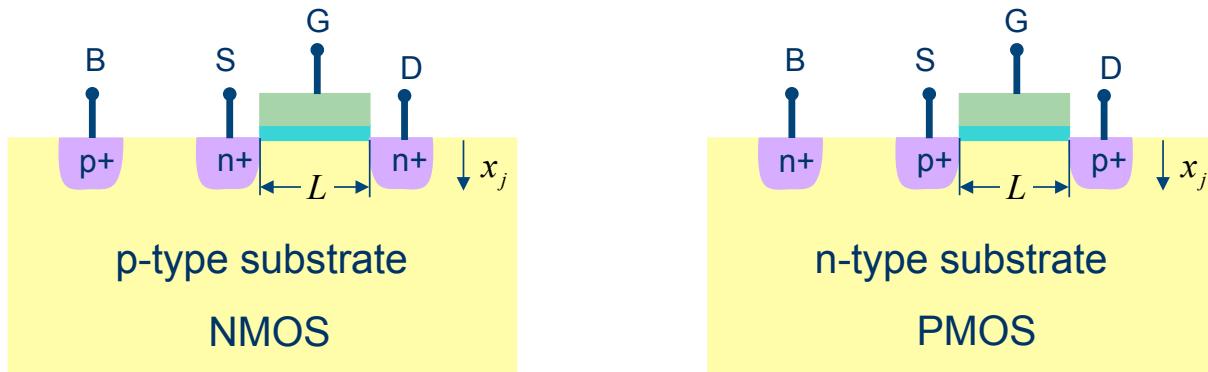
- Add two junctions around MOS capacitor
- The regions forms PN junctions with substrate
- MOSFET is a four terminal device
- The body is usually grounded (or at a DC potential)
- For ICs, the body contact is at surface

# MOSFET Layout



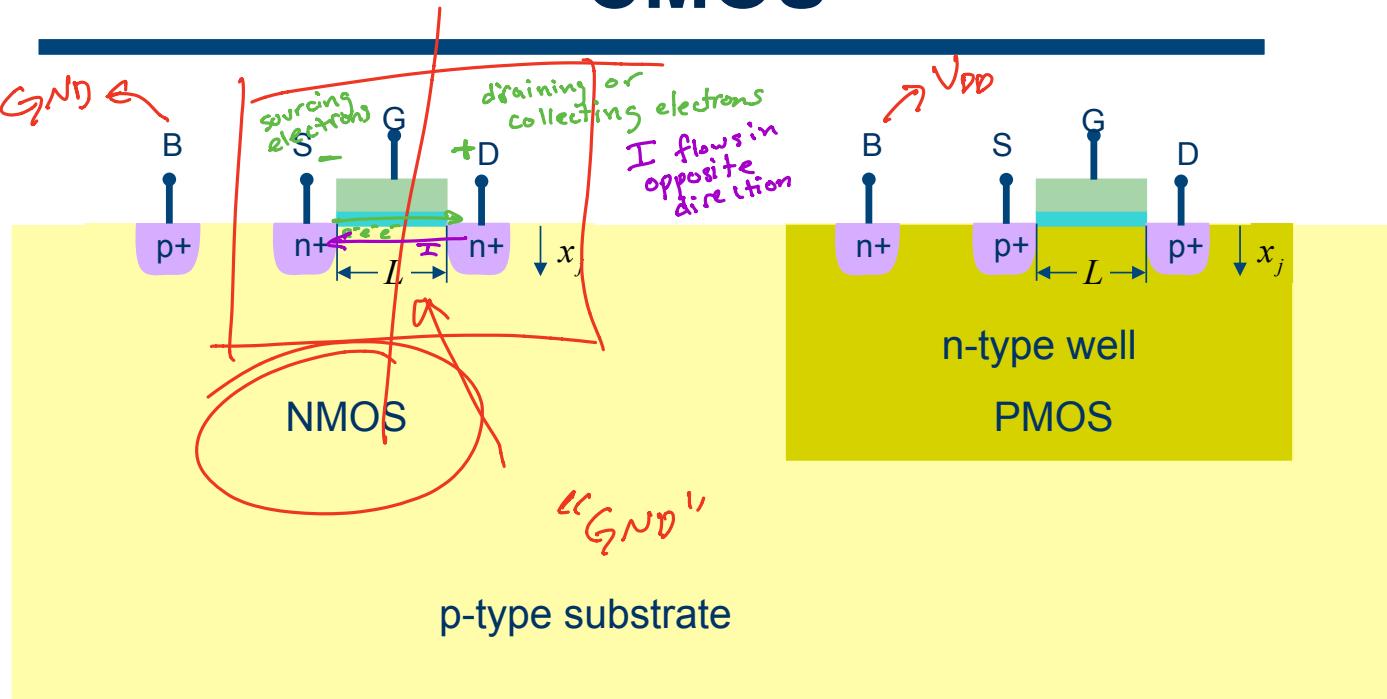
- Planar process: complete structure can be specified by a 2D layout
- Design engineer can control the transistor width  $W$  and  $L$
- Process engineer controls  $t_{ox}$ ,  $N_a$ ,  $x_j$ , etc.

# PMOS & NMOS



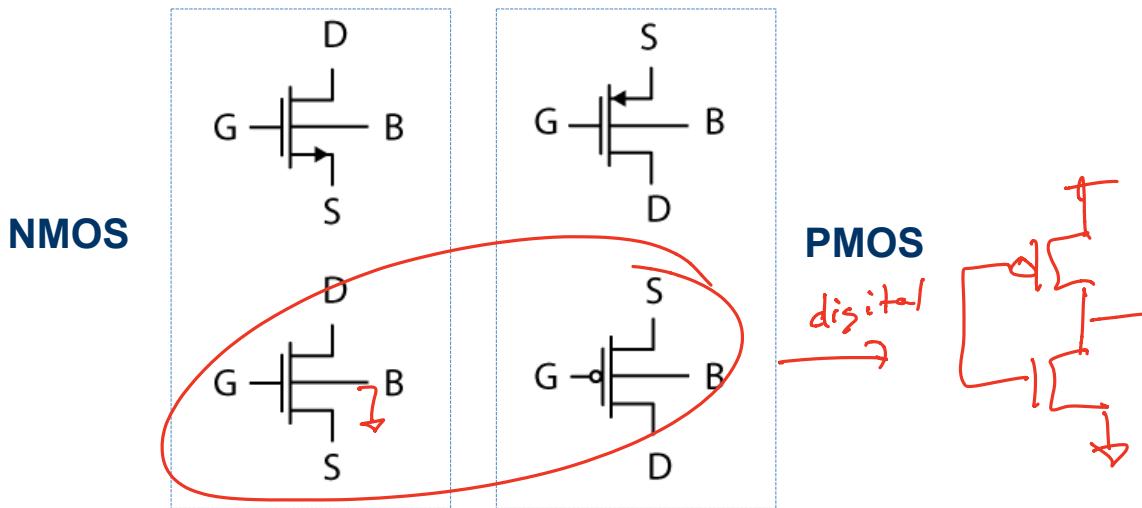
- A MOSFET by any other name is still a MOSFET:
  - NMOS, PMOS, nMOS, pMOS
  - NFET, PFET
  - IGFET
  - Other flavors: JFET, MESFET
- CMOS technology: The ability to fabricate NMOS and PMOS devices simultaneously

# CMOS



- Complementary MOS (CMOS): Both P and N type devices
- Create a n-type body in a p-type substrate through compensation. This new region is called a “well”.
- To isolate the PMOS from the NMOS, the well must be reverse biased (p-n junction)

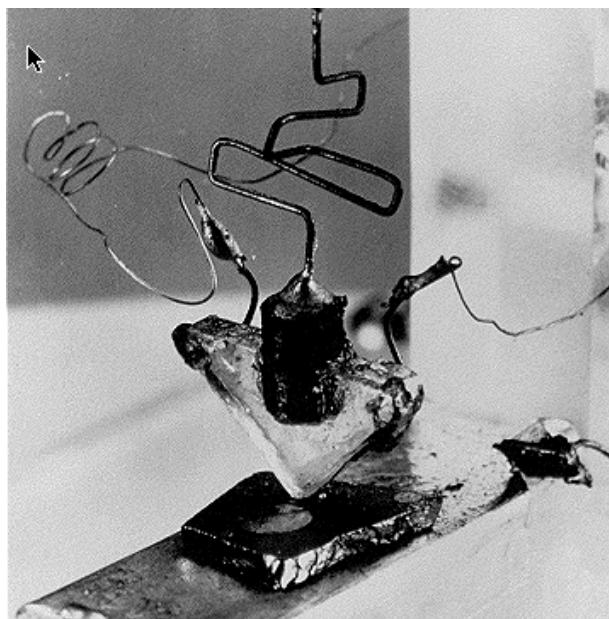
# Circuit Symbols



- The symbols with the arrows are typically used in analog applications
- The body contact is often not shown
- The source/drain can switch depending on how the device is biased (the device has inherent symmetry)

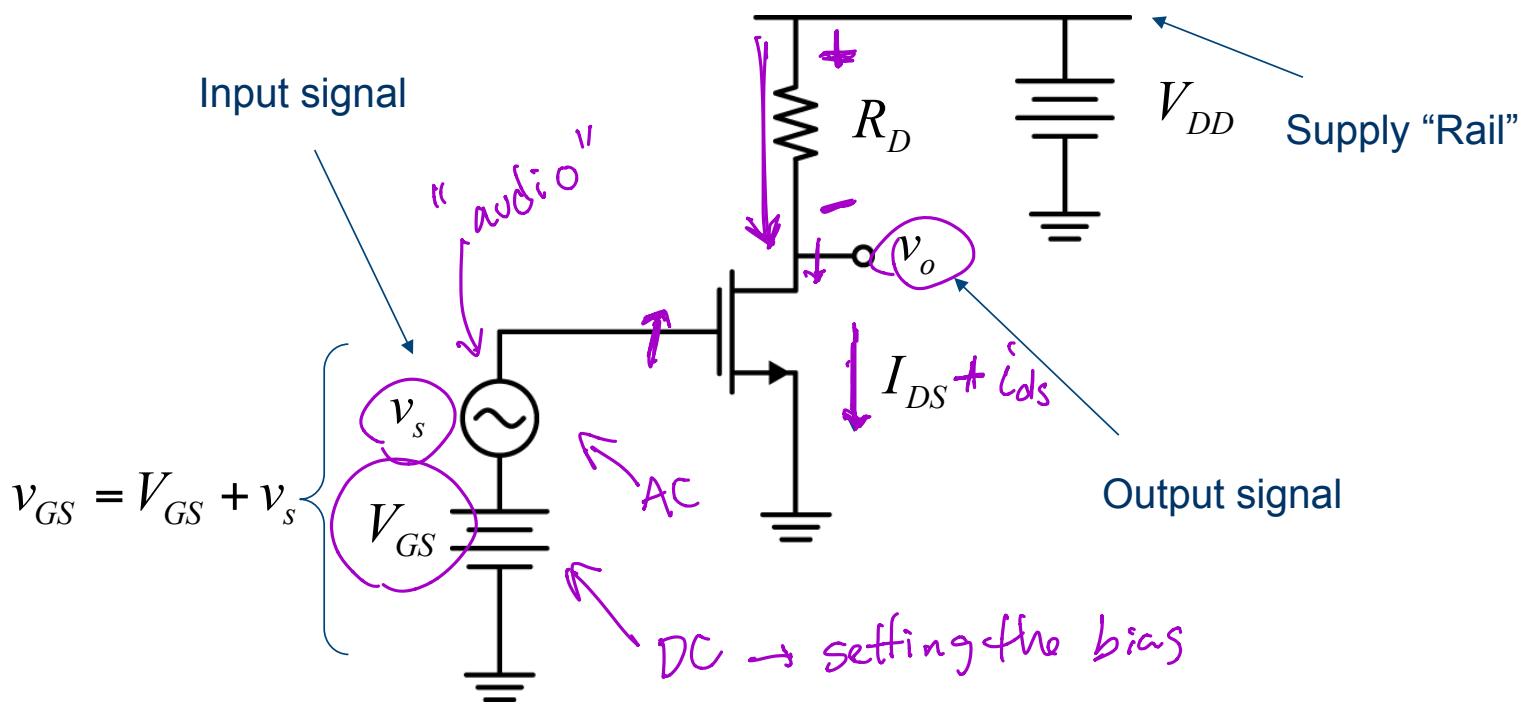
# Circuits!

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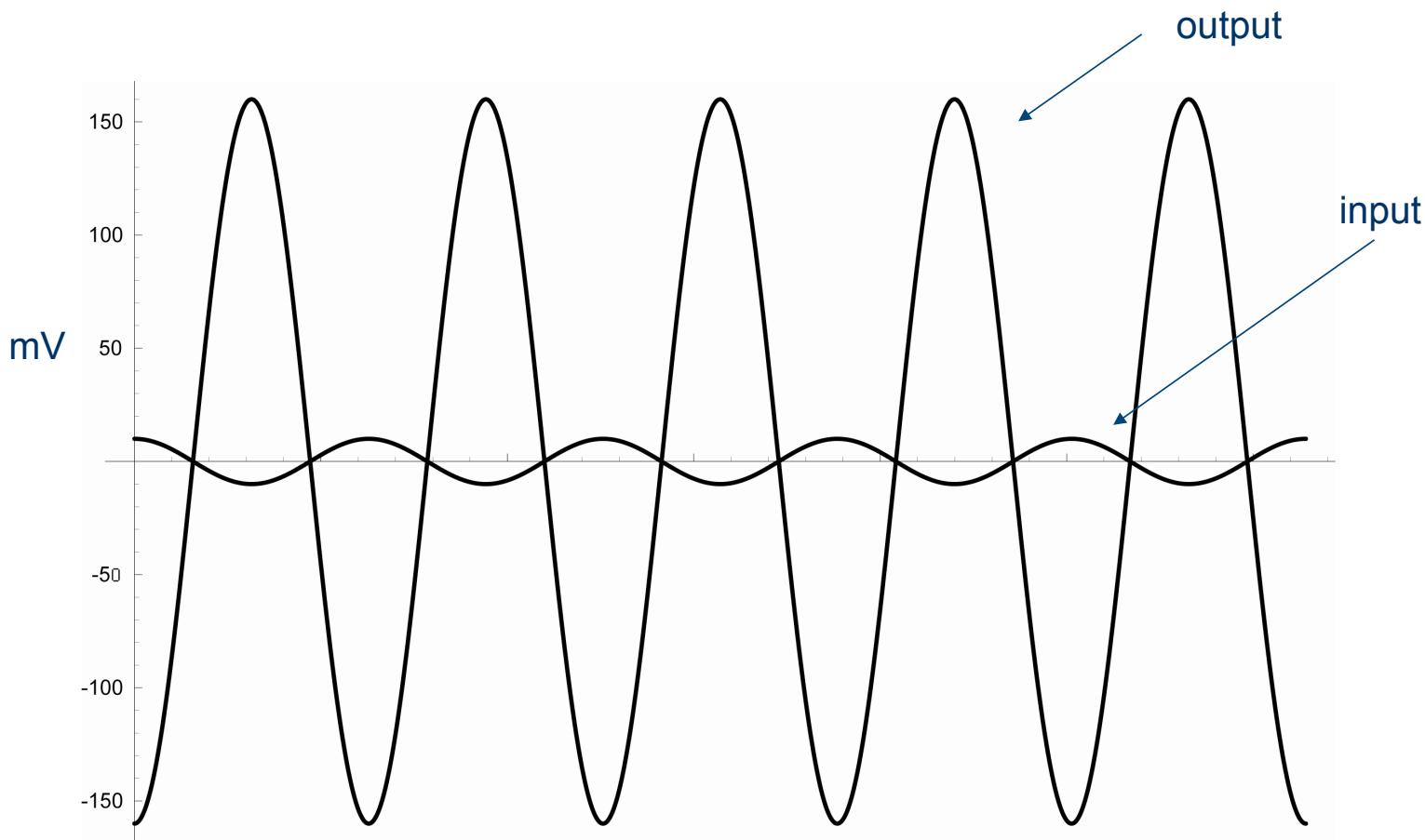


- When the inventors of the bipolar transistor first got a working device, the first thing they did was to build an audio amplifier to prove that the transistor was actually working!

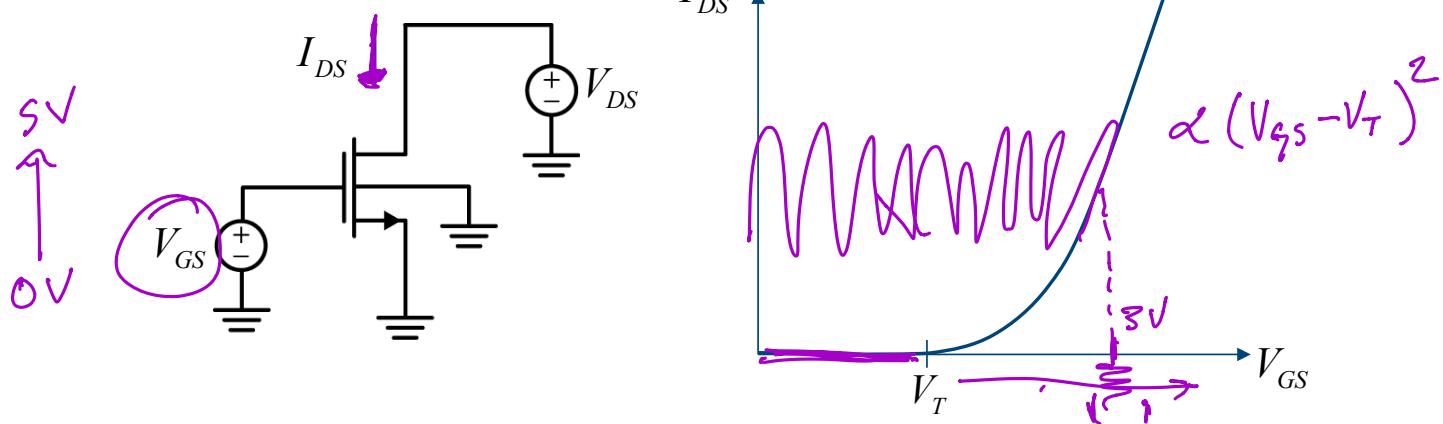
# A Simple Circuit: An MOS Amplifier



# Plot of Output Waveform (Gain!)

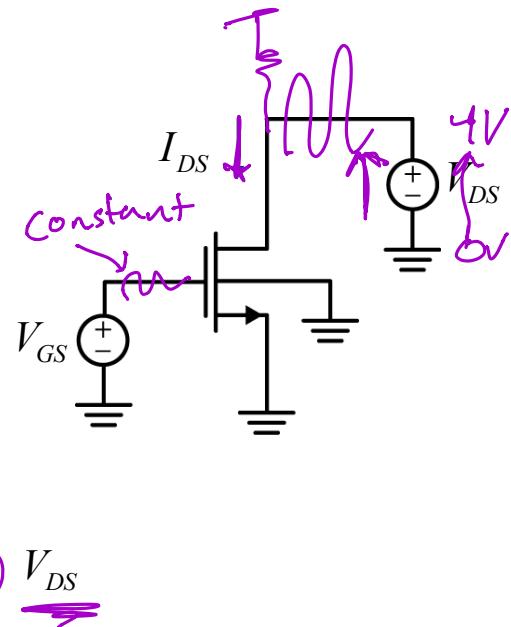
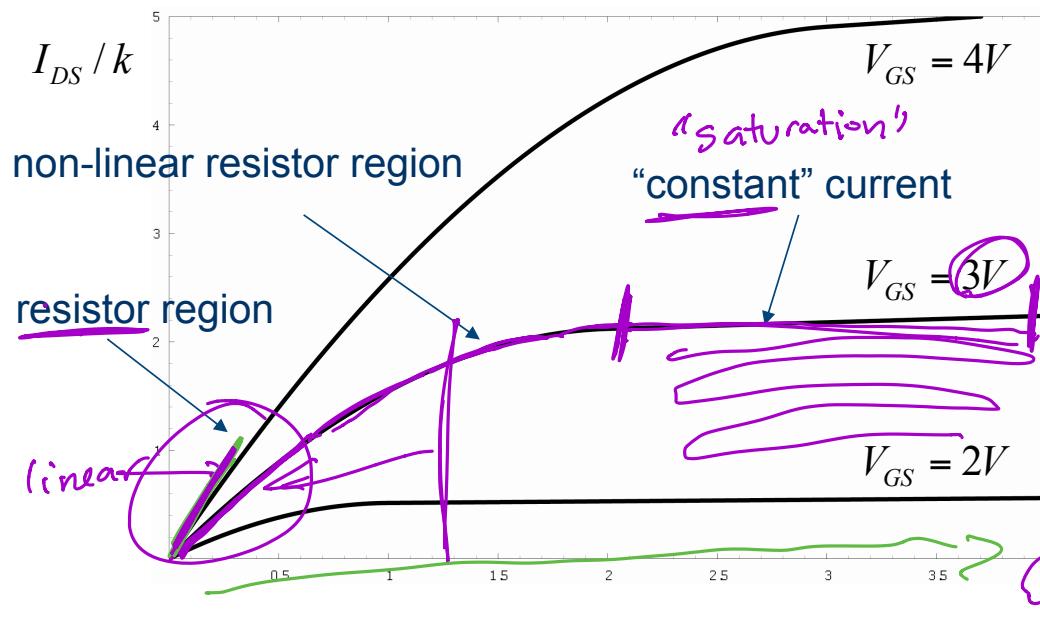


# Observed Behavior: $I_D$ - $V_{GS}$



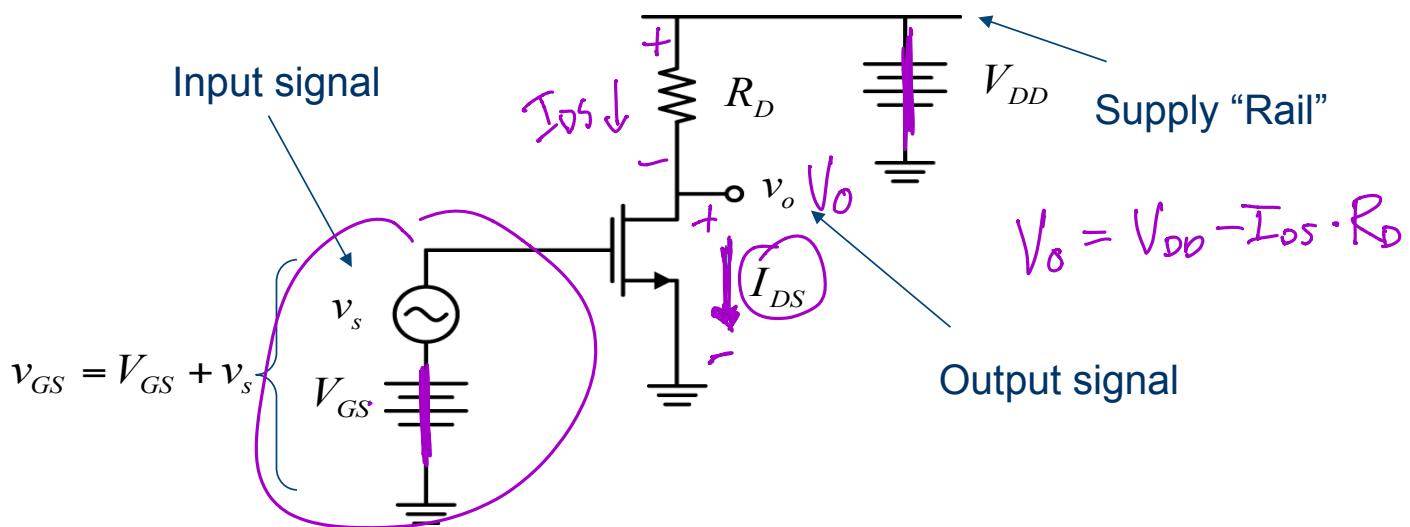
- Current zero for negative gate voltage
- Current in transistor is very low until the gate voltage crosses the threshold voltage of device (same threshold voltage as MOS capacitor)
- Current increases rapidly at first and then it finally reaches a point where it simply increases linearly

# Observed Behavior: $I_D$ - $V_{DS}$



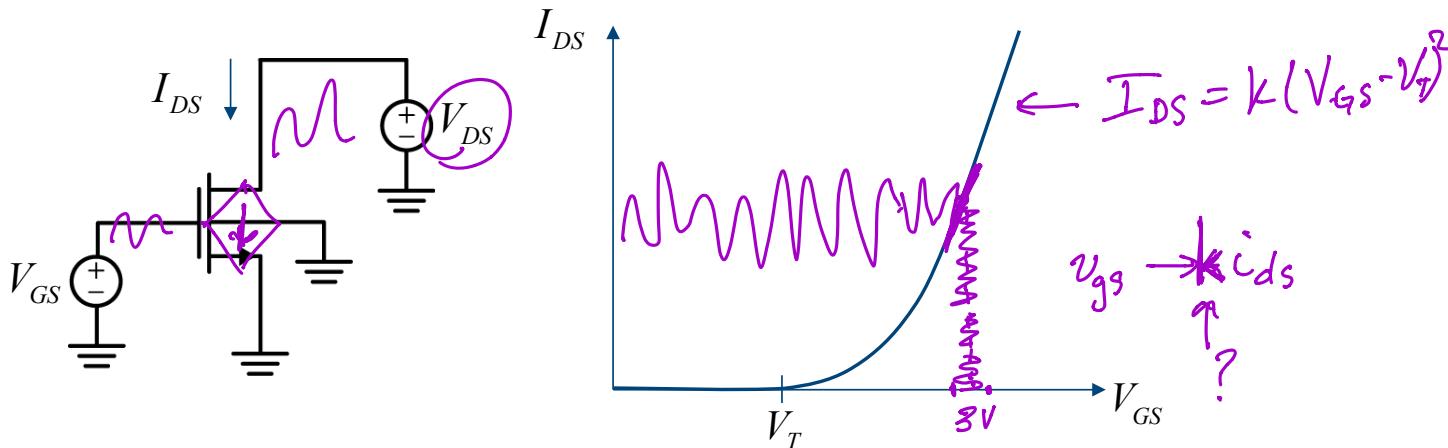
- For low values of drain voltage, the device is like a resistor
- As the voltage increases, the resistance behaves non-linearly and the rate of increase of current slows
- Eventually the current stops growing and remains essentially constant (current source)

# Operating Points



- Which bias voltages you operate the MOSFET at will make a big difference in how it functions.
- We will explore these regions of operation in this lecture.
- If we operate with a sufficiently high  $V_{GS}$  AND a sufficiently high  $V_{DS}$ , we can make a very good small-signal amplifier!

# Small Signal vs. Large Signal

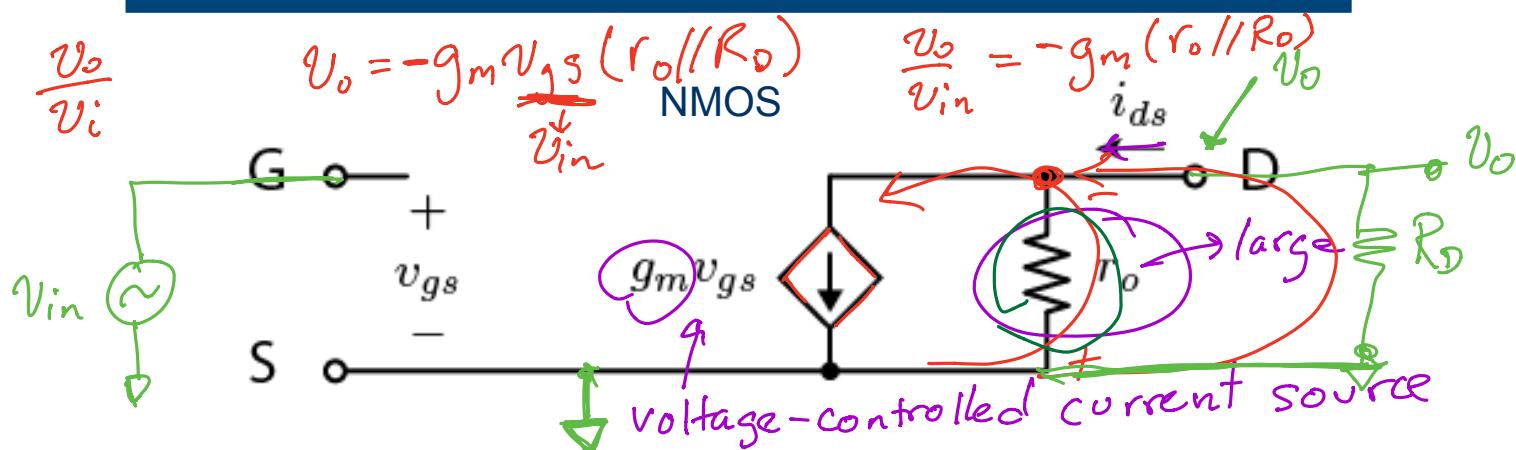


- We observe  $I_{DS}$  vs.  $V_{GS}$  to be quadratic for  $V_{GS} > V_T$
- Large changes in  $v_{GS}$  result in quadratic changes at the output
- However, for small changes in  $v_{GS}$  (denoted as  $v_{gs}$ ) will produce linear changes at the output!
- We can show this using Taylor series expansion:

$$\underline{i_{DS}} = \underline{k(v_{GS} - V_T)^2}; \underline{v_{GS}} = \underline{V_{GS}} + \underline{v_{gs}} \xrightarrow{\text{SS}}$$

$$TS : i_{DS}(v_{GS}) \Big|_{\substack{v_{GS} \\ \text{scalar}}} = \underline{k(V_{GS} - V_T)^2} + 2k(V_{GS} - V_T)\underline{v_{gs}} + \underline{kv_{gs}^2}$$

# Simple Small Signal Model for MOSFET

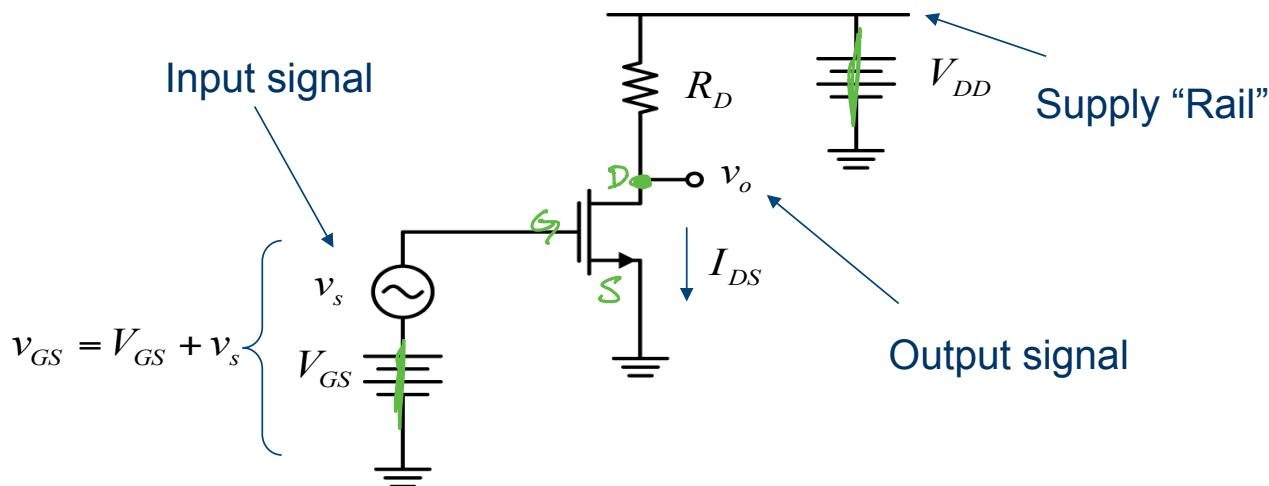


- This is a simplified, 3-terminal small-signal model for a MOSFET
- In later lectures we will develop a more complete model
- $g_m$  = transconductance
  - defined as  $di_{ds}/dv_{gs}$ , units  $[\text{Ohms}]^{-1}$
- $r_o$  = output resistance
  - defined as  $[di_{ds}/dv_{ds}]^{-1}$ , units Ohms

$$v_{gs} \cdot g_m = i_{ds}$$

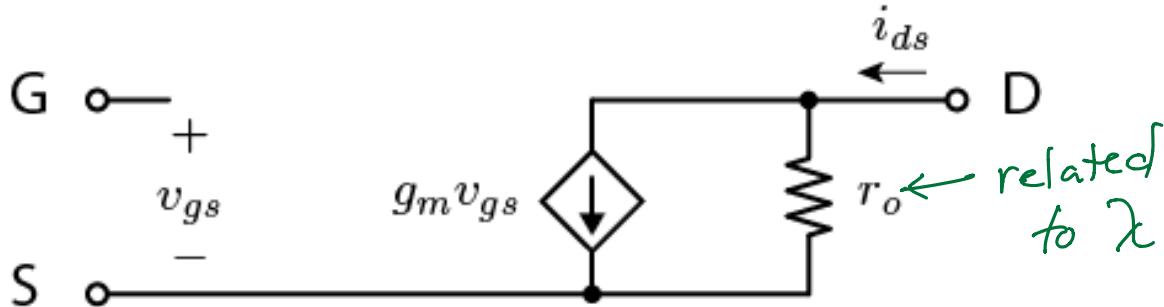
$$\hookrightarrow \text{Siemens} = \frac{1}{\Omega}$$

# Small Signal Gain Example



- Steps to analyze small signal amplifiers:
- 1. Calculate bias points using DC sources
  - As you will see in later lecture, you will use these bias points to determine the MOSFET region of operation as well as to calculate small-signal parameters
- 2. Turn off DC sources
- 3. Plug in the small-signal model for a MOSFET

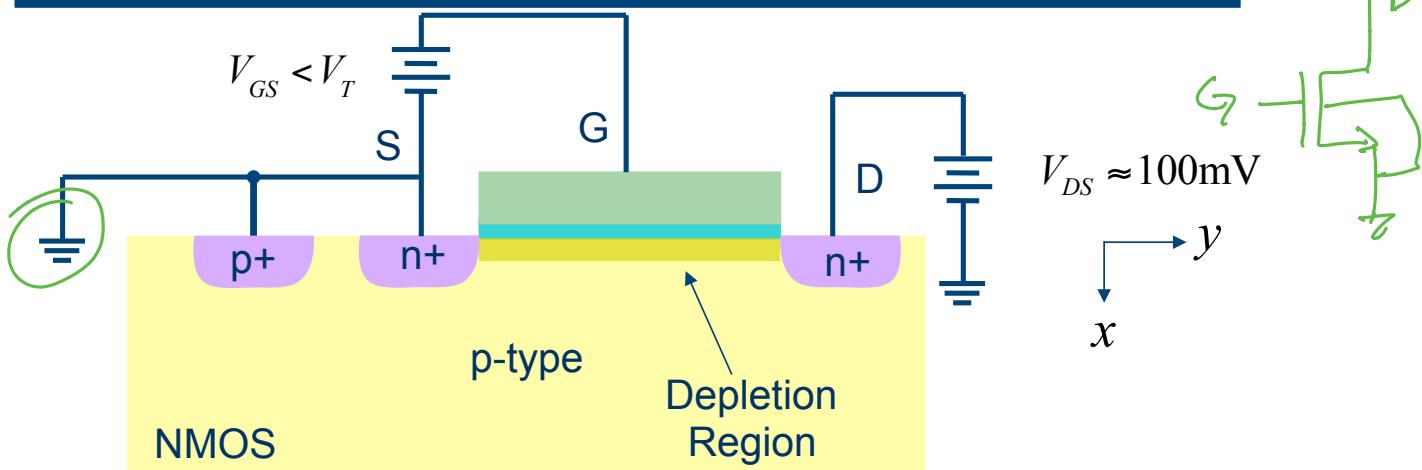
# Small Signal Gain Example



- 4. Calculate the gain ( $v_{out}/v_{in}$ ) of the circuit

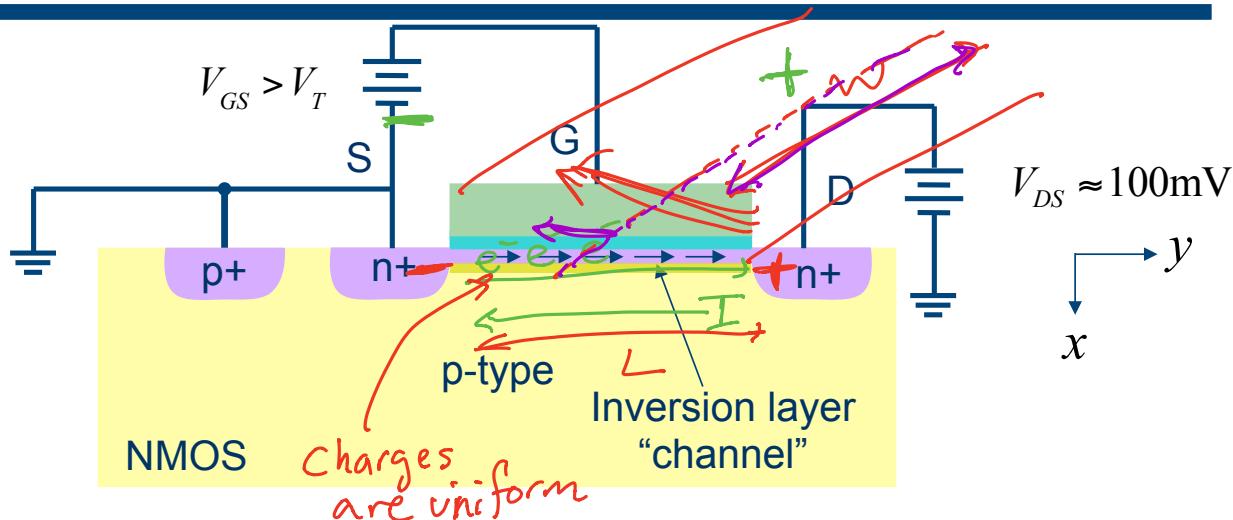
# MOSFET Large Signal Models and Regions of Operation

# Cut-off, $V_{GS} < V_T$



- This structure should look familiar! It is an MOS capacitor with two n+ diffusion regions on each side.
- When  $V_{GS} < V_T$ , the device is either in accumulation or in depletion.
- Since there are no (or few) inversion charges at the surface, therefore no current will flow regardless of the value of  $V_{DS}$ .

# “Linear” Region Current

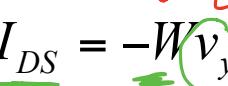


- If the gate is biased above threshold, the surface is inverted
- This inverted region forms a channel of inversion charges (in this case electrons) that connects the drain and source – inversion charges originate from n+ diffusion
- If a drain-source voltage ( $V_{DS}$ ) is applied positive, electrons will flow from source to drain
- Note: electrons flow  $S \rightarrow D$ , current flows  $D \rightarrow S$

# MOSFET “Linear” Region

- The current in this channel is given by

$$I_{DS} = -Wv_y Q_N$$

width  velocity  
 charge/area   $e^-$

- The charge proportional to the voltage applied across the oxide over threshold

$$\Rightarrow Q_N = C_{ox}(V_{GS} - V_{Tn}) \quad \text{mos capacitor}$$

$$I_{DS} = -Wv_y C_{ox}(V_{GS} - V_{Tn})$$

- If the channel is uniform density, only drift current flows

*“linear”*

$$v_y = -\mu_n E_y$$

mobility   
 electric field 

$$E_y = -\frac{V_{DS}}{L}$$

potential   
 distance 

$$I_{DS} = \frac{W}{L} \mu_n C_{ox} (V_{GS} - V_{Tn}) V_{DS}$$

small   
 $V_{DS}$

$V_{GS} > V_{Tn}$

$V_{DS} \approx 100\text{mV}$

# MOSFET: Variable Resistor

- Notice that in the linear region, the current is proportional to the voltage

$$I_{DS} = \frac{W}{L} \mu_n C_{ox} (V_{GS} - V_{Tn}) V_{DS}$$

- Can define a voltage-dependent resistor

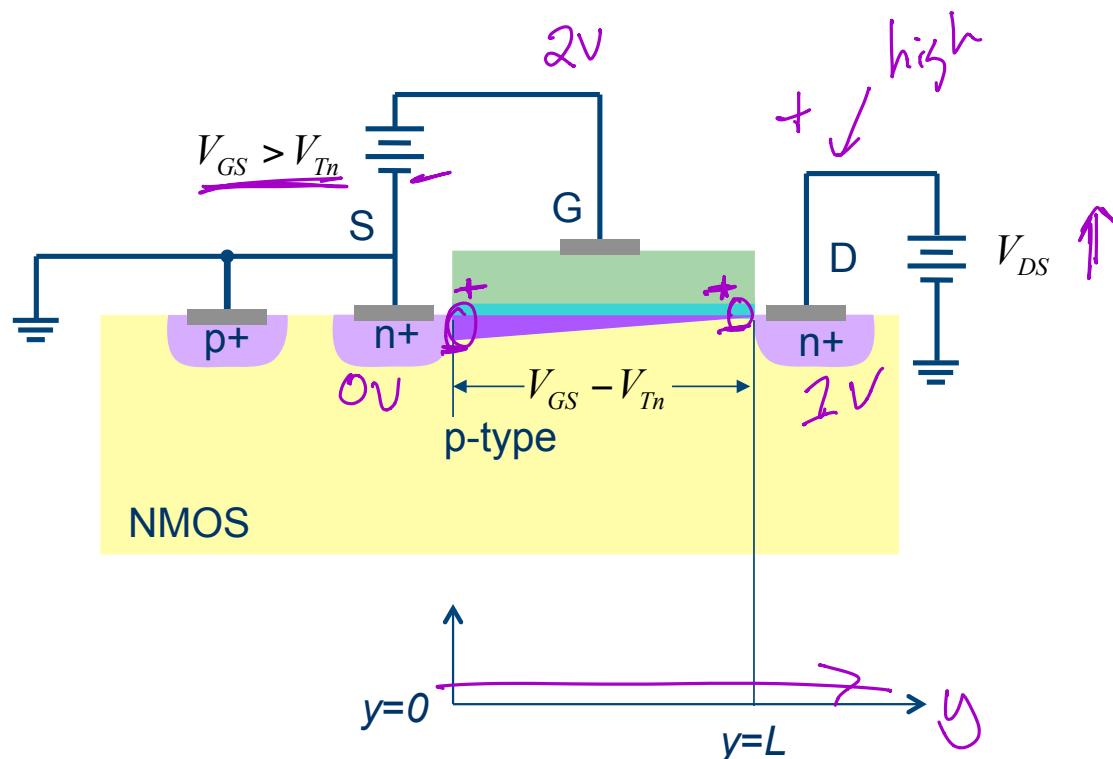
$$R_{eq} = \frac{V_{DS}}{I_{DS}} = \frac{1}{\mu_n C_{ox} (V_{GS} - V_{Tn})} \left( \frac{L}{W} \right) = R_W(V_{GS}) \frac{L}{W}$$

$R_W$

- This is a nice variable resistor, electronically tunable!

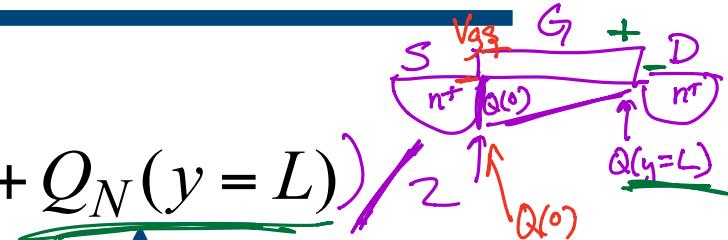
# Finding $I_D = f(V_{GS}, V_{DS})$

- Approximate inversion charge  $Q_N(y)$ : drain voltage is higher than the source  $\rightarrow$  less charge at drain end of channel



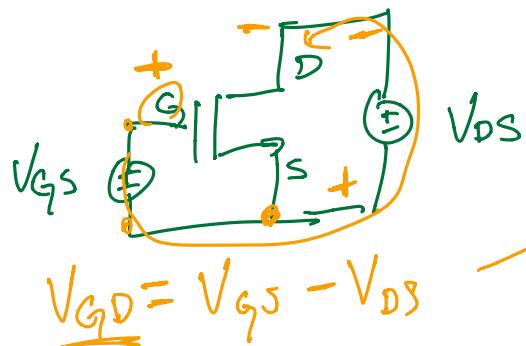
# Inversion Charge at Source/Drain

$$Q_N(y) \approx \underline{Q_N(y=0)} + \underline{Q_N(y=L)}$$



$$\underline{Q_N(y=0)} = -C_{ox}(V_{GS} - V_{Tn})$$

$$Q_N(y=L) = -C_{ox} \underline{(V_{GD} - V_{Tn})}$$



# Average Inversion Charge

Source End

Drain End

$$Q_N(y) \approx -\frac{C_{ox}(V_{GS} - V_T) + C_{ox}(V_{GD} - V_T)}{2}$$

$$Q_N(y) \approx -\frac{C_{ox}(V_{GS} - V_T) + C_{ox}(V_{GS} - V_{DS} - V_T)}{2}$$

$$Q_N(y) \approx -\frac{C_{ox}(2V_{GS} - 2V_T) - C_{ox}V_{DS}}{2} = -C_{ox}\left(V_{GS} - V_T - \frac{V_{DS}}{2}\right)$$

- Charge at drain end is lower since the vertical field is lower at that point

# Drift Velocity and Drain Current

Use mobility to find velocity  $v$

$$\underline{v(y)} = -\mu_n E(y) \approx -\mu_n (-\Delta V / \Delta y) = \frac{\mu_n V_{DS}}{L}$$

same as  
linear  
region

Substituting:

$$I_D = -WvQ_N \approx W\mu \frac{V_{DS}}{L} C_{ox} \left( V_{GS} - V_T - \frac{V_{DS}}{2} \right)$$

only term  
that's  
different

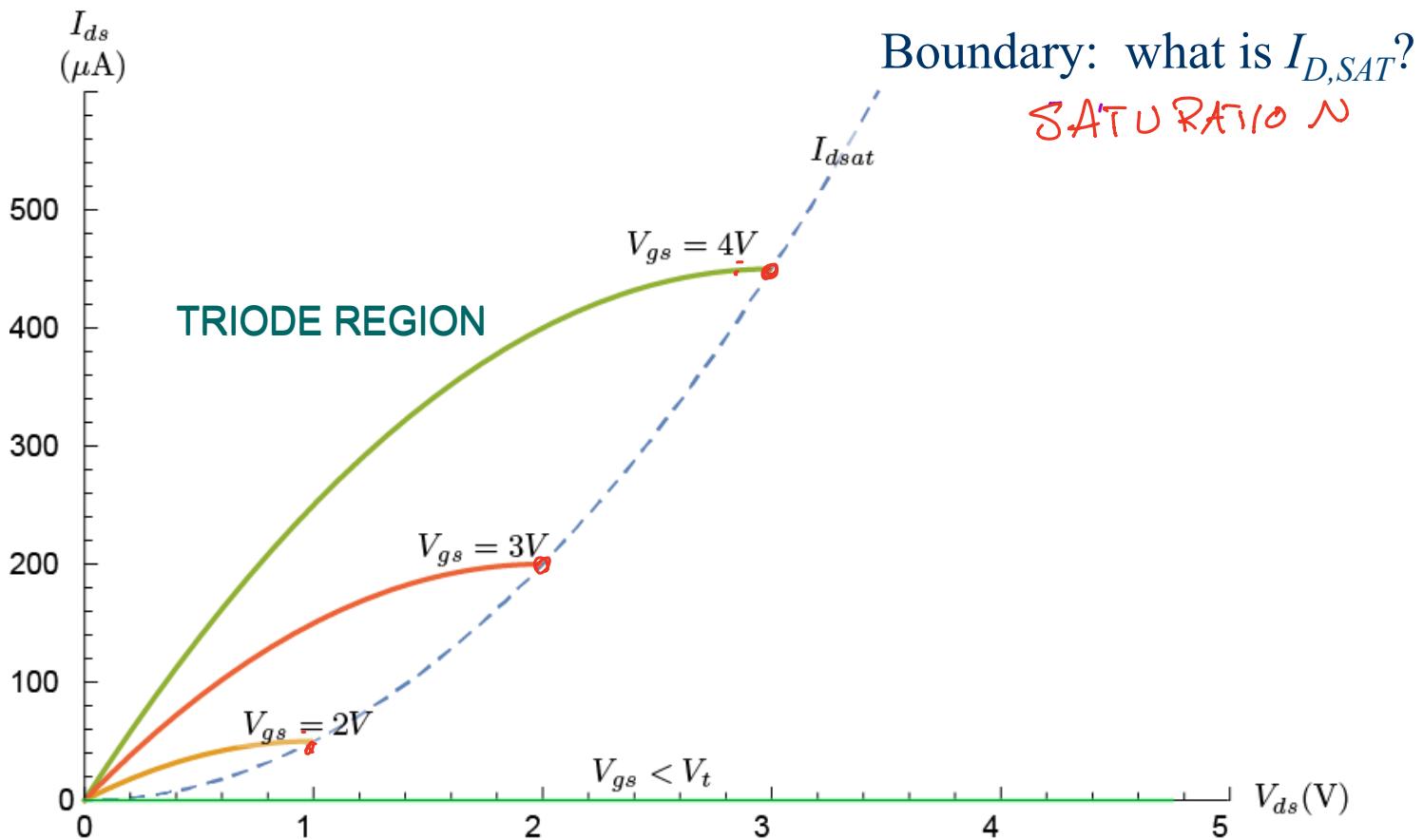
$$I_D = \frac{W}{L} \mu C_{ox} \left( V_{GS} - V_T - \frac{V_{DS}}{2} \right) V_{DS}$$

standard  
format

Family of Inverted Parabolas

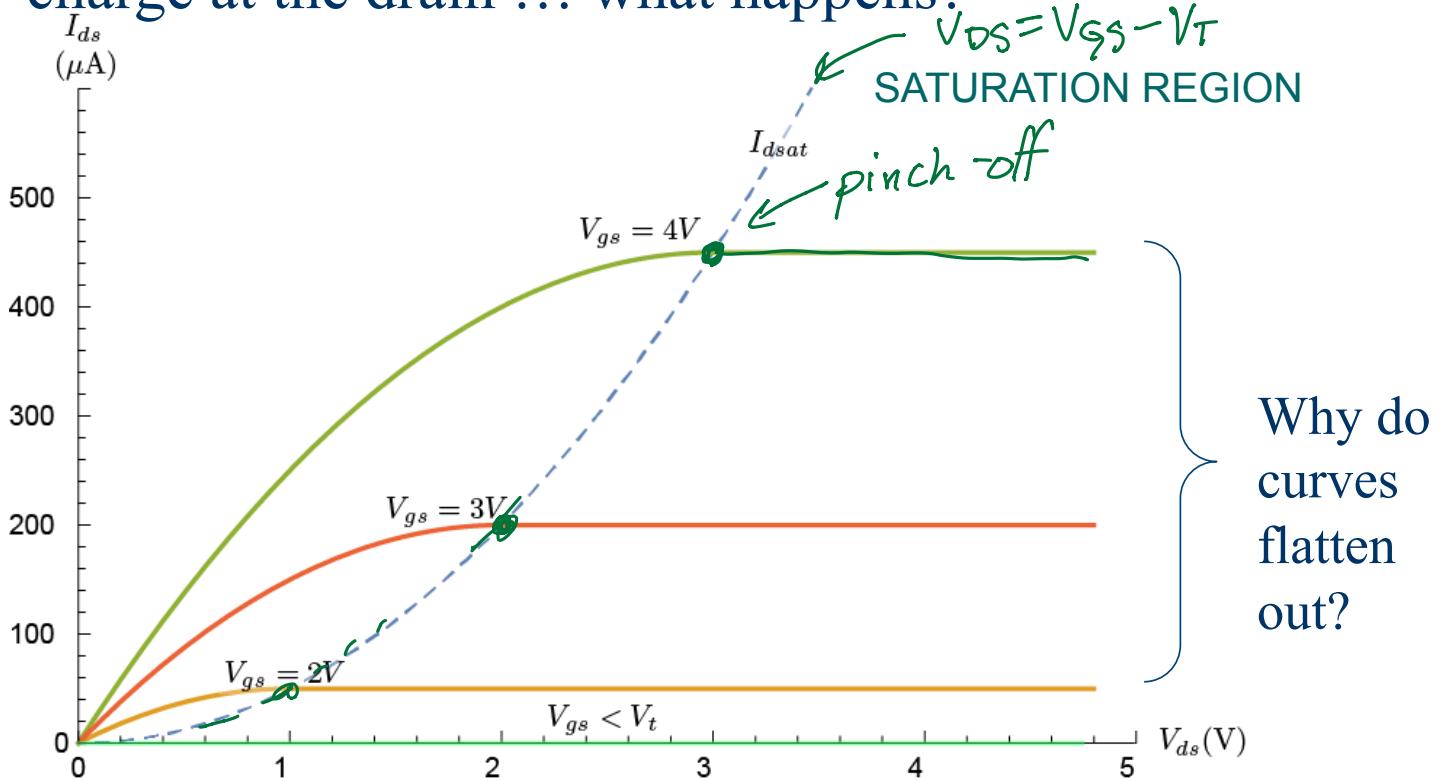
if  $V_{DS}$   
is small, this term  
goes away  $\rightarrow$  becomes  
linear eq -

# Square-Law Characteristics



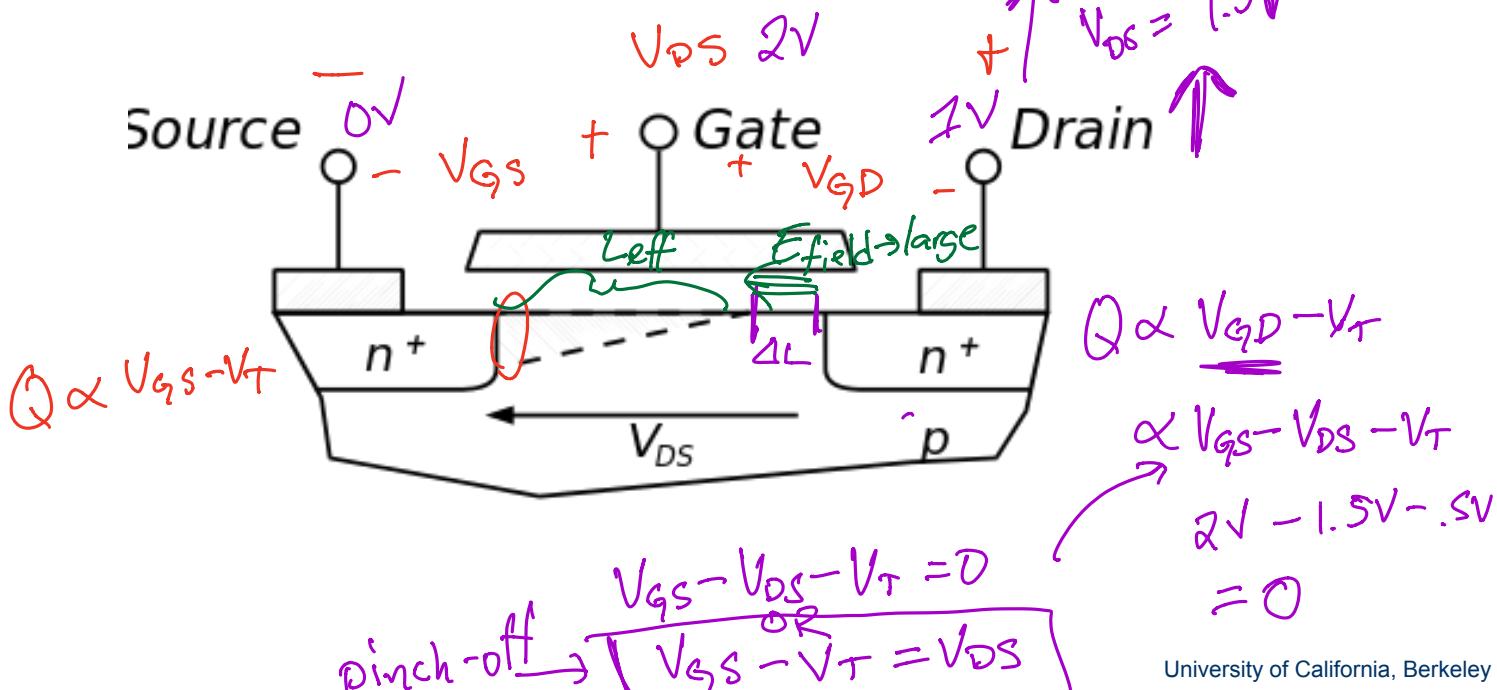
# The Saturation Region

When  $V_{DS} > V_{GS} - V_{Tn}$ , there isn't any inversion charge at the drain ... what happens?



# Why does current saturate?

- The charge at drain end goes to zero once  $V_{GD} < V_T$
- We say that the drain end is “pinched off”
  - If you pinch a hose, water flow stops !
  - But then how does current flow?



Saturation  $V_{GS} - V_T \leq V_{DS}$

# Pinch Off

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- Excess field beyond  $E_{dsat}$  drops across tiny region between drain and channel
  - Huge field means that electrons flow at very high velocity across the “high field” region.
  - They are injected from source end and are collected at the drain end
- Increasing the drain voltage does not increase current (appreciably) because the current is limited by the supply of electrons from channel side

# Square-Law Current in Saturation

Current stays at maximum (where  $\underline{\underline{V_{DS}}} = \underline{\underline{V_{GS} - V_{Tn}}} = \underline{\underline{V_{DS,SAT}}}$ )

$$I_D = \frac{W}{L} \mu C_{ox} \left( V_{GS} - V_T - \frac{V_{DS}}{2} \right) V_{DS}$$

$$I_{DS,sat} = \frac{W}{L} \mu C_{ox} \left( V_{GS} - V_T - \frac{V_{GS} - V_T}{2} \right) (V_{GS} - V_T)$$

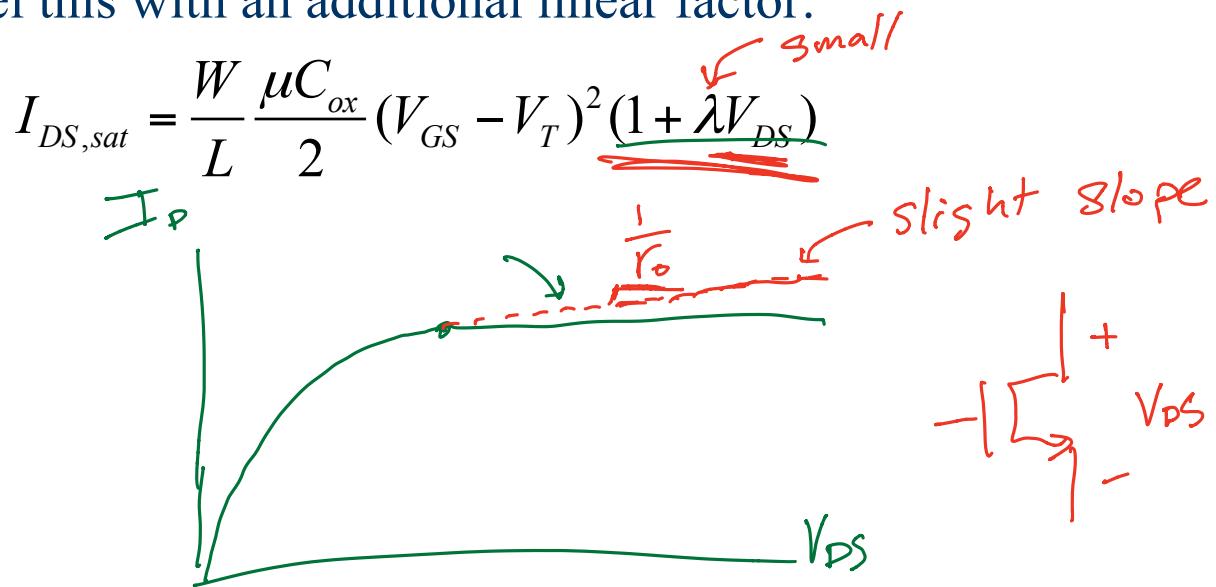
$$I_{DS,sat} = \frac{W}{L} \frac{\mu C_{ox}}{2} (V_{GS} - V_T)^2$$

*"square law"*

*Standard form*

# Actual Saturation Current

- Measurement:  $I_D$  increases slightly with increasing  $V_{DS}$ :
- The physics is complicated, but a simple way to see this is that the channel is getting shorter as the drain voltage depletes away more electrons from the drain end
- We model this with an additional linear factor:



# Channel Length Modulation

- When  $v_{DS} = v_{GS} - V_T$ , the channel pinches off near the drain. With further increase in  $v_{DS}$ , the pinch-off point moves toward the source, effectively reducing the channel length from  $L$  to  $L - \Delta L$ .

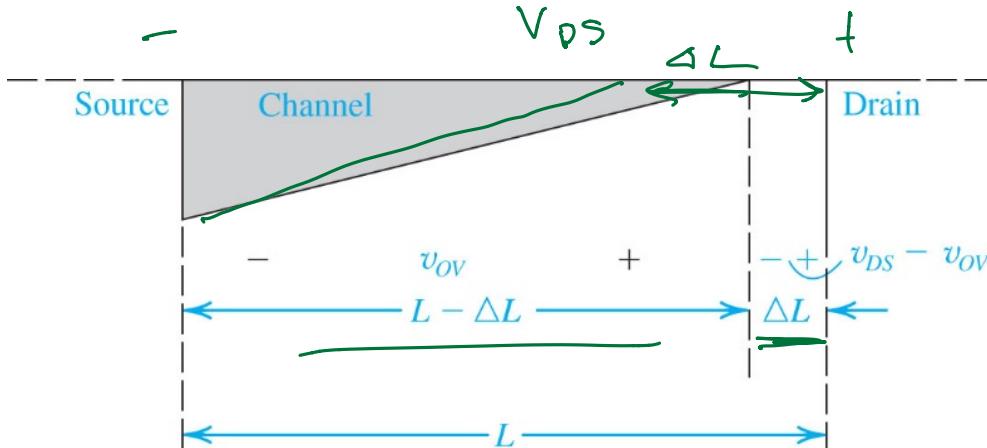
$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L - \Delta L} (V_{GS} - V_T)^2$$

*L that we drew*

*changes with  $V_{DS}$*

The continual increase of  $I_D$  with  $V_{DS}$  is modeled by

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{tn})^2 (1 + \lambda V_{DS})$$



# Summary: Regions of Operation

- Cut-off:  $V_{GS} < V_T$ 
  - $I_{DS} = 0$
  - *Note: this is an approximation we will make in EE105, in later courses you will learn about sub-threshold conduction*
- Linear:  $V_{GS} > V_T, V_{DS} \ll V_{GS} - V_T$

$$I_{DS} = \frac{W}{L} \mu_n C_{ox} (V_{GS} - V_T) V_{DS}$$

- Triode:  $V_{GS} > V_T, V_{DS} < V_{GS} - V_T$
- Saturation:  $V_{GS} > V_T, V_{DS} > V_{GS} - V_T$

$$I_{DS,sat} = \frac{W}{L} \frac{\mu C_{ox}}{2} (V_{GS} - V_T)^2 (1 + \lambda V_{DS})$$

# PMOS Device

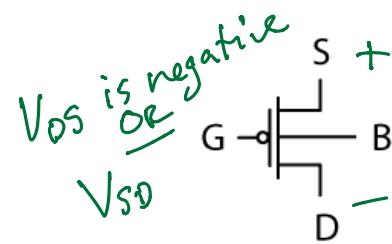
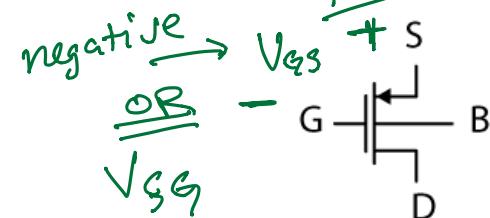
- So far, we've derived all of our equations for an NMOS device
- PMOS devices work exactly the same way, but with an n-type body and a channel made of positive charges (holes)
- The direction of the voltages and currents are inverted, for example:

$$\begin{array}{ll} \mu_n & \lambda_n \\ \mu_p & \lambda_p \\ V_{Tn} & V_{Tp} \end{array}$$

$$I_{SD,sat} = \frac{W}{L} \frac{\mu C_{ox}}{2} (V_{SG} - V_{T_p})^2 (1 + \lambda V_{SD})$$

*V<sub>Tn</sub> vs V<sub>Tp</sub>*  
*V<sub>Tn</sub> is negative*  
*V<sub>Tp</sub> is positive*  
*tox is same thickness across wafer*

*threshold is negative*  
*threshold is negative*



# Next Lecture

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- In the next lecture we will learn how to analyze small-signal amplifiers, including
  - Computation of bias points
  - How to derive and compute small signal model parameters ( $g_m$ ,  $r_o$ )
  - How to calculate small signal gain