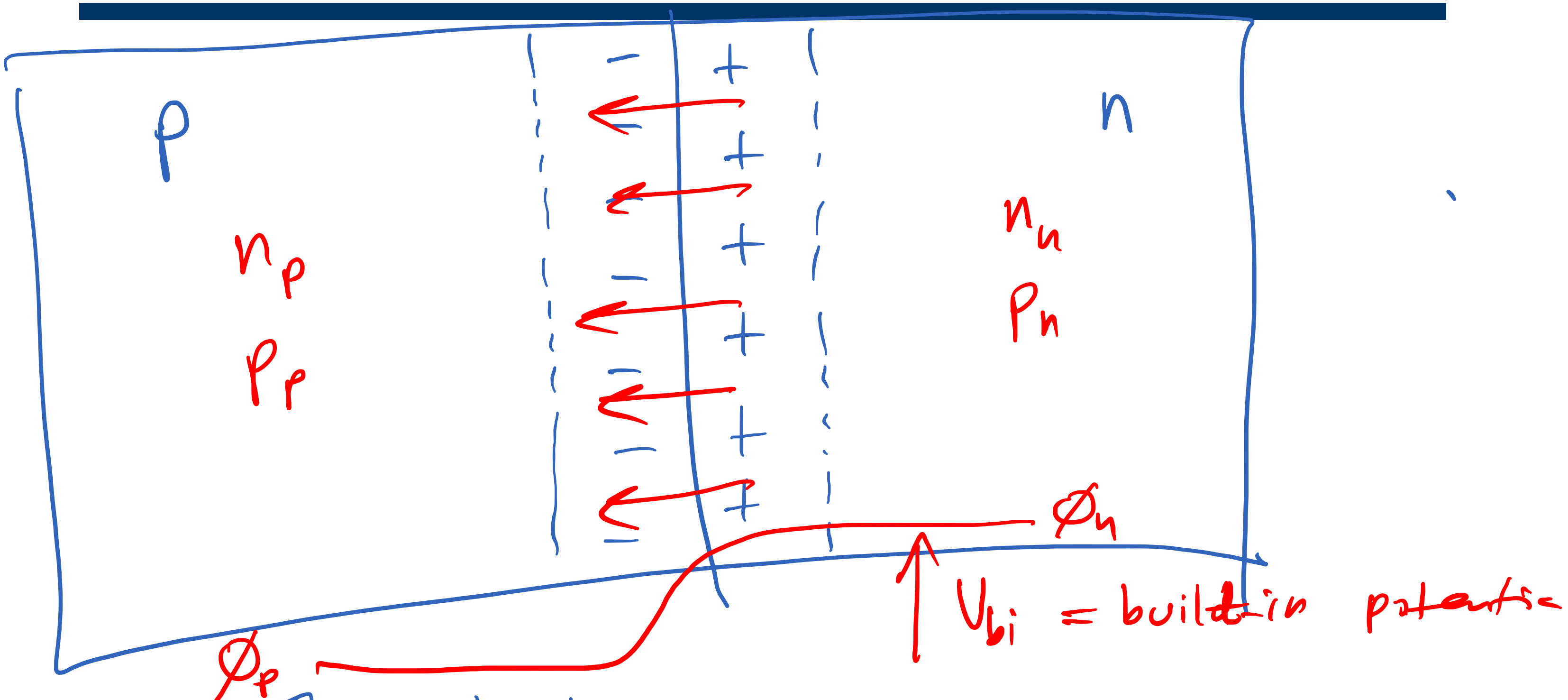
Key Ideas So Far…



Thermal Equilibrium ⟹ potential difference "build up" that causes a drift current to balance diffusion current

Solve for Depletion Lengths

- We have two equations and two unknowns. We are finally in a position to solve for the depletion depths

$$\phi_n - \frac{qN_d}{2\varepsilon_s} x_{n0}^2 = \phi_p + \frac{qN_a}{2\varepsilon_s} x_{p0}^2 \qquad (1)$$

$$qN_a x_{po} = qN_d x_{no} \qquad (2)$$

(1), (2): ϕ & E are cont. across junc.

$$x_{no} = \sqrt{\frac{2\varepsilon_s \phi_{bi}}{qN_d}\left(\frac{N_a}{N_a + N_d}\right)} \qquad x_{po} = \sqrt{\frac{2\varepsilon_s \phi_{bi}}{qN_a}\left(\frac{N_d}{N_d + N_a}\right)}$$

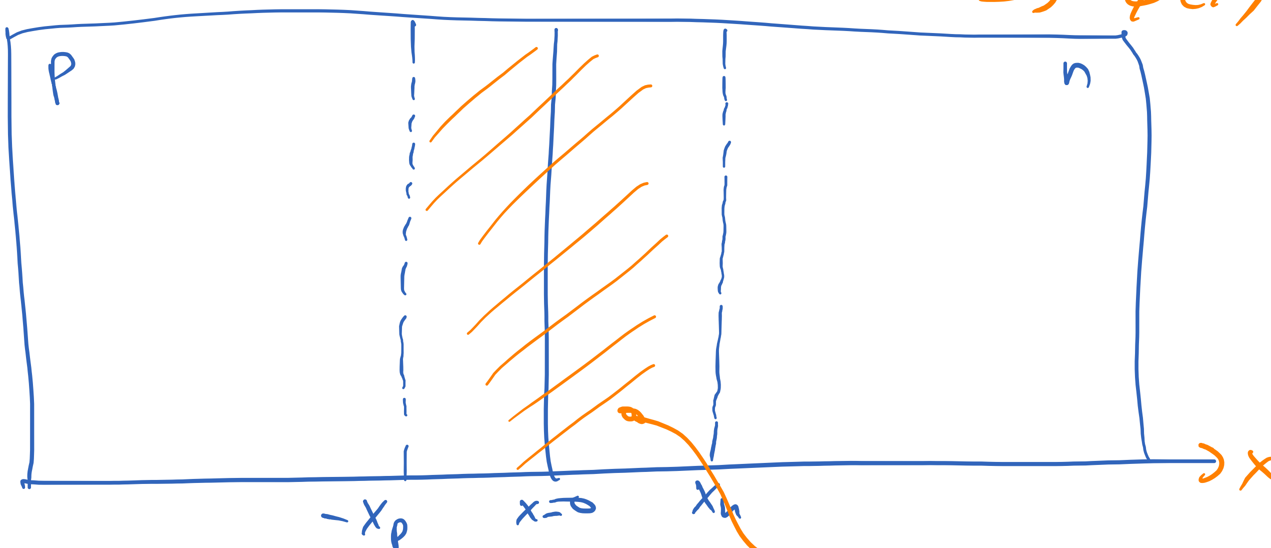
$$\phi_{bi} \equiv \phi_n - \phi_p > 0$$

$\phi_{bi} = \phi_n - \phi_p$ or $\phi'_{bi} = \phi_n + (V_d) - \phi_p$ (reverse bias)

(cont.)

Depletion Approximation ⇒ $\rho(x)$ is known

⇒ $\phi(x)$ / $\mathcal{E}(x)$



depleted of "free" carriers of charge

Key Result:

[Calculate x_n & x_p as a function of ϕ_n & ϕ_p]

depletion region depths

PN Junction Capacitor

- Under thermal equilibrium, the PN junction does not draw any (much) current
- But notice that a PN junction stores charge in the space charge region (transition region)
- Since the device is storing charge, it's acting like a capacitor

$Q \stackrel{?}{=} f(V)$ || plate $Q = C \cdot V$

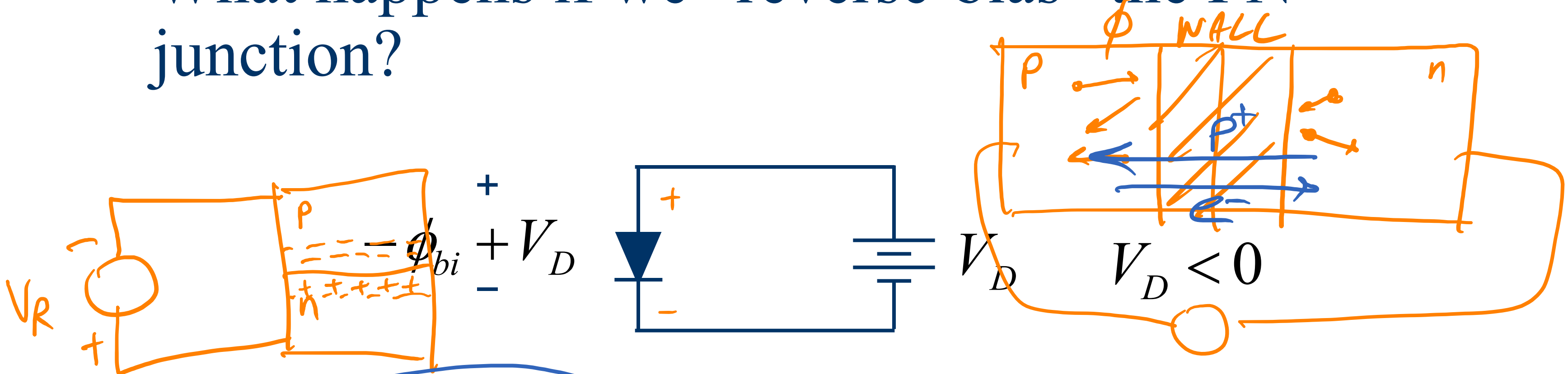
- Positive charge is stored in the n-region, and negative charge is in the p-region:

constants $f(\phi_{bi})$

$$qN_a x_{po} = qN_d x_{no}$$

Reverse Biased PN Junction

- What happens if we "reverse-bias" the PN junction?



- Since no current is flowing, the entire reverse biased potential is dropped across the transition region
- To accommodate the extra potential, the charge in these regions must increase
- If no current is flowing, the only way for the charge to increase is to grow (shrink) the depletion regions

Voltage Dependence of Depletion Width

- Can redo the math but in the end we realize that the equations are the same except we replace the built-in potential with the effective reverse bias:

$$x_n(V_D) = \sqrt{\frac{2\varepsilon_s(\phi_{bi} - V_D)}{qN_d}\left(\frac{N_a}{N_a + N_d}\right)} = x_{n0}\sqrt{1 - \frac{V_D}{\phi_{bi}}}$$

$$x_p(V_D) = \sqrt{\frac{2\varepsilon_s(\phi_{bi} - V_D)}{qN_a}\left(\frac{N_d}{N_a + N_d}\right)} = x_{p0}\sqrt{1 - \frac{V_D}{\phi_{bi}}}$$

$$X_d(V_D) = x_p(V_D) + x_n(V_D) = \sqrt{\frac{2\varepsilon_s(\phi_{bi} - V_D)}{q}\left(\frac{1}{N_a} + \frac{1}{N_d}\right)}$$

$$X_d(V_D) = X_{d0}\sqrt{1 - \frac{V_D}{\phi_{bi}}}$$

Charge Versus Bias

- As we increase the reverse bias, the depletion region grows to accommodate more charge

$$Q_J(V_D) = -qN_a x_p(V_D) = -qN_a \sqrt{1 - \frac{V_D}{\phi_{bi}}} \; (X_{p0})$$

- Charge is *not* a linear function of voltage
- This is a non-linear capacitor
- We can define a small signal capacitance for small signals by breaking up the charge into two terms

DC BIAS — AC VOLTAGE (SMALL)

$$Q_J(V_D + v_D) = Q_J(V_D) + q(v_D)$$

typo

Derivation of Small Signal Capacitance

- Do a Taylor Series expansion:

$$Q_J(V_D + v_D) = Q_J(V_D) + \left.\frac{dQ_D}{dV}\right|_{V_D} v_D + \cdots$$

$$C_j = C_j(V_D) = \left.\frac{dQ_j}{dV}\right|_{V=V_D} = \frac{d}{dV}\left[\left(-qN_a x_{p0}\sqrt{1-\frac{V}{\phi_{bi}}}\right)\right]_{V=V_R}$$

$$C_j = \frac{qN_a x_{p0}}{2\phi_{bi}\sqrt{1-\frac{V_D}{\phi_{bi}}}} = \frac{C_{j0}}{\sqrt{1-\frac{V_D}{\phi_{bi}}}}$$

- Notice that

$$C_{j0} = \frac{qN_a x_{p0}}{2\phi_{bi}} = \frac{qN_a}{2\phi_{bi}}\sqrt{\left(\frac{2\varepsilon_s\phi_{bi}}{qN_a}\right)\left(\frac{N_d}{N_a+N_d}\right)} = \sqrt{\frac{q\varepsilon_s}{2\phi_{bi}}\frac{N_a N_d}{N_a+N_d}}$$

Physical Interpretation of Depletion Cap

$$C_{j0} = \sqrt{\frac{q\varepsilon_s}{2\phi_{bi}} \frac{N_a N_d}{N_a + N_d}}$$

- Notice that the expression on the right-hand-side is just the depletion width in thermal equilibrium

permittivity of Si

Looks like a // plate Cap for small-signals

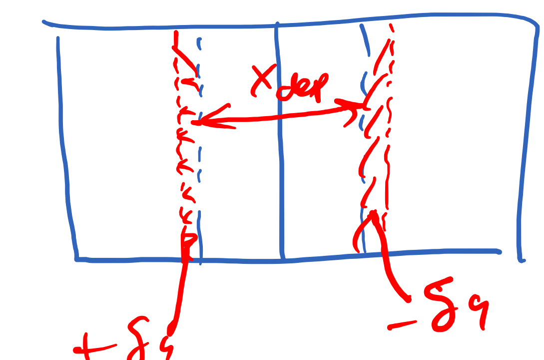
$$C_{j0} = \varepsilon_s \sqrt{\frac{q}{2\varepsilon_s \phi_{bi}} \left(\frac{1}{N_a} + \frac{1}{N_d} \right)^{-1}} = \frac{\varepsilon_s}{X_{d0}}$$

Cap / Area

$$C = \frac{\varepsilon A}{d}$$

- This looks like a parallel plate capacitor!

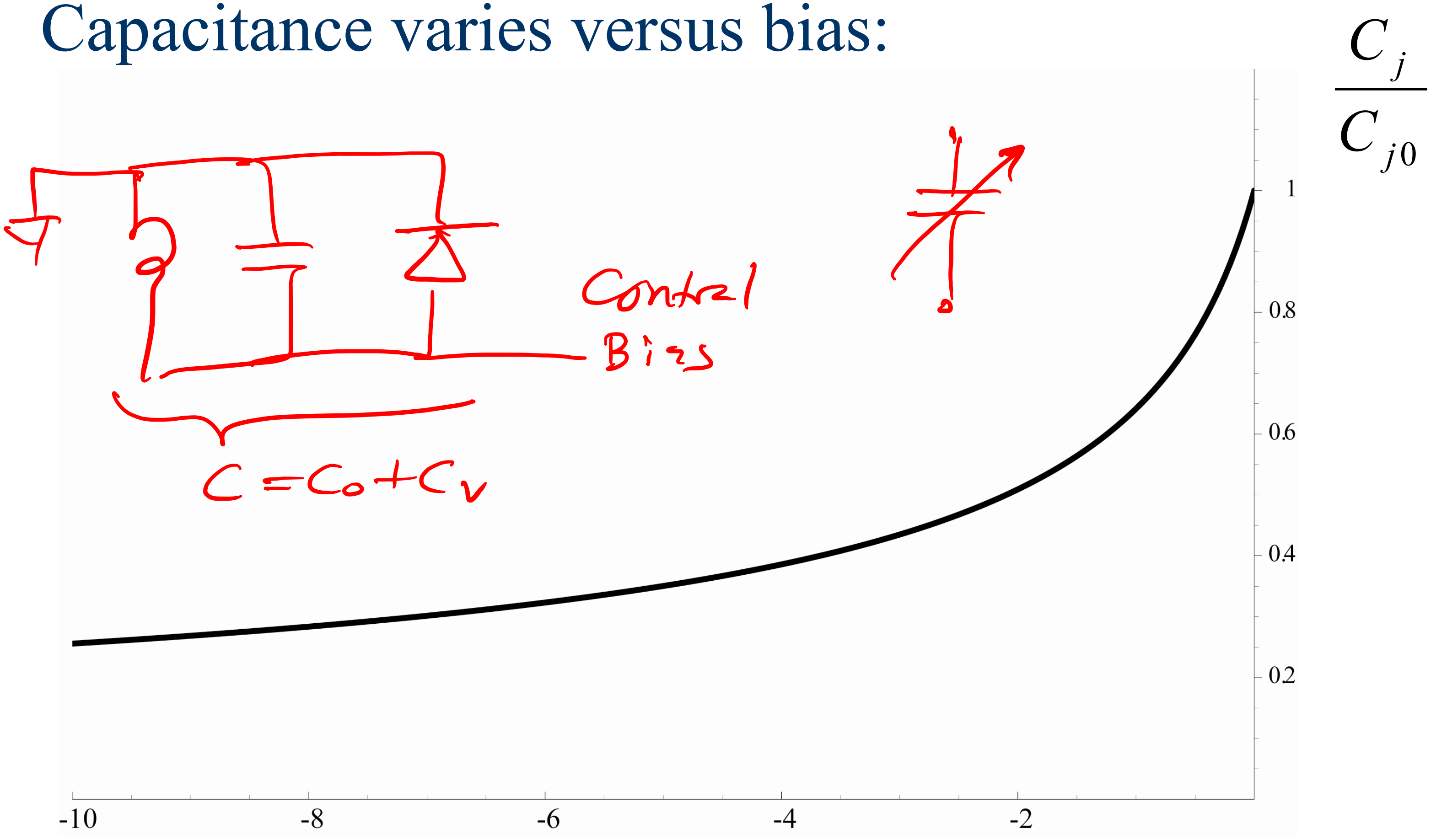
$\varepsilon_s = 11.9 \times \varepsilon_0$



$$C_j(V_D) = \frac{\varepsilon_s}{X_d(V_D)}$$

A Variable Capacitor (Varactor)

- Capacitance varies versus bias:

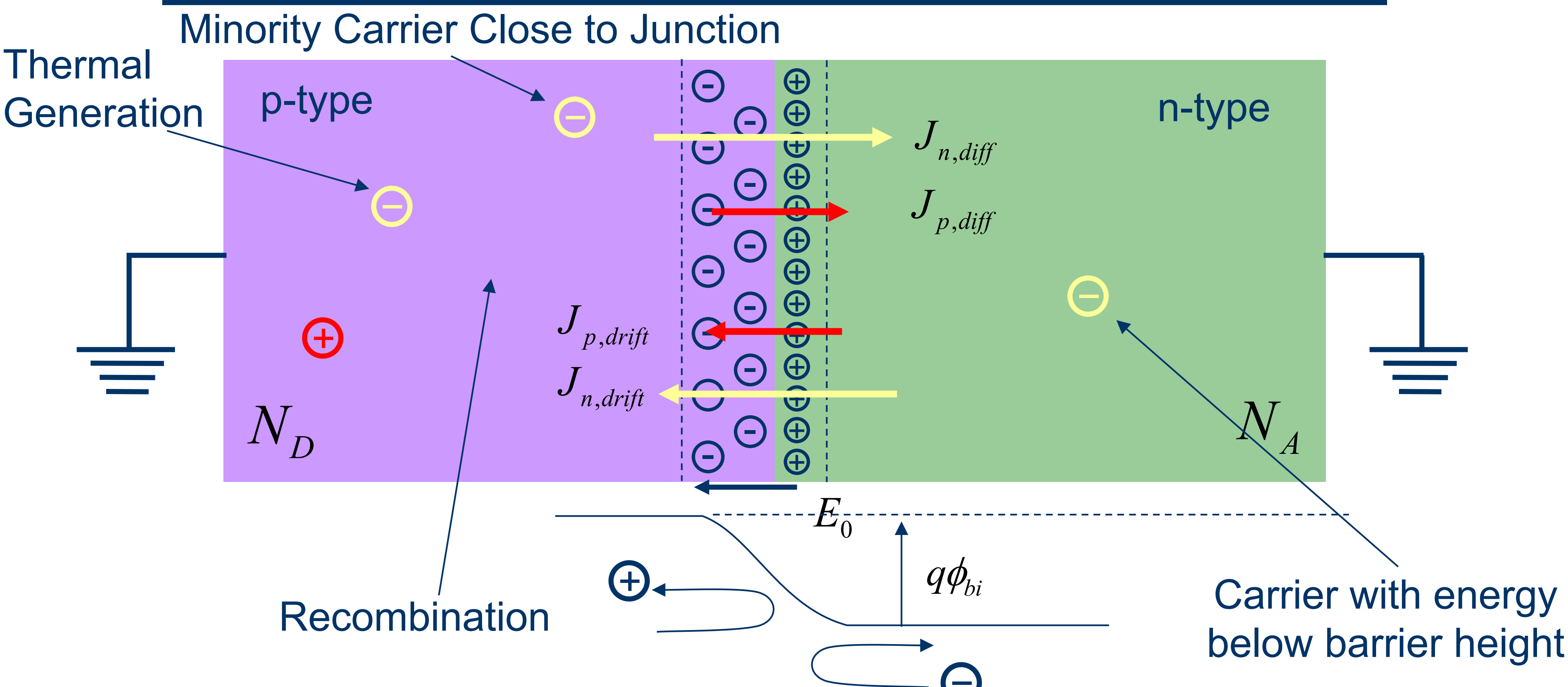


- Application: Radio Tuner

Currents in PN Junctions

Prof. Ali M. Niknejad
Prof. Rikky Muller

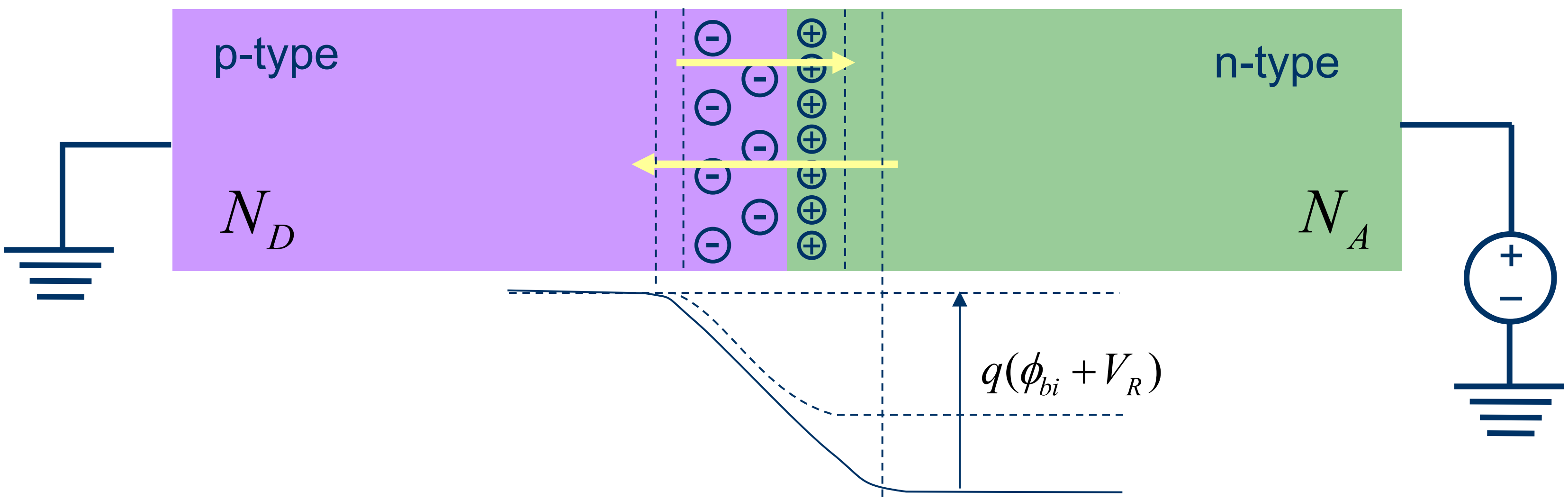
Diode under Thermal Equilibrium



- Diffusion small since few carriers have enough energy to penetrate barrier
- Drift current is small since minority carriers are few and far between: Only minority carriers generated within a diffusion length can contribute current
- **Important Point: Minority drift current independent of barrier!**
- **Diffusion current strong (exponential) function of barrier**

Reverse Bias

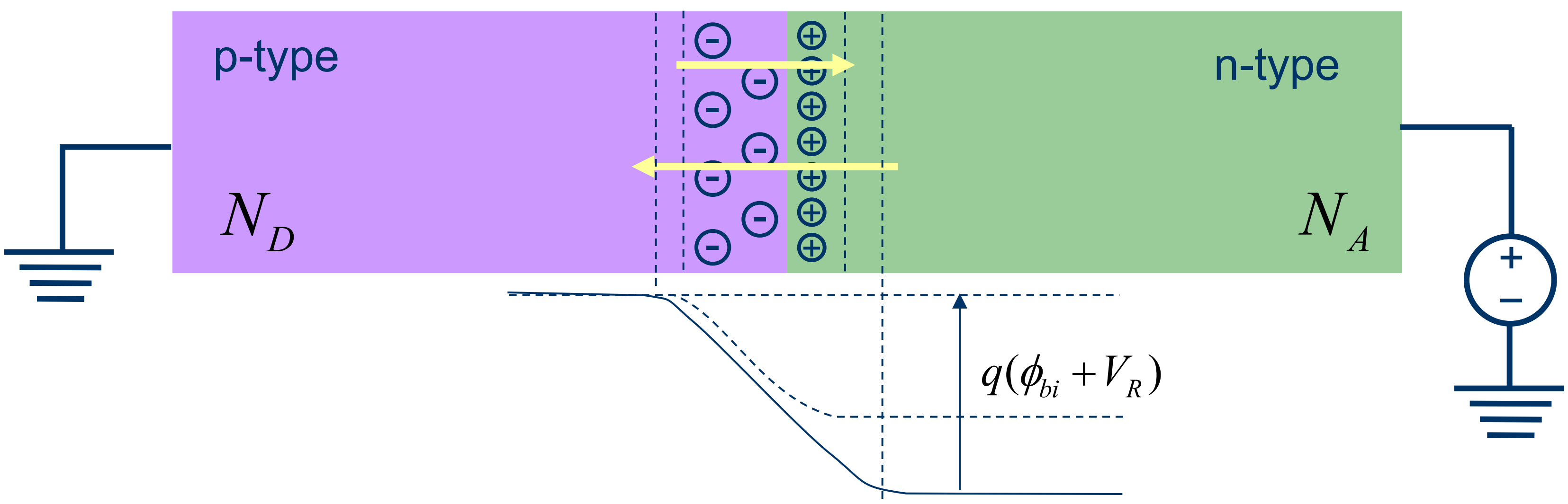
- Reverse Bias causes an increases barrier to diffusion
- Diffusion current is reduced exponentially



- Drift current does not change
- Net result: Small reverse current

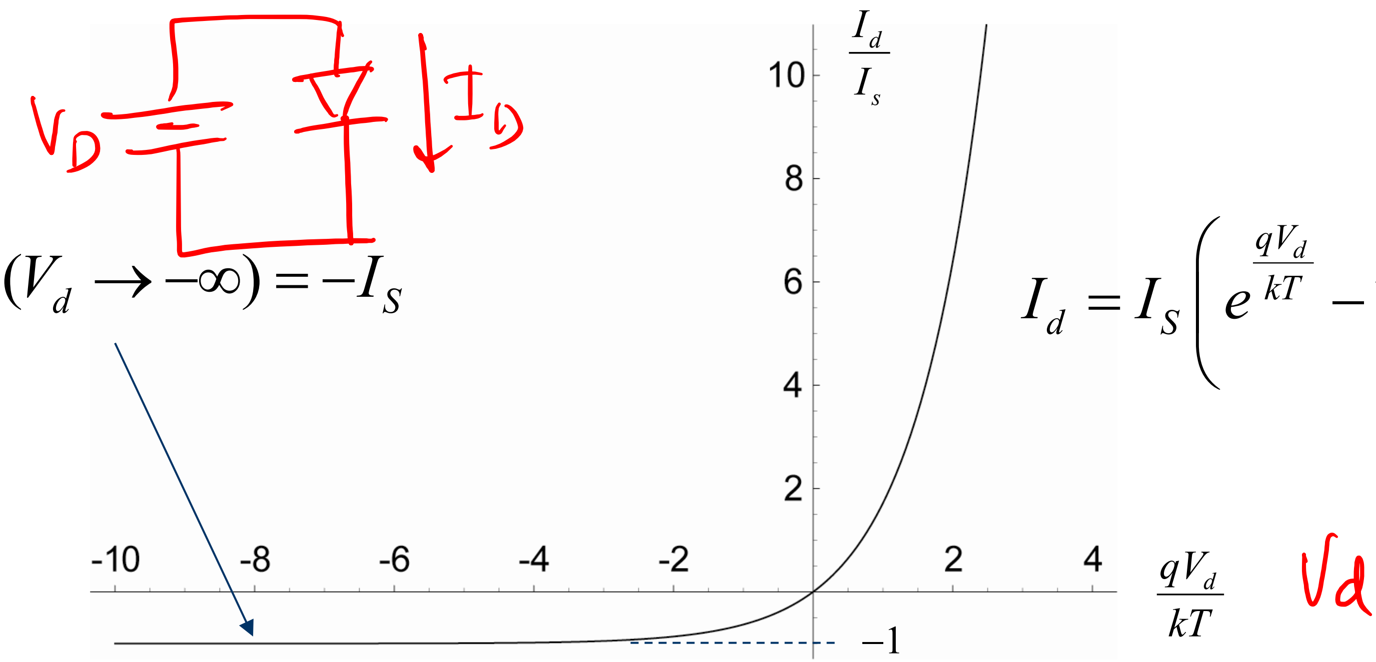
Forward Bias

- Forward bias causes an exponential increase in the number of carriers with sufficient energy to penetrate barrier
- Diffusion current ***increases*** exponentially



- Drift current does not change
- Net result: Large forward current

Diode I-V Curve



$$I_d(V_d \to -\infty) = -I_S$$

$$I_d = I_S\left(e^{\frac{qV_d}{kT}} - 1\right)$$

- Diode IV relation is an exponential function
- This exponential is due to the Boltzmann distribution of carriers versus energy
- For reverse bias the current saturations to the drift current due to minority carriers

Minority Carriers at Junction Edges

Minority carrier concentration at boundaries of depletion region increase as barrier lowers … the function is

$$\frac{p_n(x = x_n)}{p_p(x = -x_p)} = \frac{\text{(minority) hole conc. on n-side of barrier}}{\text{(majority) hole conc. on p-side of barrier}}$$

$$= e^{-(Barrier\ Energy)/kT}$$

$$\boxed{\frac{p_n(x = x_n)}{N_A} = e^{-q(\phi_B - V_D)/kT}}$$

(Boltzmann's Law)

"Law of the Junction"

Minority carrier concentrations at the edges of the depletion region are given by:

$$p_n(x = x_n) = N_A e^{-q(\phi_B - V_D)/kT}$$

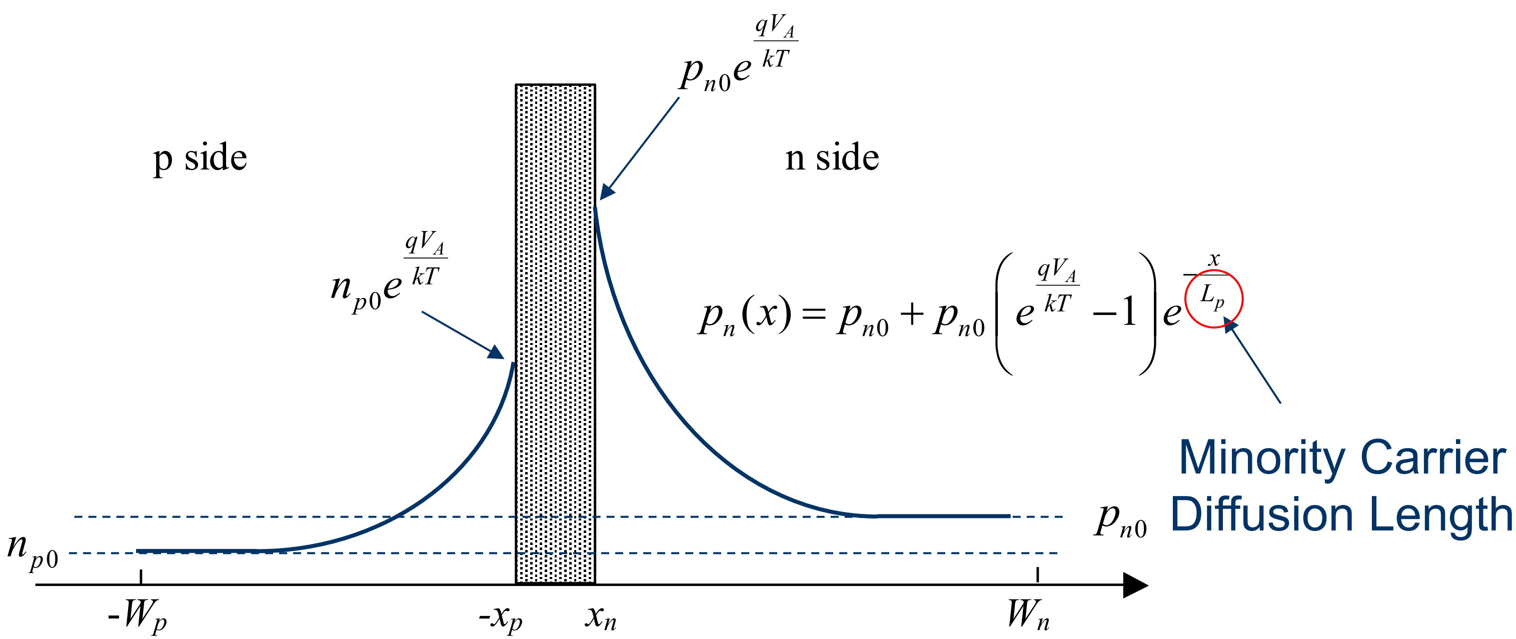
$$n_p(x = -x_p) = N_D e^{-q(\phi_B - V_D)/kT}$$

Note 1: N_A and N_D are the majority carrier concentrations on the *other* side of the junction

Note 2: we can reduce these equations further by substituting $V_D = 0$ V (thermal equilibrium)

Note 3: assumption that $p_n << N_D$ and $n_p << N_A$

Minority Carrier Concentration



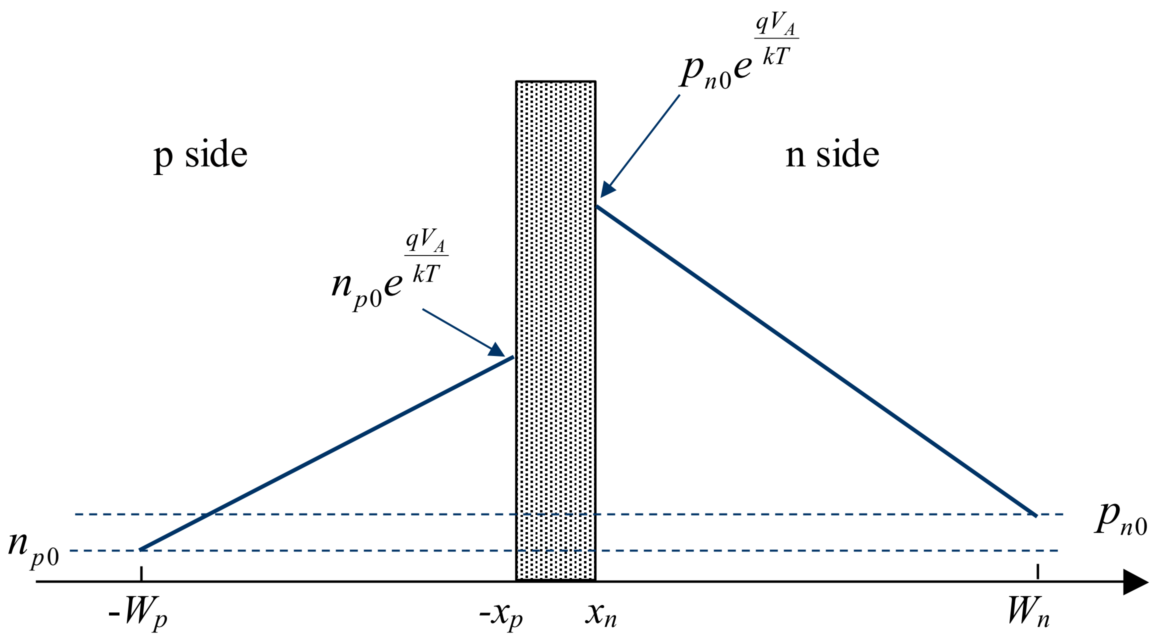
$$p_n(x) = p_{n0} + p_{n0}\left(e^{\frac{qV_A}{kT}} - 1\right)e^{-\frac{x}{L_p}}$$

The minority carrier concentration in the bulk region for forward bias is a decaying exponential due to recombination

Derived in EE130

Steady-State Concentrations

Assume that none of the diffusing holes and electrons recombine → get straight lines …

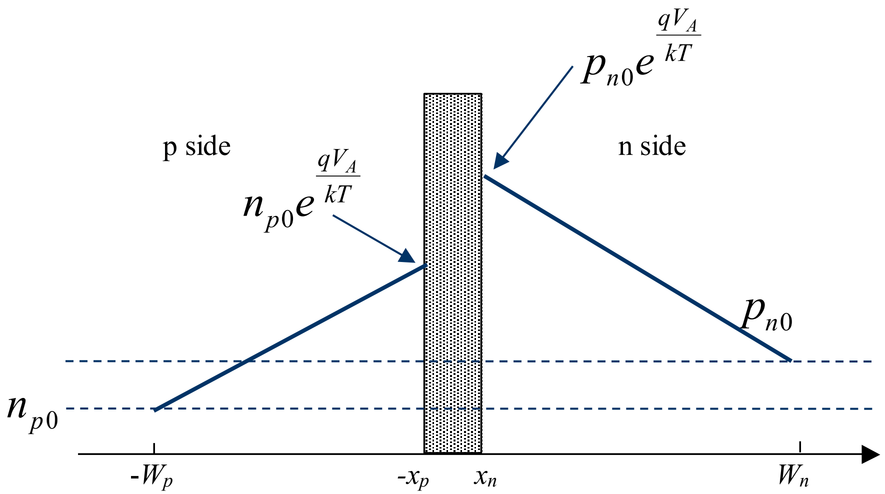


This also happens if the minority carrier diffusion lengths are much larger than $W_{n,p}$

$$L_{n,p} >> W_{n,p}$$

Derived in EE 130

Diode Current Densities



$$\frac{dn_p}{dx}(x) \approx \frac{n_{p0}e^{\frac{qV_A}{kT}} - n_{p0}}{-x_p - (-W_p)}$$

$$n_{p0} = \frac{n_i^2}{N_a}$$

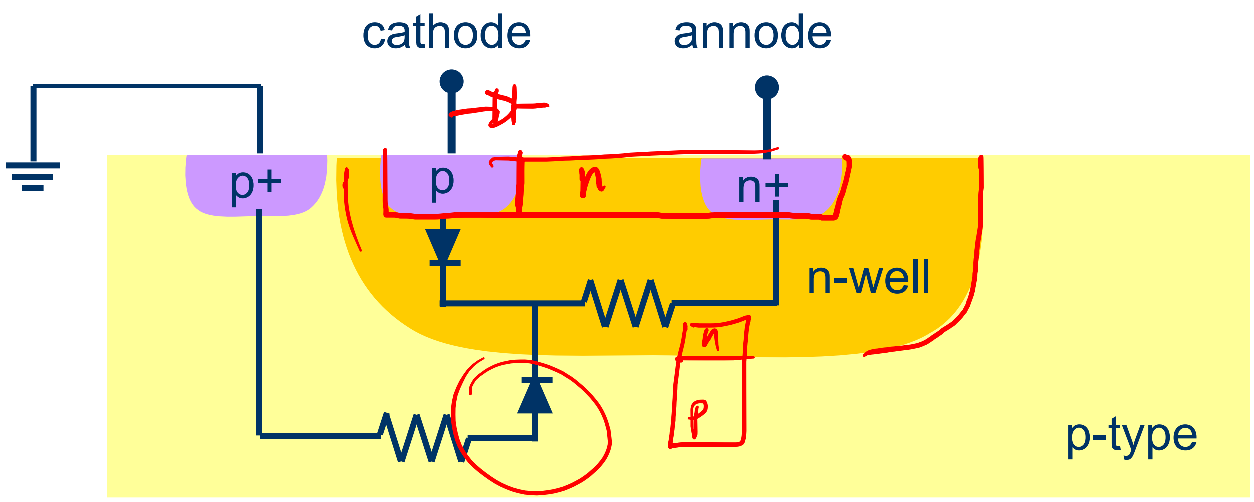
$$J_n^{diff} = qD_n \left.\frac{dn_p}{dx}\right|_{x=-x_p} \approx q\frac{D_n}{W_p} n_{p0}\left(e^{\frac{qV_A}{kT}} - 1\right)$$

$$J_p^{diff} = -qD_p \left.\frac{dp_n}{dx}\right|_{x=x_n} \approx -q\frac{D_p}{W_n} p_{n0}\left(1 - e^{\frac{qV_A}{kT}}\right)$$

$$J^{diff} = qn_i^2\left(\frac{D_p}{N_d W_n} + \frac{D_n}{N_a W_p}\right)\left(e^{\frac{qV_A}{kT}} - 1\right)$$

Derived in EE 130

Fabrication of IC Diodes



- Start with p-type substrate
- Create n-well to house diode
- p and n+ diffusion regions are the cathode and annode
- N-well must be reverse biased from substrate isolation
- Parasitic resistance due to well resistance

Diode Small Signal Model

- The I-V relation of a diode can be linearized

BIAS, small signal

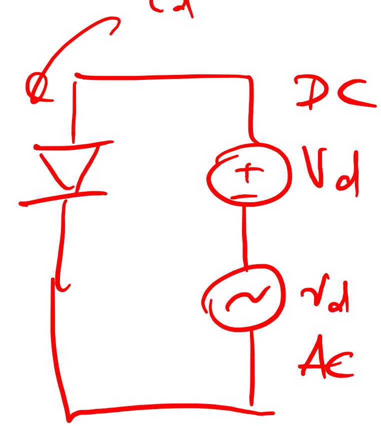
$$I_D + i_D = I_S\left(e^{\frac{q(V_d+v_d)}{kT}} - 1\right) \approx I_S e^{\frac{qV_d}{kT}} e^{\frac{qv_d}{kT}}$$

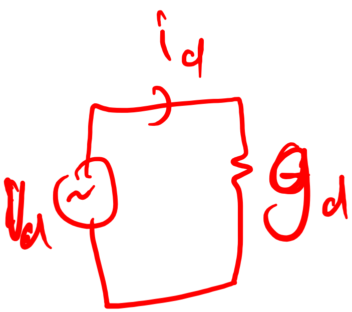
Total current

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$I_D + i_D \approx I_D\left(1 + \frac{q(V_d+v_d)}{kT} + \cdots\right)$$

$$i_D \approx \frac{qv_d}{kT} I_D = g_d v_d$$





$\frac{kT}{q} = 26\text{mV}$

$v_d \ll 26\text{mV}$

$g_d \triangleq \frac{qI_D}{kT}$

Small-Signal Low Frequency Model

⇒ must include capacitance too

typo X

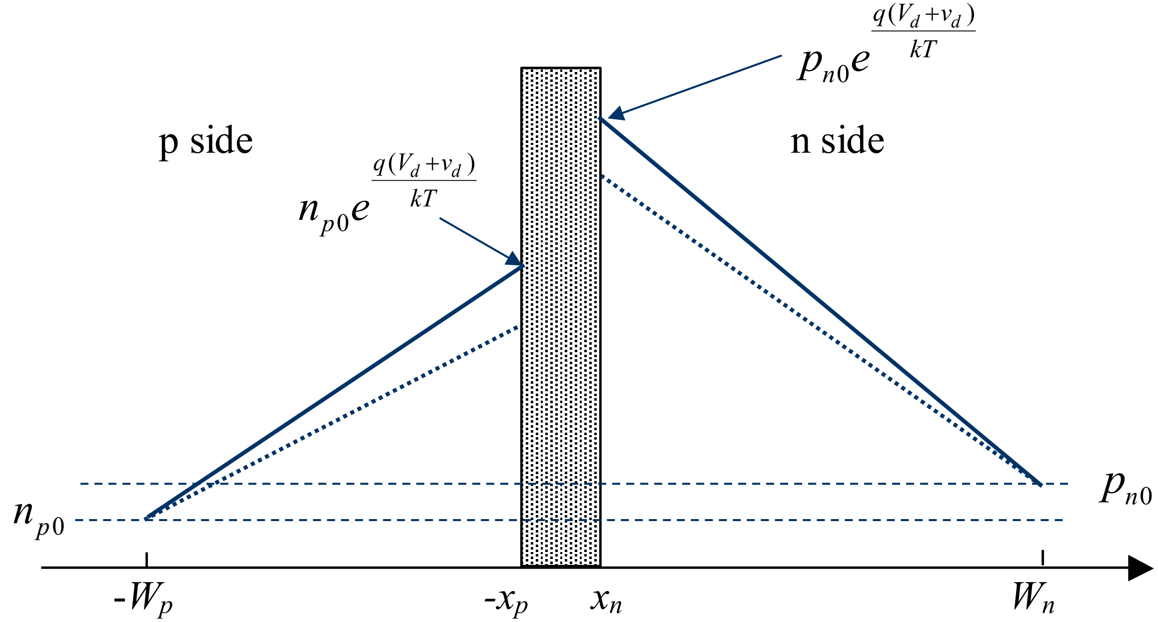
Diode Capacitance

- We have already seen that a reverse biased diode acts like a capacitor since the depletion region grows and shrinks in response to the applied field. The capacitance in forward bias is given by

$$C_j = A\frac{\varepsilon_S}{X_{dep}} \approx 1.4C_{j0}$$

- But another charge storage mechanism comes into play in forward bias
- Minority carriers injected into p and n regions "stay" in each region for a while
- On average additional charge is stored in diode

Charge Storage



- Increasing forward bias increases minority charge density
- By charge neutrality, the source voltage must supply equal and opposite charge
- A detailed analysis yields: $C_d = \frac{1}{2}\frac{qI_d}{kT}\tau$

Time to cross junction (or minority carrier lifetime)

Diode Circuits

- Rectifier (AC to DC conversion)
- Average value circuit
- Peak detector (AM demodulator)
- DC restorer
- Voltage doubler / quadrupler /…