# Module 2.3: PN Junctions

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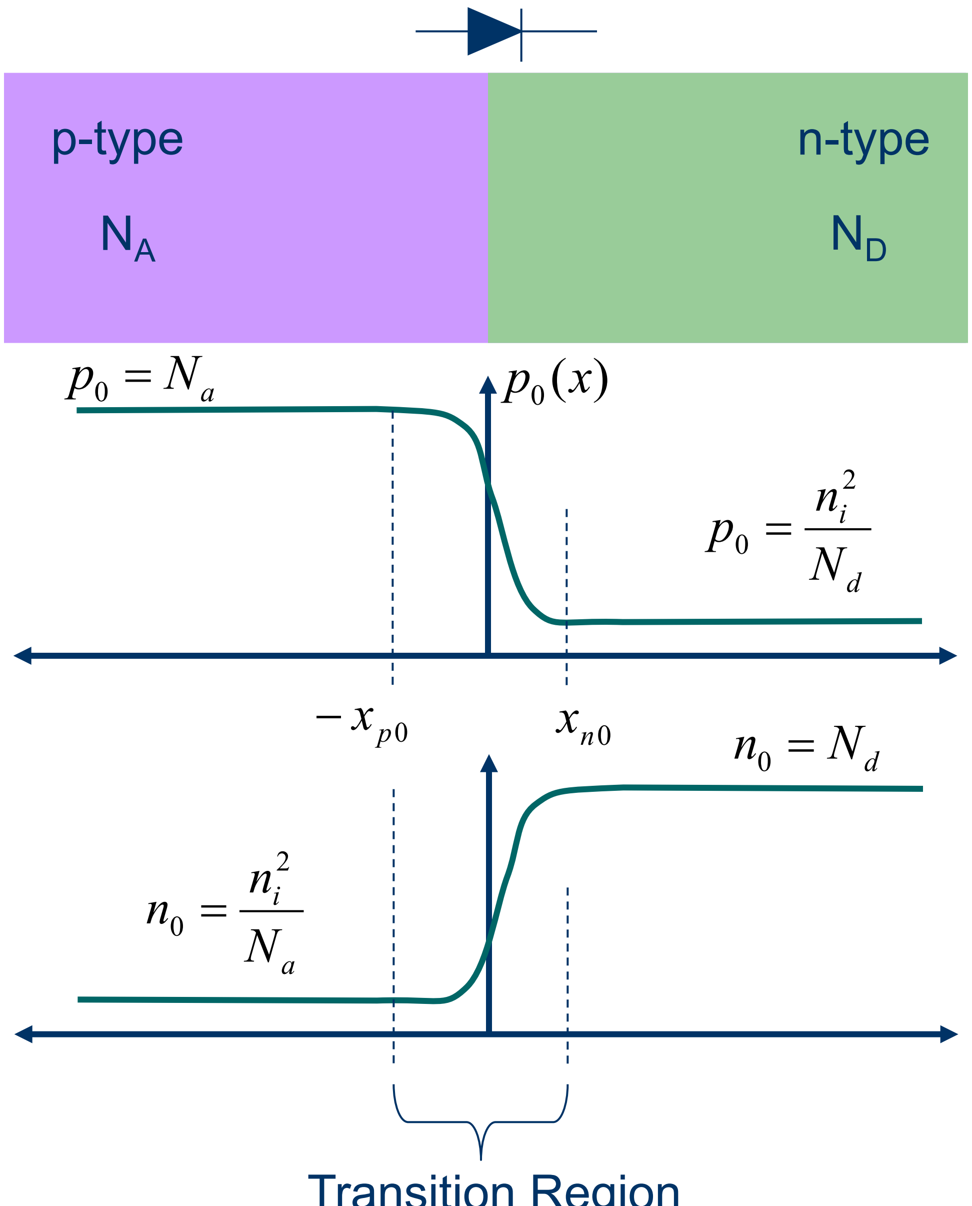
# What is a pn-junction?




AKA: diode

# Module 2.3 Outline

- Part 1: Carrier concentration variation and potential
  - Show that any time there's a variation in carrier concentration, then at thermal equilibrium there must be a variation in potential
- Part 2: Apply this to a pn-junction at thermal equilibrium
  - Extend result to a reverse biased junction
- Part 3: Look at a forward biased pn-junction

# Carrier Concentration and Potential

- In thermal equilibrium, there are no external fields and we thus expect the electron and hole current densities to be zero:

$$J_n = 0 = qn_0 \mu_n E_0 + qD_n \frac{dn_o}{dx}$$

$$\frac{dn_o}{dx} = -\left(\frac{\mu_n}{D_n}\right) n_o E_0 = \left(\frac{q}{kT}\right) n_o \frac{d\phi_0}{dx}$$

$$d\phi_0 = \left(\frac{kT}{q}\right) \frac{dn_o}{n_0} = V_{th} \frac{dn_0}{n_0}$$

# Carrier Concentration and Potential (2)

- We have an equation relating the potential to the carrier concentration

$$d\phi_0 = \left(\frac{kT}{q}\right)\frac{dn_o}{n_0} = V_{th}\frac{dn_0}{n_0}$$

- If we integrate the above equation we have

$$\phi_0(x) - \phi_0(x_0) = V_{th}\ln\frac{n_0(x)}{n_0(x_0)}$$

- We define the potential reference to be intrinsic Si:

$$\phi_0(x_0) = 0 \qquad n_0(x_0) = n_i$$

# Carrier Concentration Versus Potential

- The carrier concentration is thus a function of potential

$$n_0(x) = n_i e^{\phi_0(x)/V_{th}}$$

- Check that for zero potential, we have intrinsic carrier concentration (reference).
- If we do a similar calculation for holes, we arrive at a similar equation

$$p_0(x) = n_i e^{-\phi_0(x)/V_{th}}$$

- Note that the law of mass action is upheld

$$n_0(x) p_0(x) = n_i^2 e^{-\phi_0(x)/V_{th}} e^{\phi_0(x)/V_{th}} = n_i^2$$

# The Doping Changes Potential

- Due to the log nature of the potential, the potential changes linearly for exponential increase in doping:

$$\phi_0(x) = V_{th} \ln \frac{n_0(x)}{n_i(x_0)} = 26\text{mV} \ln \frac{n_0(x)}{n_i(x_0)} \approx 26\text{mV} \ln 10 \log \frac{n_0(x)}{10^{10}}$$

$$\phi_0(x) \approx 60\text{mV} \log \frac{n_0(x)}{10^{10}}$$

$$\phi_0(x) \approx -60\text{mV} \log \frac{p_0(x)}{10^{10}}$$

- Quick calculation aid: For a p-type concentration of $10^{16}$ $\text{cm}^{-3}$, the potential is -360 mV
- N-type materials have a positive potential with respect to intrinsic Si

# Revesed Biased PN Junctions

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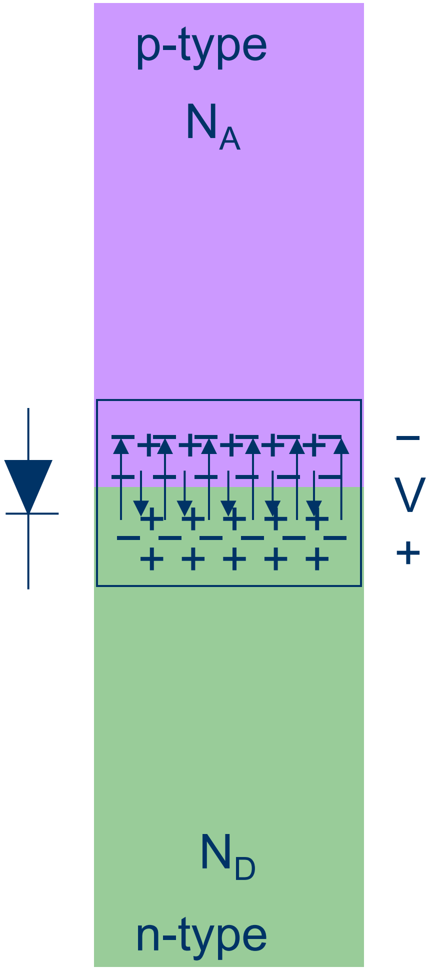
# PN Junctions: Overview

- The most important device is a junction between a p-type region and an n-type region
- When the junction is first formed, due to the concentration gradient, mobile charges transfer near junction
- Electrons leave n-type region and holes leave p-type region
- These mobile carriers become minority carriers in new region (can't penetrate far due to recombination)
- Due to charge transfer, a voltage difference occurs between regions
- This creates a field at the junction that causes drift currents to oppose the diffusion current
- In thermal equilibrium, drift current and diffusion must balance



# PN Junction Currents

- Consider the PN junction in thermal equilibrium
- Again, the currents have to be zero, so we have

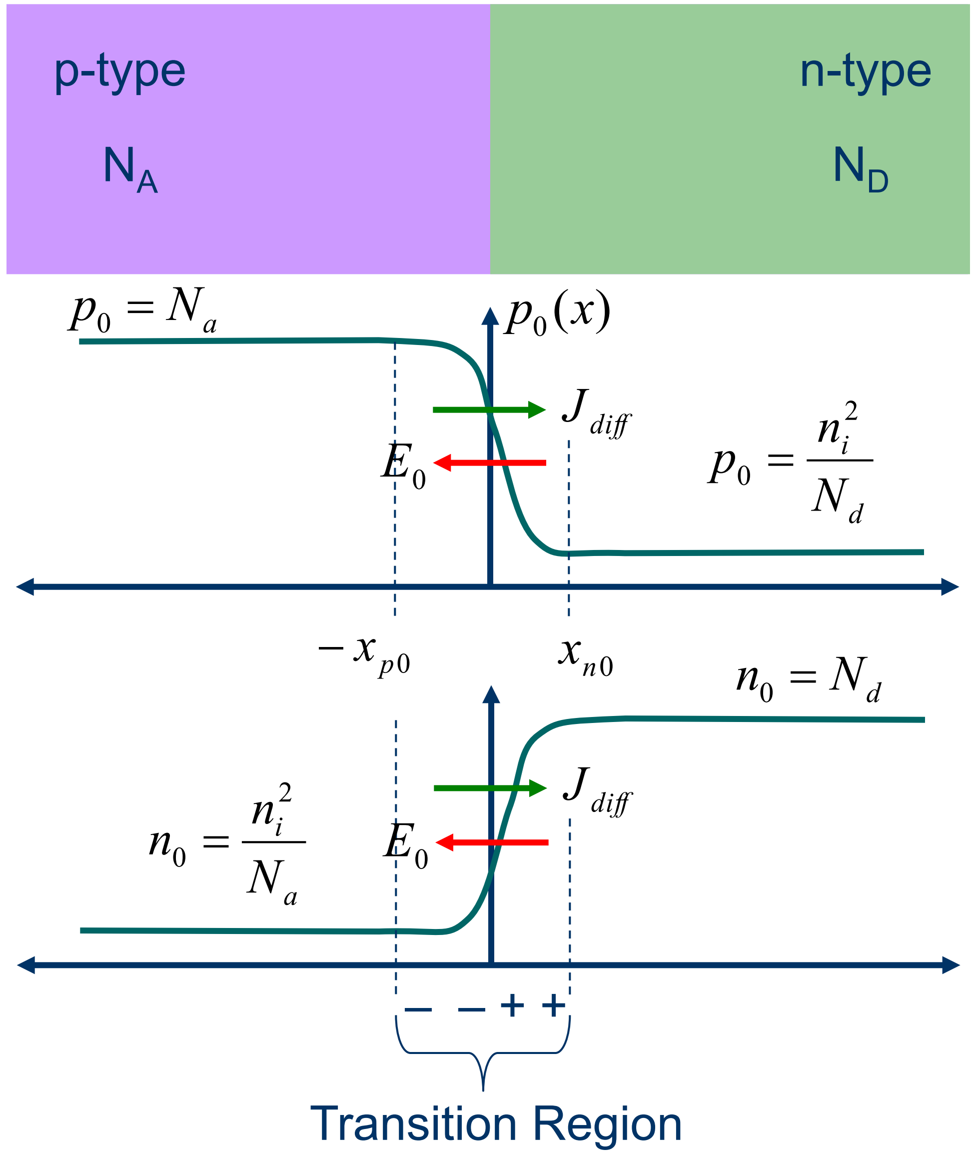
$$J_n = 0 = qn_0\mu_n E_0 + qD_n \frac{dn_o}{dx}$$

$$qn_0\mu_n E_0 = -qD_n \frac{dn_o}{dx}$$

$$E_0 = \frac{-D_n \dfrac{dn_o}{dx}}{n_0\mu_n} = -\frac{kT}{q}\frac{1}{n_0}\frac{dn_0}{dx}$$

$$E_0 = \frac{D_p \dfrac{dp_o}{dx}}{n_0\mu_p} = -\frac{kT}{q}\frac{1}{p_0}\frac{dp_0}{dx}$$

# PN Junction Fields



# Total Charge in Transition Region

- To solve for the electric fields, we need to write down the charge density in the transition region:

$$\rho_0(x) = q(p_0 - n_0 + N_d - N_a)$$

- In the p-side of the junction, there are very few electrons and only acceptors:

$$\rho_0(x) \approx q(p_0 - N_a) \qquad -x_{p0} < x < 0$$

- Since the hole concentration is decreasing on the p-side, the net charge is negative:

$$N_a > p_0 \qquad \rho_0(x) < 0$$

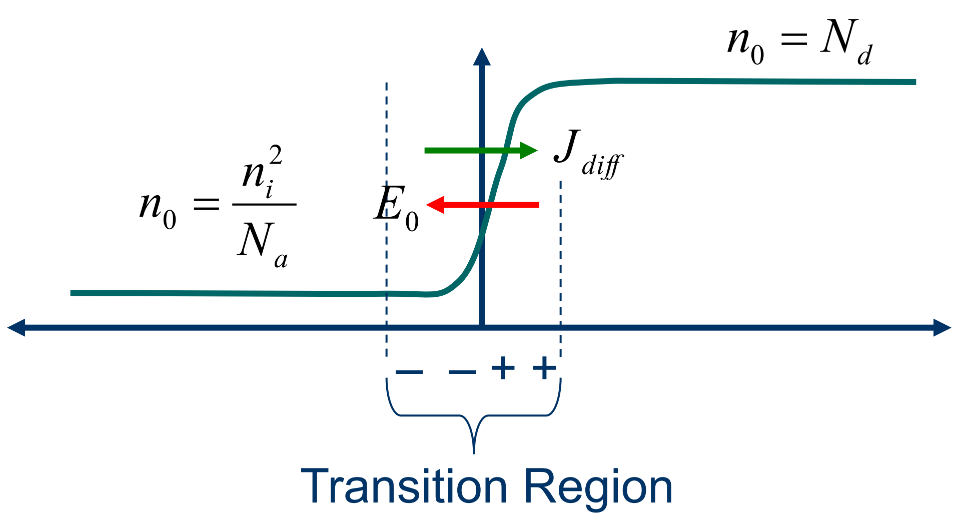
# Charge on N-Side

- Analogous to the p-side, the charge on the n-side is given by:

$$\rho_0(x) \approx q(-n_0 + N_d) \qquad 0 < x < x_{n0}$$

- The net charge here is positive since:

$$N_d > n_0 \qquad \rho_0(x) > 0$$



# "Exact" Solution for Fields

- Given the above approximations, we now have an expression for the charge density

$$\rho_0(x) \cong \begin{cases} q(n_i e^{-\phi_0(x)/V_{th}} - N_a) & -x_{po} < x < 0 \\ q(N_d - n_i e^{\phi_0(x)/V_{th}}) & 0 < x < x_{n0} \end{cases}$$

- We also have the following result from electrostatics

$$\frac{dE_0}{dx} = -\frac{d^2\phi}{dx^2} = \frac{\rho_0(x)}{\varepsilon_s}$$

- Notice that the potential appears on both sides of the equation… difficult problem to solve
- A much simpler way to solve the problem…

# Depletion Approximation

- Let's assume that the transition region is completely depleted of free carriers (only immobile dopants exist)
- Then the charge density is given by

$$\rho_0(x) \cong \begin{cases} -qN_a & -x_{po} < x < 0 \\ +qN_d & 0 < x < x_{n0} \end{cases}$$

- The solution for electric field is now easy

$$\frac{dE_0}{dx} = \frac{\rho_0(x)}{\varepsilon_s}$$

$$E_0(x) = \int_{-x_{p0}}^{x} \frac{\rho_0(x')}{\varepsilon_s} dx' + E_0(-x_{p0})$$

Field zero outside transition region

# Depletion Approximation (2)

- Since charge density is a constant

$$E_0(x) = \int_{-x_{p0}}^{x} \frac{\rho_0(x')}{\varepsilon_s} dx' = -\frac{qN_a}{\varepsilon_s}(x + x_{po})$$

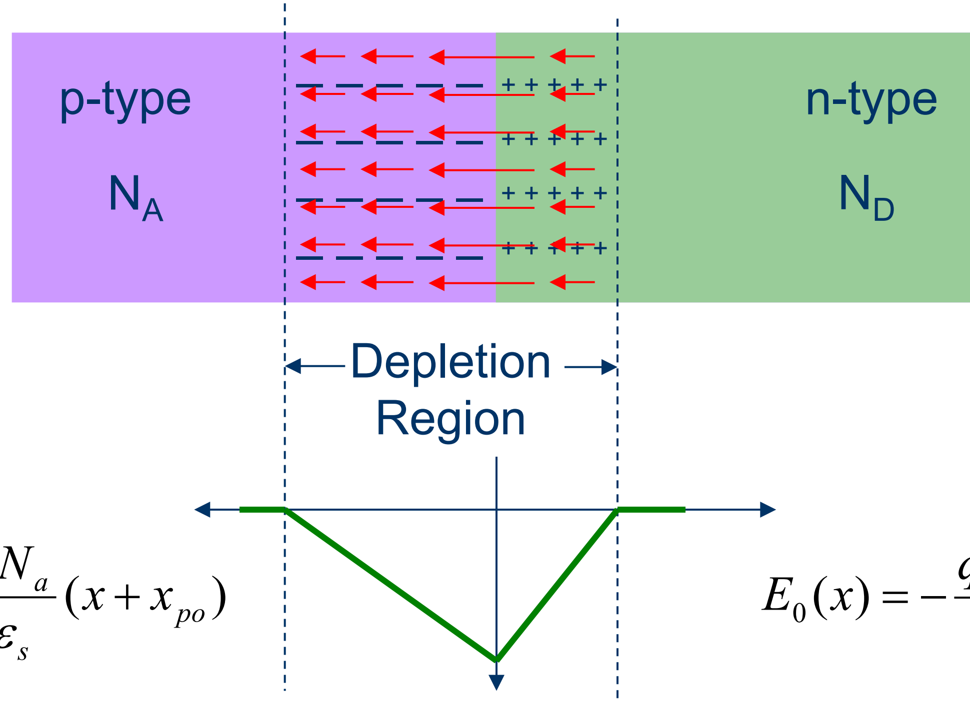
- If we start from the n-side we get the following result

$$E_0(x_{n0}) = \int_{x}^{x_{n0}} \frac{\rho_0(x')}{\varepsilon_s} dx' + E_0(x) = \frac{qN_d}{\varepsilon_s}(x_{n0} - x) + E_0(x)$$

Field zero outside transition region

$$E_0(x) = -\frac{qN_d}{\varepsilon_s}(x_{n0} - x)$$

# Plot of Fields In Depletion Region




$$E_0(x) = -\frac{qN_a}{\varepsilon_s}(x + x_{po})$$

$$E_0(x) = -\frac{qN_d}{\varepsilon_s}(x_{n0} - x)$$

- E-Field zero outside of depletion region
- Note the asymmetrical depletion widths
- Which region has higher doping?
- Slope of E-Field larger in n-region. Why?
- Peak E-Field at junction. Why continuous?

# Continuity of E-Field Across Junction

- Recall that E-Field diverges on charge. For a sheet charge at the interface, the E-field could be discontinuous
- In our case, the depletion region is only populated by a background density of fixed charges so the E-Field is continuous
- What does this imply?

$$E_0^n(x=0) = -\frac{qN_a}{\varepsilon_s}x_{po} = -\frac{qN_d}{\varepsilon_s}x_{no} = E_0^p(x=0)$$

$$qN_a x_{po} = qN_d x_{no}$$

- Total fixed charge in n-region equals fixed charge in p-region! Somewhat obvious result.

# Potential Across Junction

- From our earlier calculation we know that the potential in the n-region is higher than p-region
- The potential has to smoothly transition from high to low in crossing the junction
- Physically, the potential difference is due to the charge transfer that occurs due to the concentration gradient
- Let's integrate the field to get the potential:

$$\phi(x) = \phi(-x_{po}) + \int_{-x_{p0}}^{x} \frac{qN_a}{\varepsilon_s}(x' + x_{po})dx'$$

$$\phi(x) = \phi_p + \frac{qN_a}{\varepsilon_s}\left(\frac{x'^2}{2} + x' x_{po}\right)\Bigg|_{-x_{p0}}^{x}$$

# Potential Across Junction

- We arrive at potential on p-side (parabolic)

$$\phi_o^p(x) = \phi_p + \frac{qN_a}{2\varepsilon_s}(x + x_{p0})^2$$

- Do integral on n-side

$$\phi_n(x) = \phi_n - \frac{qN_d}{2\varepsilon_s}(x - x_{n0})^2$$

- Potential *must* be continuous at interface (field finite at interface)

$$\phi_n(0) = \phi_n - \frac{qN_d}{2\varepsilon_s}x_{n0}^2 = \phi_p + \frac{qN_a}{2\varepsilon_s}x_{p0}^2 = \phi_p(0)$$

# Solve for Depletion Lengths

- We have two equations and two unknowns. We are finally in a position to solve for the depletion depths

$$\phi_n - \frac{qN_d}{2\varepsilon_s} x_{n0}^2 = \phi_p + \frac{qN_a}{2\varepsilon_s} x_{p0}^2 \qquad (1)$$

$$qN_a x_{po} = qN_d x_{no} \qquad (2)$$

$$x_{no} = \sqrt{\frac{2\varepsilon_s \phi_{bi}}{qN_d}\left(\frac{N_a}{N_a + N_d}\right)} \qquad x_{po} = \sqrt{\frac{2\varepsilon_s \phi_{bi}}{qN_a}\left(\frac{N_d}{N_d + N_a}\right)}$$

$$\phi_{bi} \equiv \phi_n - \phi_p > 0$$

# Sanity Check

- Does the above equation make sense?
- Let's say we dope one side very highly. Then physically we expect the depletion region width for the heavily doped side to approach zero:

$$x_{n0} = \lim_{N_d \to \infty} \sqrt{\frac{2\varepsilon_s \phi_{bi}}{qN_d} \frac{N_d}{N_d + N_a}} = 0 \quad \boxed{\checkmark}$$

$$x_{p0} = \lim_{N_d \to \infty} \sqrt{\frac{2\varepsilon_s \phi_{bi}}{qN_a} \left( \frac{N_d}{N_d + N_a} \right)} = \sqrt{\frac{2\varepsilon_s \phi_{bi}}{qN_a}}$$

- Entire depletion width dropped across p-region

# Total Depletion Width

- The sum of the depletion widths is the "space charge region"

$$X_{d0} = x_{p0} + x_{n0} = \sqrt{\frac{2\varepsilon_s \phi_{bi}}{q}\left(\frac{1}{N_a} + \frac{1}{N_d}\right)}$$

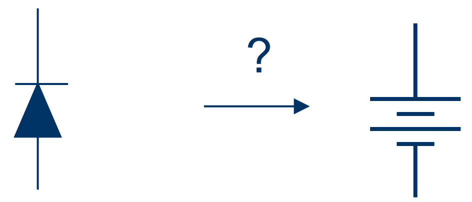
- This region is essentially depleted of all mobile charge
- Due to high electric field, carriers move across region at velocity saturated speed

$$X_{d0} = \sqrt{\frac{2\varepsilon_s \phi_{bi}}{q}\left(\frac{1}{10^{15}}\right)} \approx 1\mu \qquad E_{pn} \approx \frac{1\text{V}}{1\mu} = 10^4 \frac{\text{V}}{\text{cm}}$$

# Have we invented a battery?

- Can we harness the PN junction and turn it into a battery?

$$\phi_{bi} \equiv \phi_n - \phi_p = V_{th}\left( \ln \frac{N_D}{n_i} + \ln \frac{N_A}{n_i} \right) = V_{th} \ln \frac{N_D N_A}{n_i^2}$$

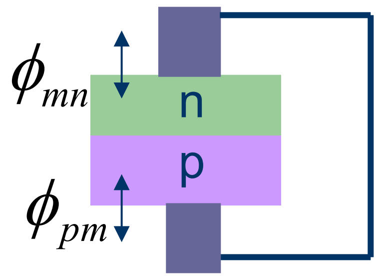



- Numerical example:

$$\phi_{bi} = 26\text{mV} \ln \frac{N_D N_A}{n_i^2} = 60\text{mV} \times \log \frac{10^{15} 10^{15}}{10^{20}} = 600\text{mV}$$

# Contact Potential

- The contact between a PN junction creates a potential difference
- Likewise, the contact between two dissimilar metals creates a potential difference (proportional to the difference between the work functions)
- When a metal semiconductor junction is formed, a contact potential forms as well
- If we short a PN junction, the sum of the voltages around the loop must be zero:


$$0 = \phi_{bi} + \phi_{pm} + \phi_{mn}$$

$$\phi_{bi} = -(\phi_{pm} + \phi_{mn})$$

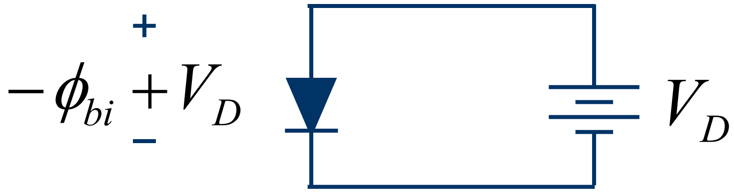
# PN Junction Capacitor

- Under thermal equilibrium, the PN junction does not draw any (much) current
- But notice that a PN junction stores charge in the space charge region (transition region)
- Since the device is storing charge, it's acting like a capacitor
- Positive charge is stored in the n-region, and negative charge is in the p-region:

$$qN_a x_{po} = qN_d x_{no}$$

# Reverse Biased PN Junction

- What happens if we "reverse-bias" the PN junction?




$V_D < 0$

- Since no current is flowing, the entire reverse biased potential is dropped across the transition region
- To accommodate the extra potential, the charge in these regions must increase
- If no current is flowing, the only way for the charge to increase is to grow (shrink) the depletion regions

# Voltage Dependence of Depletion Width

- Can redo the math but in the end we realize that the equations are the same except we replace the built-in potential with the effective reverse bias:

$$x_n(V_D) = \sqrt{\frac{2\varepsilon_s(\phi_{bi} - V_D)}{qN_d}\left(\frac{N_a}{N_a + N_d}\right)} = x_{n0}\sqrt{1 - \frac{V_D}{\phi_{bi}}}$$

$$x_p(V_D) = \sqrt{\frac{2\varepsilon_s(\phi_{bi} - V_D)}{qN_a}\left(\frac{N_d}{N_a + N_d}\right)} = x_{p0}\sqrt{1 - \frac{V_D}{\phi_{bi}}}$$

$$X_d(V_D) = x_p(V_D) + x_n(V_D) = \sqrt{\frac{2\varepsilon_s(\phi_{bi} - V_D)}{q}\left(\frac{1}{N_a} + \frac{1}{N_d}\right)}$$

$$X_d(V_D) = X_{d0}\sqrt{1 - \frac{V_D}{\phi_{bi}}}$$

# Charge Versus Bias

- As we increase the reverse bias, the depletion region grows to accommodate more charge

$$Q_J(V_D) = -qN_a x_p(V_D) = -qN_a \sqrt{1 - \frac{V_D}{\phi_{bi}}}$$

- Charge is *not* a linear function of voltage
- This is a non-linear capacitor
- We can define a small signal capacitance for small signals by breaking up the charge into two terms

$$Q_J(V_D + v_D) = Q_J(V_D) + q(v_D)$$

# Derivation of Small Signal Capacitance

- Do a Taylor Series expansion:

$$Q_J(V_D + v_D) = Q_J(V_D) + \left.\frac{dQ_D}{dV}\right|_{V_D} v_D + \cdots$$

$$C_j = C_j(V_D) = \left.\frac{dQ_j}{dV}\right|_{V=V_D} = \frac{d}{dV}\left(-qN_a x_{p0}\sqrt{1-\frac{V}{\phi_{bi}}}\right)\Bigg]_{V=V_R}$$

$$C_j = \frac{qN_a x_{p0}}{2\phi_{bi}\sqrt{1-\frac{V_D}{\phi_{bi}}}} = \frac{C_{j0}}{\sqrt{1-\frac{V_D}{\phi_{bi}}}}$$

- Notice that

$$C_{j0} = \frac{qN_a x_{p0}}{2\phi_{bi}} = \frac{qN_a}{2\phi_{bi}}\sqrt{\left(\frac{2\varepsilon_s\phi_{bi}}{qN_a}\right)\left(\frac{N_d}{N_a+N_d}\right)} = \sqrt{\frac{q\varepsilon_s}{2\phi_{bi}}\frac{N_a N_d}{N_a+N_d}}$$

# Physical Interpretation of Depletion Cap

$$C_{j0} = \sqrt{\frac{q\varepsilon_s}{2\phi_{bi}} \frac{N_a N_d}{N_a + N_d}}$$

- Notice that the expression on the right-hand-side is just the depletion width in thermal equilibrium

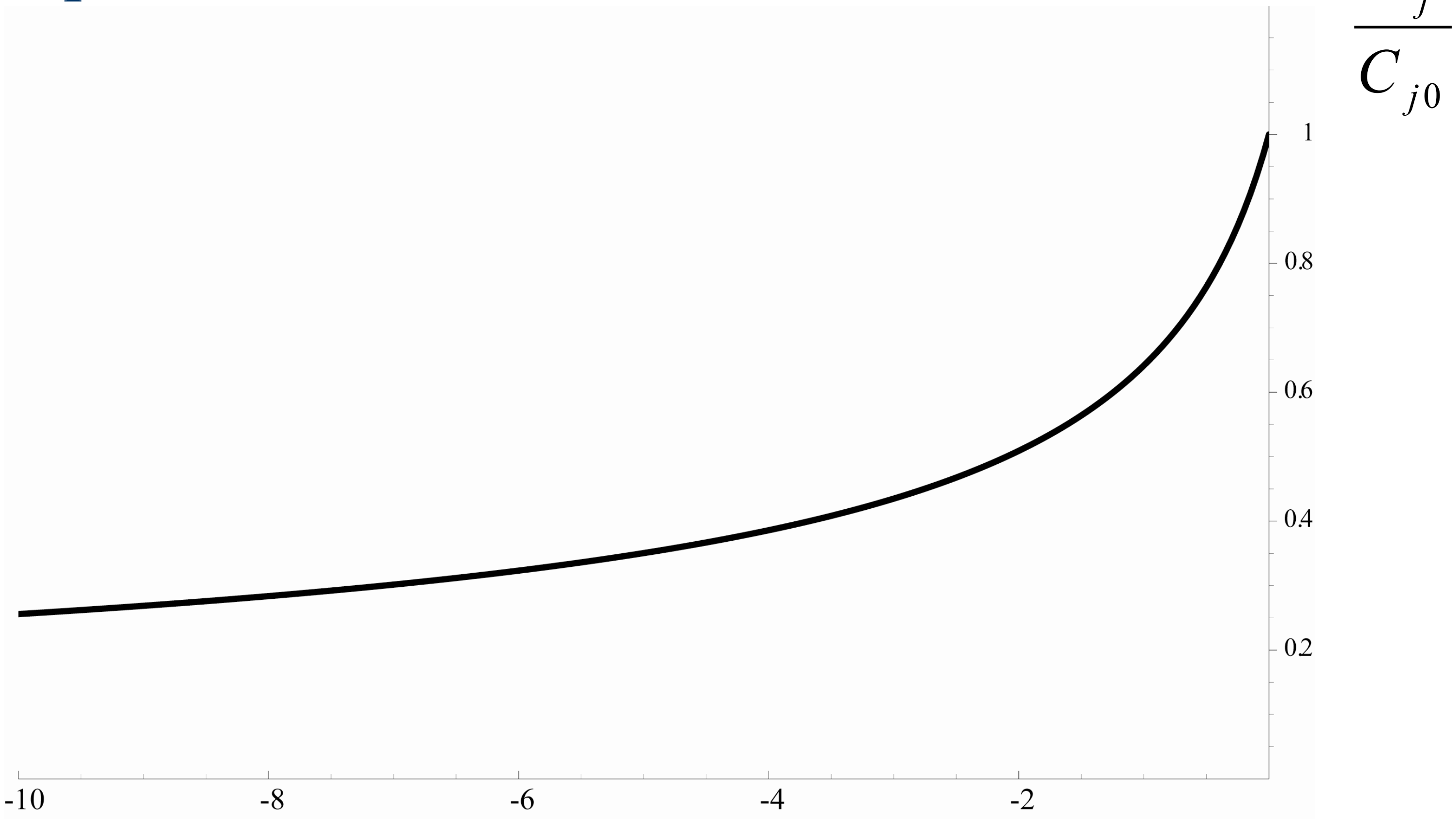
$$C_{j0} = \varepsilon_s \sqrt{\frac{q}{2\varepsilon_s \phi_{bi}} \left( \frac{1}{N_a} + \frac{1}{N_d} \right)^{-1}} = \frac{\varepsilon_s}{X_{d0}}$$

- This looks like a parallel plate capacitor!

$$C_j(V_D) = \frac{\varepsilon_s}{X_d(V_D)}$$

# A Variable Capacitor (Varactor)

- Capacitance varies versus bias:




- Application: Radio Tuner

# Currents in PN Junctions

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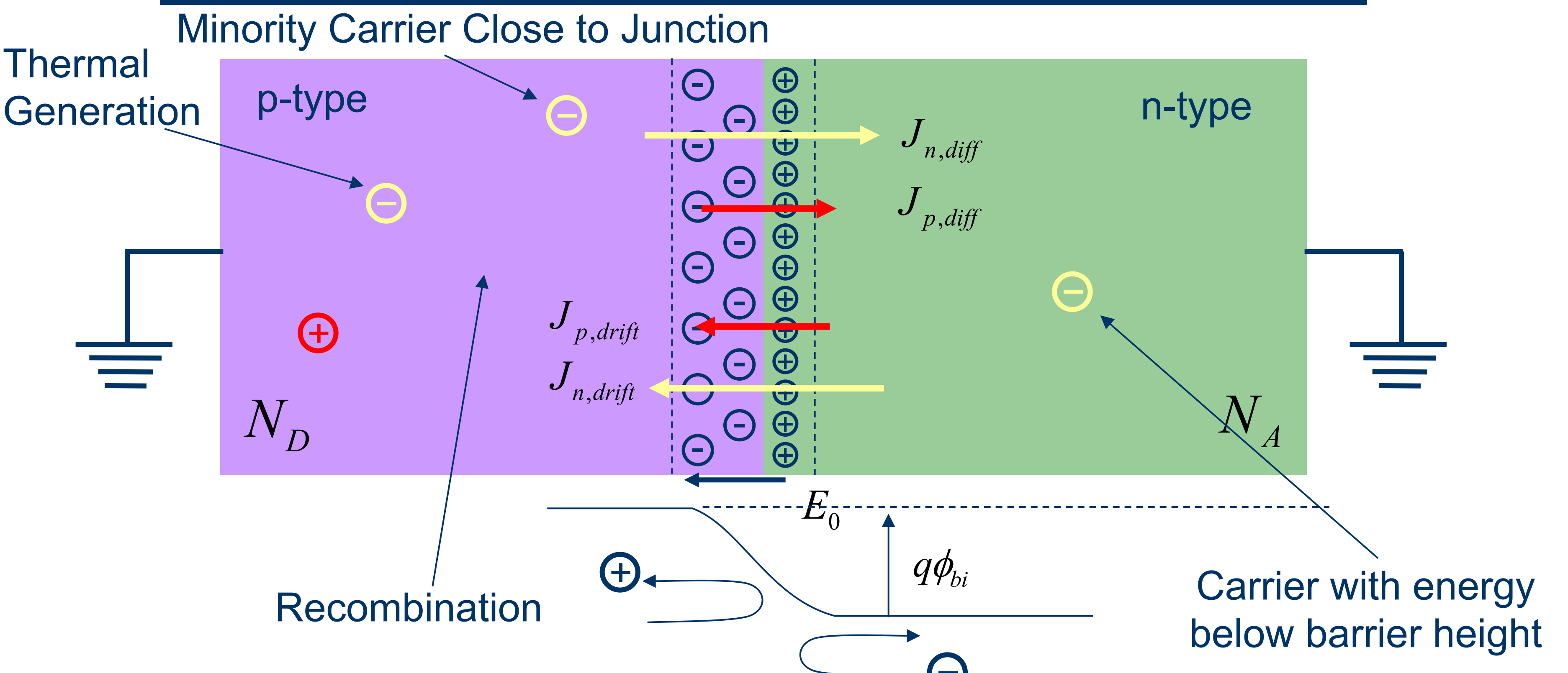
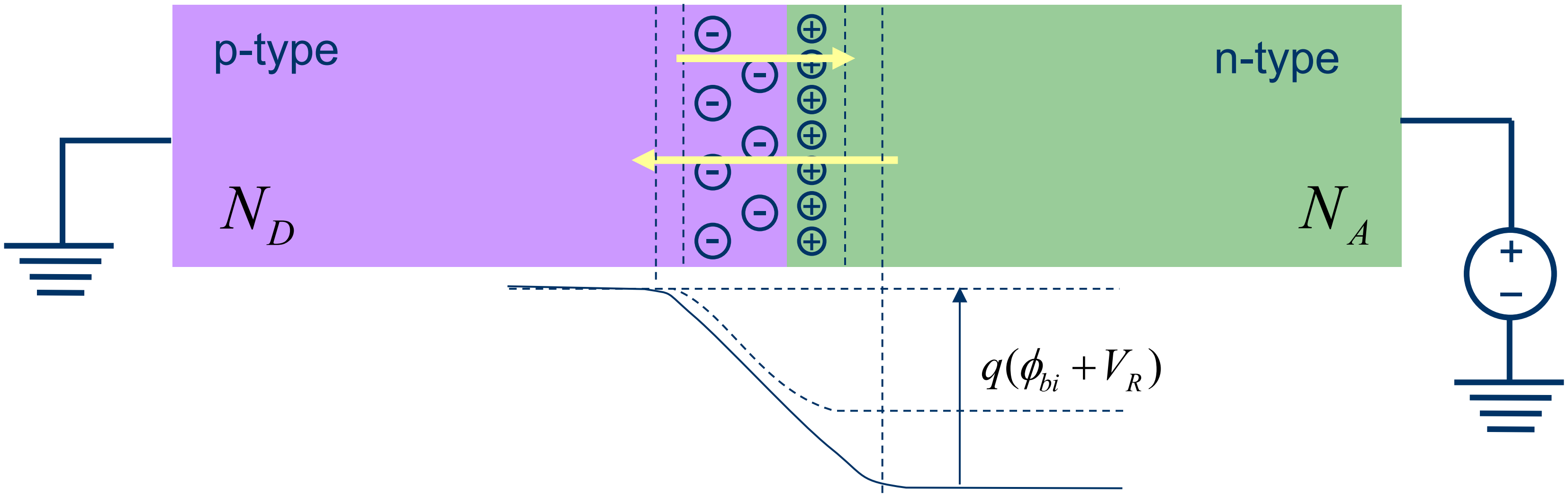
# Diode under Thermal Equilibrium




- Diffusion small since few carriers have enough energy to penetrate barrier
- Drift current is small since minority carriers are few and far between: Only minority carriers generated within a diffusion length can contribute current
- **Important Point: Minority drift current independent of barrier!**
- **Diffusion current strong (exponential) function of barrier**

# Reverse Bias

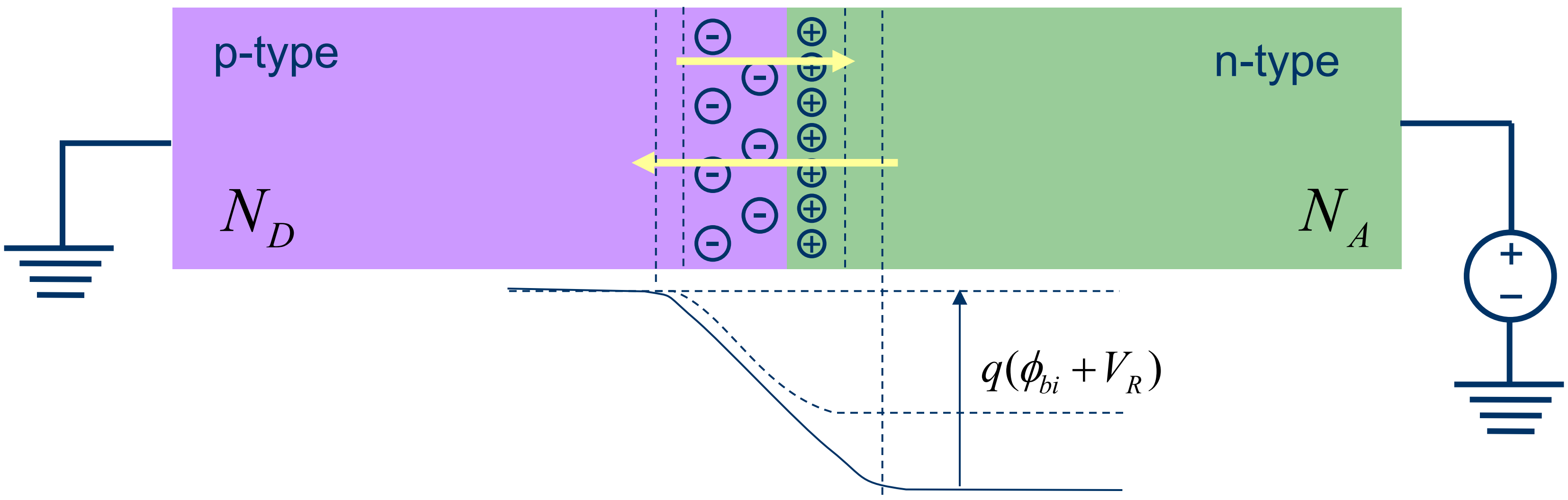
- Reverse Bias causes an increases barrier to diffusion
- Diffusion current is reduced exponentially




- Drift current does not change
- Net result:  Small reverse current

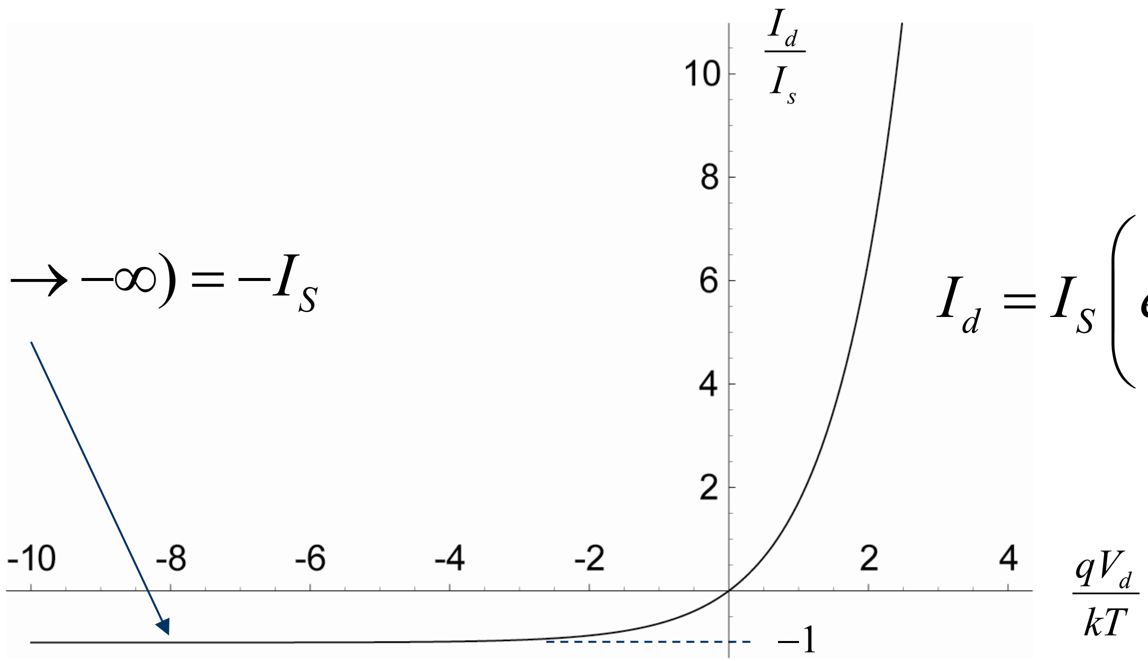
# Forward Bias

- Forward bias causes an exponential increase in the number of carriers with sufficient energy to penetrate barrier
- Diffusion current ***increases*** exponentially




- Drift current does not change
- Net result: Large forward current

# Diode I-V Curve




$$I_d(V_d \to -\infty) = -I_S$$

$$I_d = I_S\left(e^{\frac{qV_d}{kT}} - 1\right)$$

- Diode IV relation is an exponential function
- This exponential is due to the Boltzmann distribution of carriers versus energy
- For reverse bias the current saturations to the drift current due to minority carriers

# Minority Carriers at Junction Edges

Minority carrier concentration at boundaries of depletion region increase as barrier lowers … the function is

$$\frac{p_n(x=x_n)}{p_p(x=-x_p)} = \frac{\text{(minority) hole conc. on n-side of barrier}}{\text{(majority) hole conc. on p-side of barrier}}$$

$$= e^{-(Barrier\ Energy)/kT}$$

$$\boxed{\frac{p_n(x=x_n)}{N_A} = e^{-q(\phi_B - V_D)/kT}}$$

(Boltzmann's Law)

# "Law of the Junction"

Minority carrier concentrations at the edges of the depletion region are given by:

$$p_n(x = x_n) = N_A e^{-q(\phi_B - V_D)/kT}$$

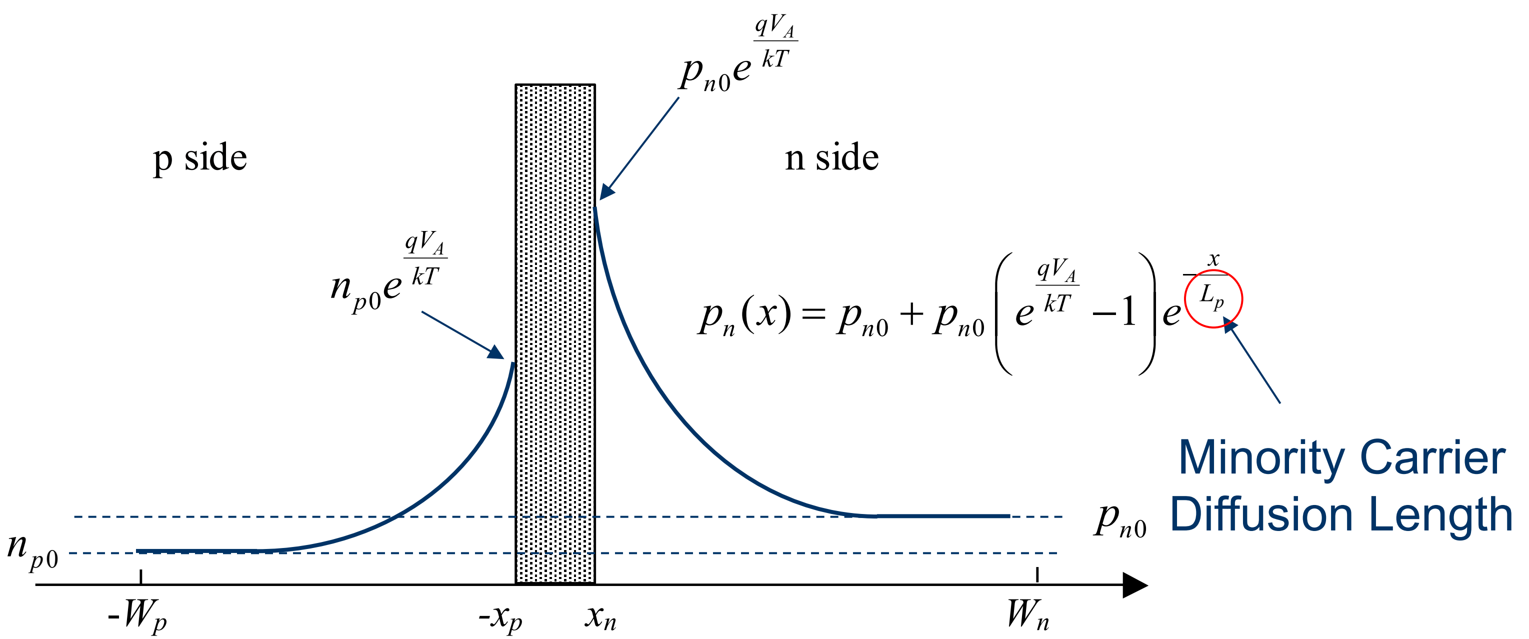
$$n_p(x = -x_p) = N_D e^{-q(\phi_B - V_D)/kT}$$

Note 1: $N_A$ and $N_D$ are the majority carrier concentrations on the *other* side of the junction

Note 2: we can reduce these equations further by substituting $V_D = 0$ V (thermal equilibrium)

Note 3: assumption that $p_n << N_D$ and $n_p << N_A$

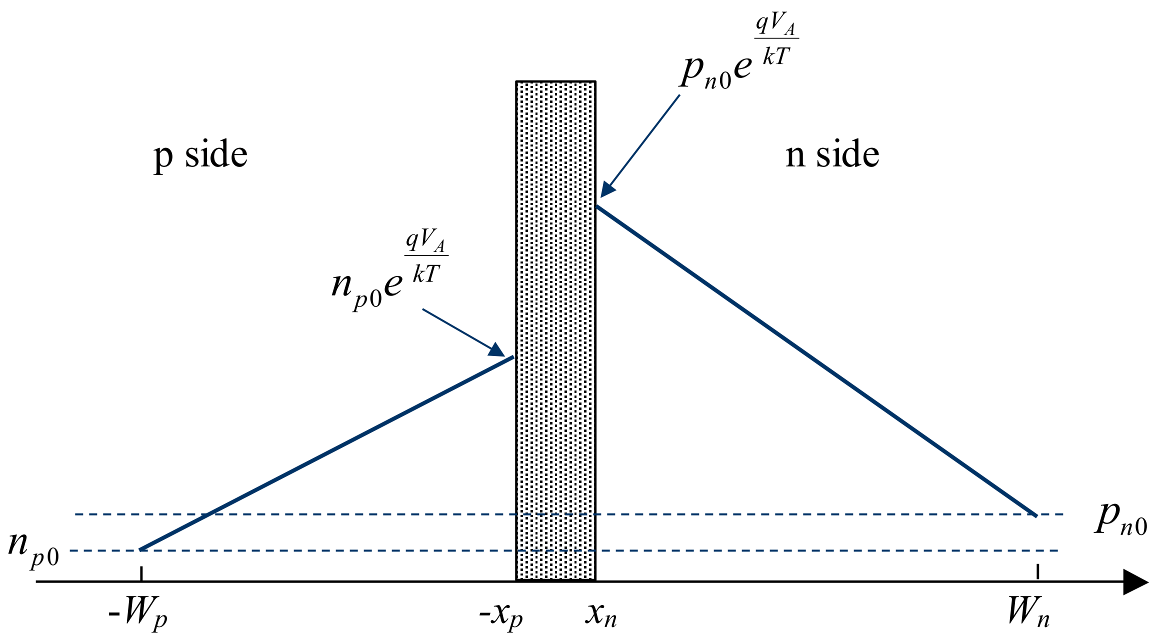
# Minority Carrier Concentration




The minority carrier concentration in the bulk region for forward bias is a decaying exponential due to recombination

# Steady-State Concentrations

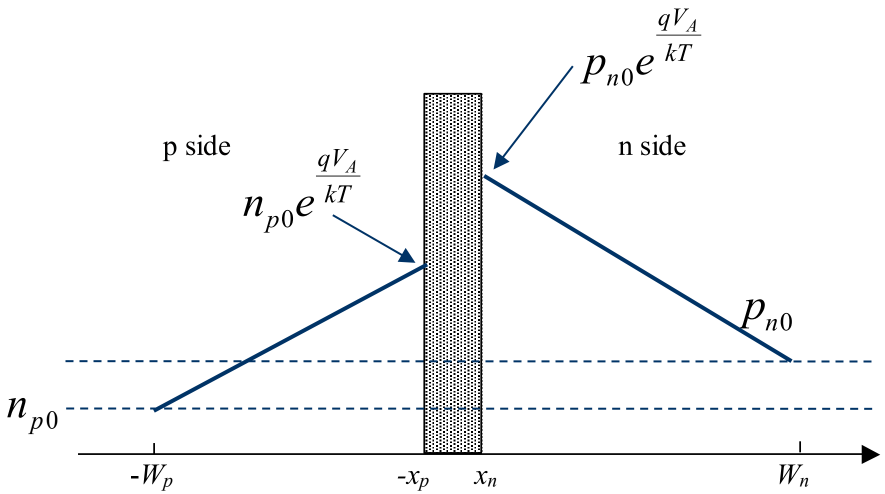
Assume that none of the diffusing holes and electrons recombine → get straight lines …




This also happens if the minority carrier diffusion lengths are much larger than $W_{n,p}$

$$L_{n,p} >> W_{n,p}$$

# Diode Current Densities




$$\frac{dn_p}{dx}(x) \approx \frac{n_{p0}e^{\frac{qV_A}{kT}} - n_{p0}}{-x_p - (-W_p)}$$

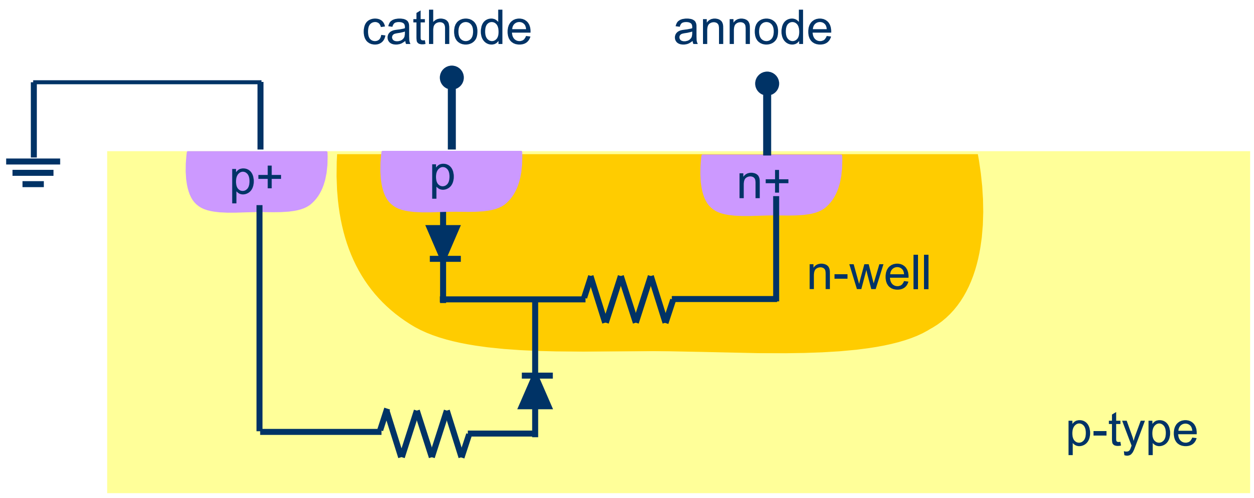
$$n_{p0} = \frac{n_i^2}{N_a}$$

$$J_n^{diff} = qD_n \left.\frac{dn_p}{dx}\right|_{x=-x_p} \approx q\frac{D_n}{W_p} n_{p0}\left(e^{\frac{qV_A}{kT}} - 1\right)$$

$$J_p^{diff} = -qD_p \left.\frac{dp_n}{dx}\right|_{x=x_n} \approx -q\frac{D_p}{W_n} p_{n0}\left(1 - e^{\frac{qV_A}{kT}}\right)$$

$$J^{diff} = qn_i^2\left(\frac{D_p}{N_d W_n} + \frac{D_n}{N_a W_p}\right)\left(e^{\frac{qV_A}{kT}} - 1\right)$$

# Fabrication of IC Diodes




- Start with p-type substrate
- Create n-well to house diode
- p and n+ diffusion regions are the cathode and annode
- N-well must be reverse biased from substrate
- Parasitic resistance due to well resistance

# Diode Small Signal Model

- The I-V relation of a diode can be linearized

$$I_D + i_D = I_S\left(e^{\frac{q(V_d+v_d)}{kT}} - 1\right) \approx I_S e^{\frac{qV_d}{kT}} e^{\frac{qv_d}{kT}}$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$I_D + i_D \approx I_D\left(1 + \frac{q(V_d + v_d)}{kT} + \cdots\right)$$

$$i_D \approx \frac{qv_d}{kT} = g_d v_d$$

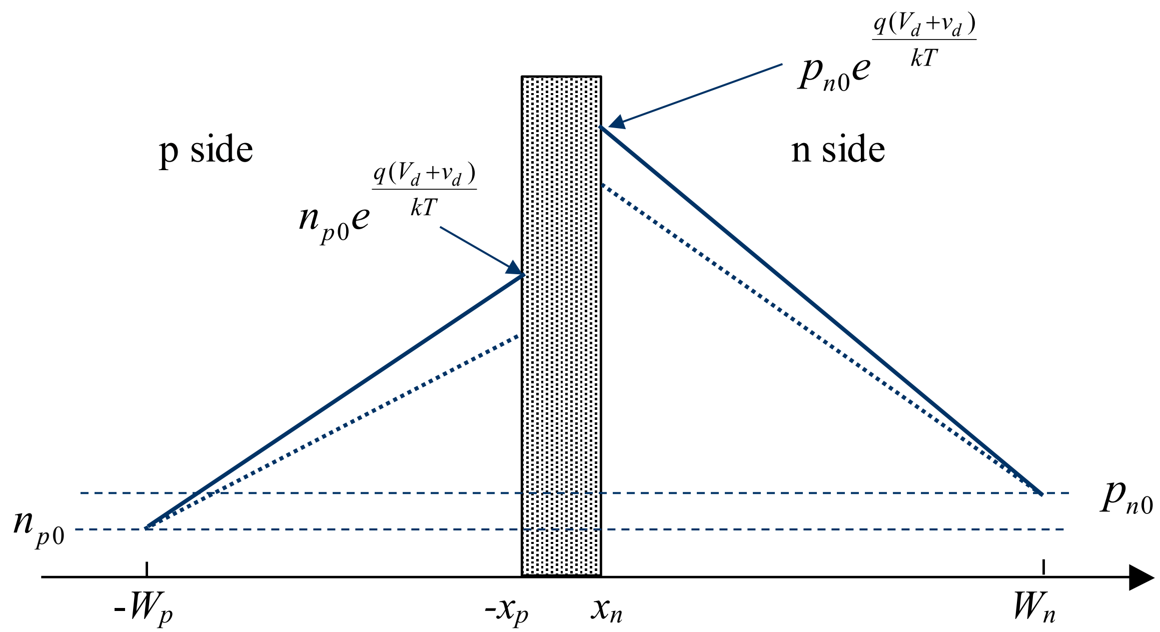
# Diode Capacitance

- We have already seen that a reverse biased diode acts like a capacitor since the depletion region grows and shrinks in response to the applied field. The capacitance in forward bias is given by

$$C_j = A\frac{\varepsilon_S}{X_{dep}} \approx 1.4 C_{j0}$$

- But another charge storage mechanism comes into play in forward bias
- Minority carriers injected into p and n regions “stay” in each region for a while
- On average additional charge is stored in diode

# Charge Storage




- Increasing forward bias increases minority charge density
- By charge neutrality, the source voltage must supply equal and opposite charge
- A detailed analysis yields: $$C_d = \frac{1}{2}\frac{qI_d}{kT}\tau$$

Time to cross junction (or minority carrier lifetime)

# Diode Circuits

- Rectifier (AC to DC conversion)
- Average value circuit
- Peak detector (AM demodulator)
- DC restorer
- Voltage doubler / quadrupler /…