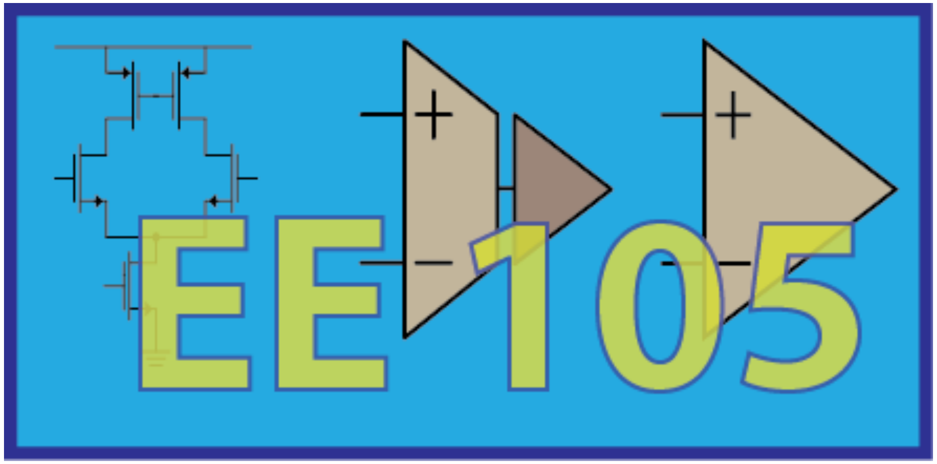


Lecture 6:
Semiconductor Conduction

Prof. Ali M. Niknejad

Lecture Outline

- Physics of conduction
- Semiconductors
  - Si Diamond Structure
  - Bond Model
- Intrinsic Carrier Concentration
  - Doping by Ion Implantation
- Drift
  - Velocity Saturation
- Diffusion

Physics of Conduction

Ohm's Law

- One of the first things we learn as EECS majors is:

$$V = I \times R$$

- Is this trivial? Maybe what's really going on is the following:

$$V = f(I) = f(0) + f'(0)I + f''(0)I^2/2 + \ldots \approx f'(0)I$$

- In the above Taylor exansion, if the voltage is zero for zero current, then this is generally valid
- The range of validity (radius of convergence) is the important question. It turns out to be VERY large!

Ohm's Law Revisited

- In Physics we learned:

$$J = \sigma E$$

- Is this also trivial? Well, it's the same as Ohm's law, so the questions are related. For a rectangular solid:

$$J = \frac{I}{A} = \sigma \frac{V}{L} \qquad V = \frac{L}{\sigma A} I = R I$$

- Isn't it strange that current (velocity) is proportional to Force?
- Where does conductivity come from?

Conductivity of a Gas

- Electrical conduction is due to the motion of positive and negative charges
- For water with pH=7, the concentration of hydrogen H^+ ions (and OH^-) is:

$$10^{-7}\,\text{mole/L} = 10^{-10}\,\text{mole/cm}^3 = 10^{-10} \times 6.02 \times 10^{23}\,\text{cm}^{-3} = 6 \times 10^{13}\,\text{cm}^{-3}$$

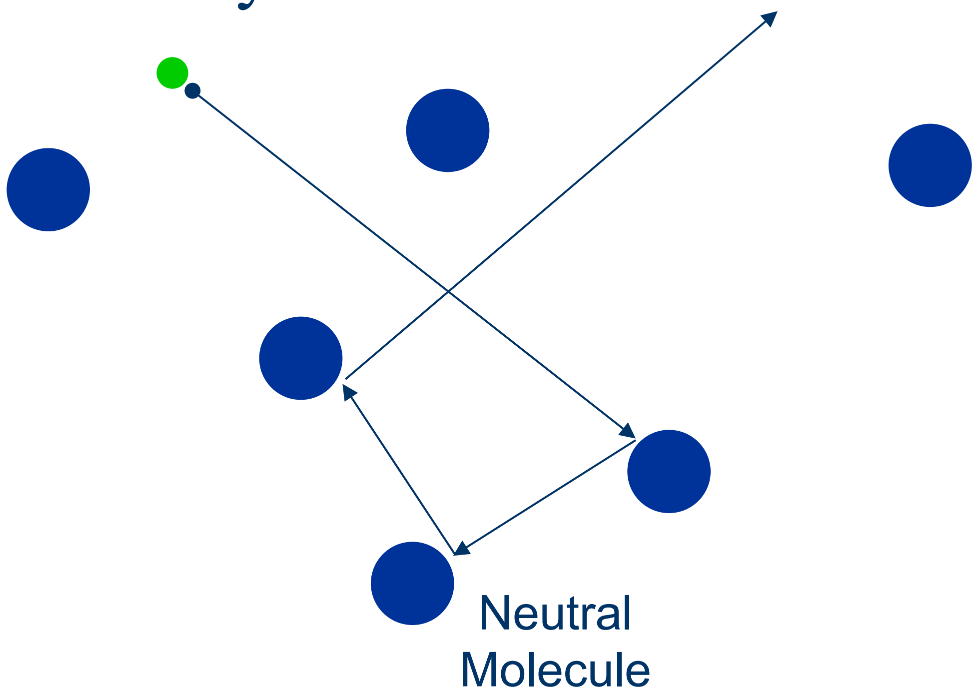
- Typically, the concentration of charged carriers is much smaller than the concentration of neutral molecules
- The motion of the charged carriers (electrons, ions, molecules) gives rise to electrical conduction

Collisions in Gas

- At a temperate T, each charged carrier will move in a random direction and velocity until it encounters a neutral molecule or another charged carrier
- Since the concentration of charged carriers is much less than molecules, it will most likely encounter a molecule
- For a gas, the molecules are widely separated (~ 10 molecular diameters)
- After colliding with the molecule, there is some energy exchange and the charge carrier will come out with a new velocity and new direction

Memory Loss in Collisions

- Schematically



- Key Point: The initial velocity and direction is lost (randomized) after a few collisions

Application of Field

- When we apply an electric field, during each "free flight", the carriers will gain a momentum of $\mathbf{E}qt$
- Therefore, after t seconds, the momentum is given by:

$$M\mathbf{u} + \mathbf{E}qt$$

- If we take the average momentum of all particles at any given time, we have:

$$M\overline{\mathbf{u}} = \frac{1}{N}\sum_j \left(M\mathbf{u}_j + \underline{\mathbf{E}qt_j} \right)$$

Time from Last Collision

Number of Carriers

Initial Momentum Before Collision

Momentum Gained from Field

Random Things Sum to Zero!

- When we sum over all the random velocities of the particles, we are averaging over a large number of random variables with zero mean, the average is zero

$$M\bar{\mathbf{u}} = \frac{1}{N}\sum_j \left(\cancel{M\mathbf{u}_j} + \mathbf{E}qt_j\right)$$

$$M\bar{\mathbf{u}} = \frac{1}{N}\sum_j \mathbf{E}qt_j = \mathbf{E}q\tau$$

Average Time Between Collisions

$$\mathbf{J} = Nq\bar{\mathbf{u}} = Nq\left(\frac{\mathbf{E}q\tau}{M}\right) = Nq^2\frac{\tau}{M}\mathbf{E} = \sigma\mathbf{E}$$

Negative and Positive Carriers

- Since current is contributed by positive and negative charge carriers:

$$\mathbf{J} = \mathbf{J}^{+} - \mathbf{J}^{-} = e\left(\frac{N^{+} e \tau^{+}}{M^{+}} - \frac{-N^{-} e \tau^{-}}{M^{-}} \right)\mathbf{E}$$

$$\sigma = e^{2}\left(\frac{N^{+} \tau^{+}}{M^{+}} + \frac{N^{-} \tau^{-}}{M^{-}} \right)$$

Conduction in Metals

- High conductivity of metals is due to large concentration of free electrons
- These electrons are not attached to the solid but are free to move about the solid
- In metal sodium, each atom contributes a free electron: $N = 2.5 \times 10^{22} \text{ atoms/cm}^3$
- From the measured value of conductivity (easy to do), we can back calculate the mean free time:

$$\tau = \frac{\sigma m}{Ne^2} = \frac{\left(1.9 \times 10^{17}\right)\left(9 \times 10^{-28}\right)}{\left(2.5 \times 10^{22}\right)\left(23 \times 10^{-20}\right)} = 3 \times 10^{-14} \text{ sec}$$

A Deep Puzzle

- This value of mean free time is surprisingly long
- The mean velocity for an electron at room temperature is about:

$$\frac{mv^2}{2} = \frac{3}{2}kT \qquad v = 3\times10^{7}\,\text{cm}/\text{sec}$$

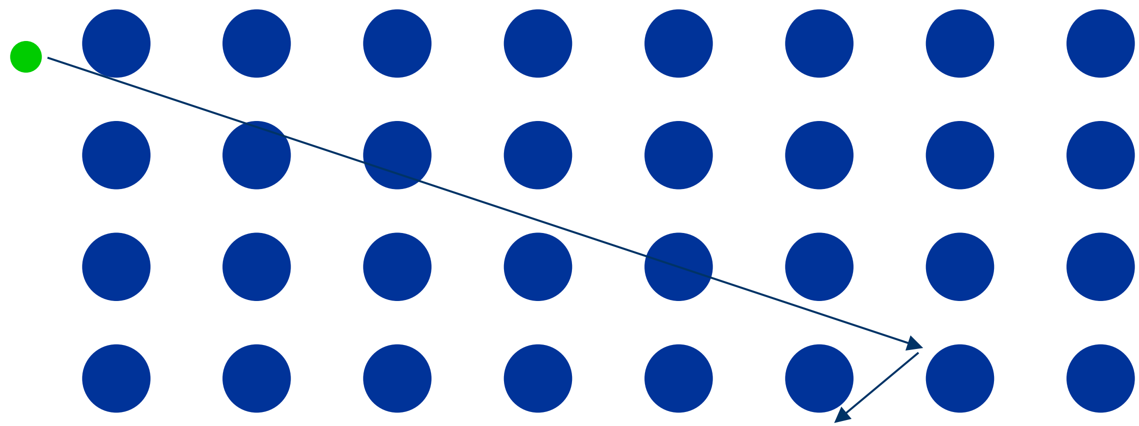
- At this speed, the electron travels $v\tau = 3\times10^{-7}\,\text{cm}$

- The molecular spacing between adjacent ions is only

$$3.8\times10^{-8}\,\text{cm}$$

- Why is it that the electron is on average zooming by 10 positively charged ions?

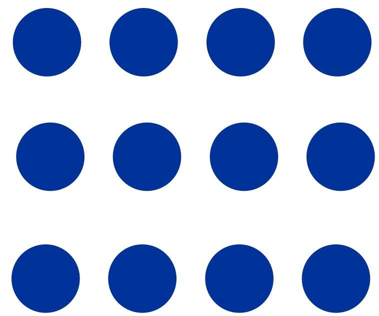
Wave Nature of Electron



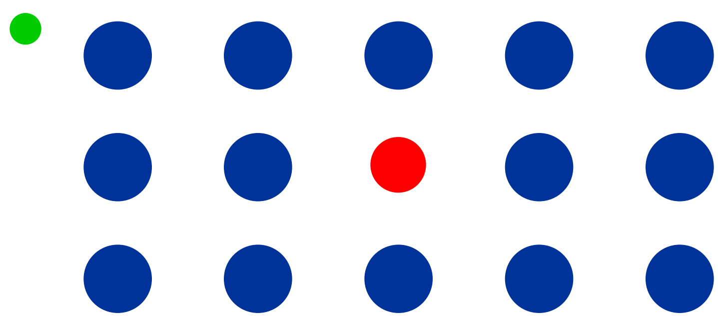
- The free carrier can penetrate right through positively charged host atoms!
- Quantum mechanics explains this! (Take Physics 117A/B)
- For a periodic arrangement of potential functions, the electron does not scatter. The influence of the crystal is that it will travel freely with an effective mass.
- So why does it scatter at all?

Scattering in Metals

- At temperature T, the atoms are in random motion and thus upset periodicity



- Even at extremely low temperatures, the presence of an impurity upsets periodicity



Summary of Conduction

$$\sigma = e^2\left(\frac{N^+\tau^+}{M^+}+\frac{N^-\tau^-}{M^-}\right)$$

Conductivity determined by:

- Density of free charge carriers (both positive and negative)
- Charge of carrier (usually just *e*)
- Effective mass of carrier (different inside solid)
- Mean relaxation time (time for memory loss … usually the time between collisions)
  - This is determined by several mechanisms, e.g.:
    - Scattering by impurities
    - Scattering due to vibrations in crystal

Semiconductors

Resistivity for a Few Materials

• Pure copper, 273K	1.56×10^{-6} ohm-cm
• Pure copper, 373 K	2.24×10^{-6} ohm-cm
• Pure germanium, 273 K	200 ohm-cm
• Pure germanium, 500 K	.12 ohm-cm
• Pure water, 291 K	2.5×10^{7} ohm-cm
• Seawater	25 ohm-cm

What gives rise to this enormous range?

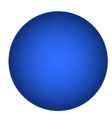
Why are some materials semi-conductive?

Why the strong temp dependence?

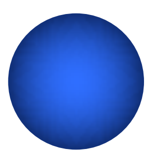
Electronic Properties of Silicon

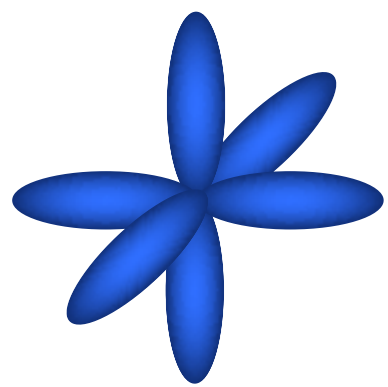
- Silicon is in Group IV
  - Atom electronic structure: $1s^2 2s^2 2p^6 3s^2 3p^2$
  - Crystal electronic structure: $1s^2 2s^2 2p^6 3(sp)^4$
  - Diamond lattice, with 0.235 nm bond length
- Very poor conductor at room temperature: why?

$(1s)^2$

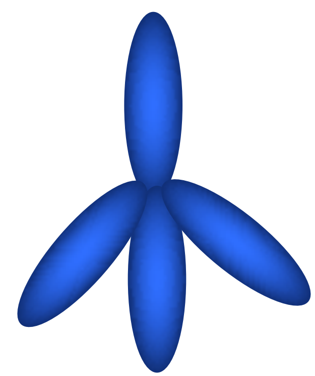


$(2s)^2$





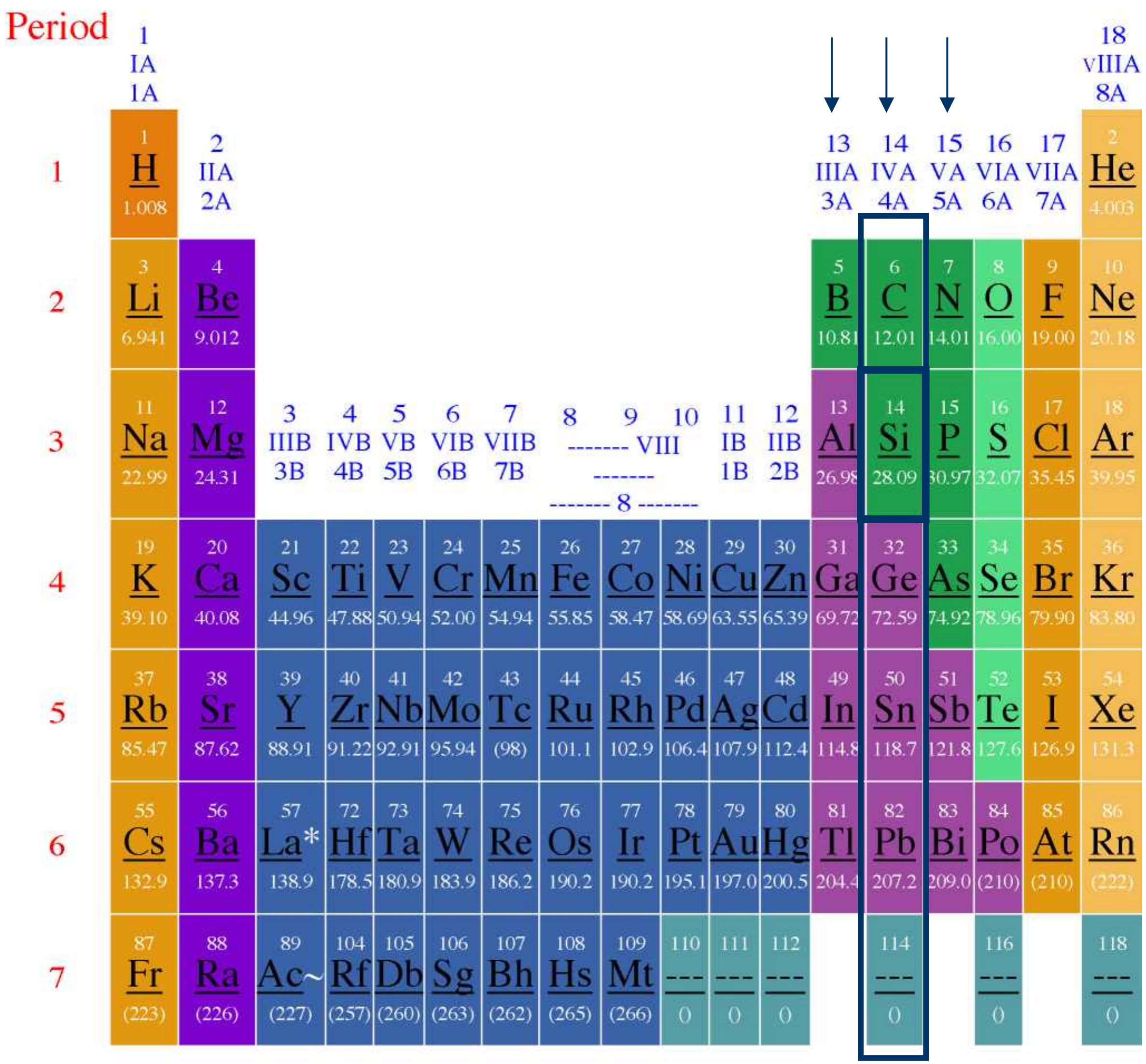
$(2p)^6$



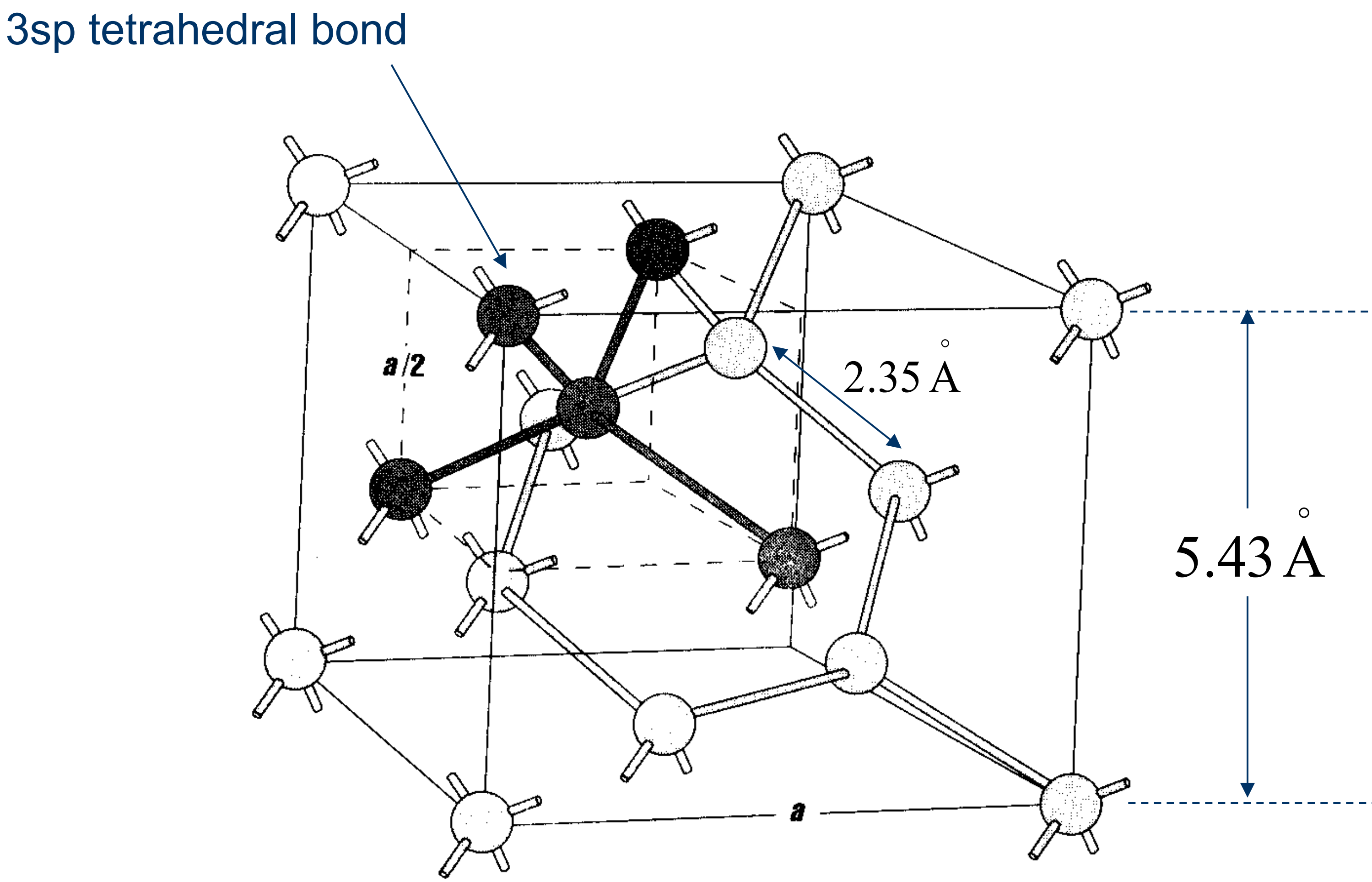
$(3sp)^4$

Hybridized State

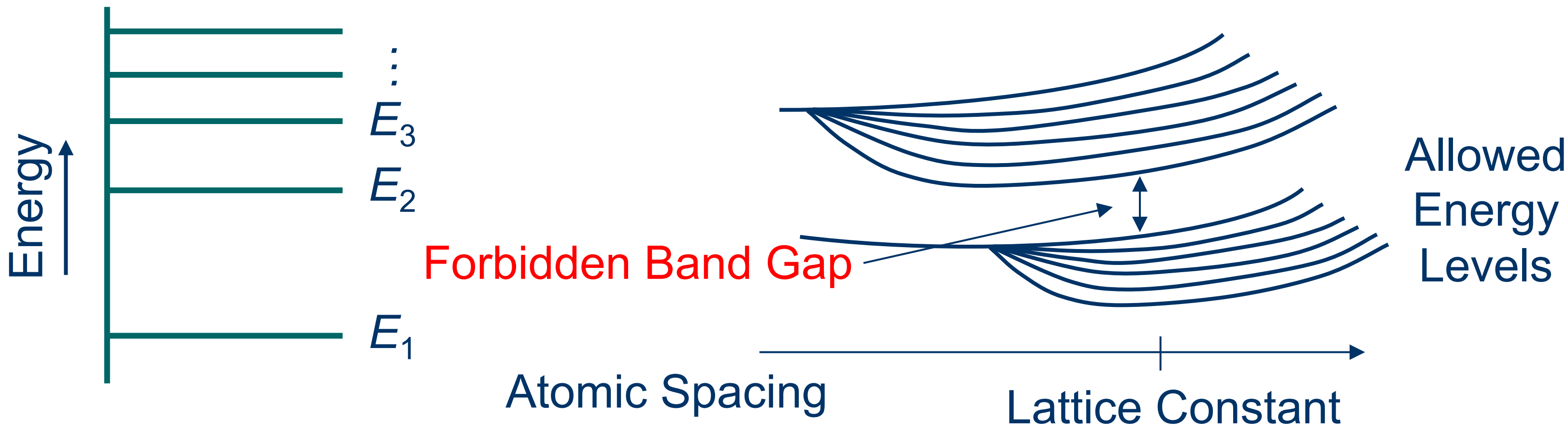
Periodic Table of Elements



The Diamond Structure



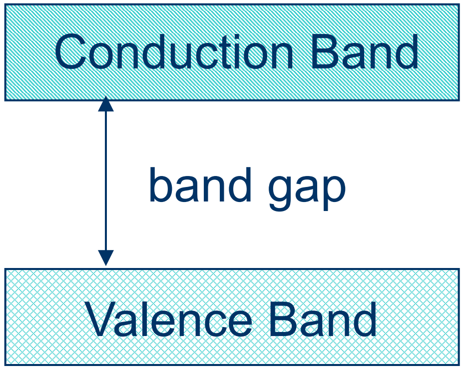
States of an Atom

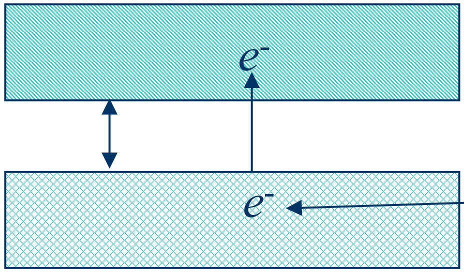


- Quantum Mechanics: The allowed energy levels for an atom are discrete (2 electrons can occupy a state since with opposite spin)
- When atoms are brought into close contact, these energy levels split
- If there are a large number of atoms, the discrete energy levels form a “continuous” band

Energy Band Diagram

- The gap between the conduction and valence band determines the conductive properties of the material
- Metal
  - Partially filled band
- Insulator
  - large band gap, ~ 8 eV
- Semiconductor
  - medium sized gap, ~ 1 eV

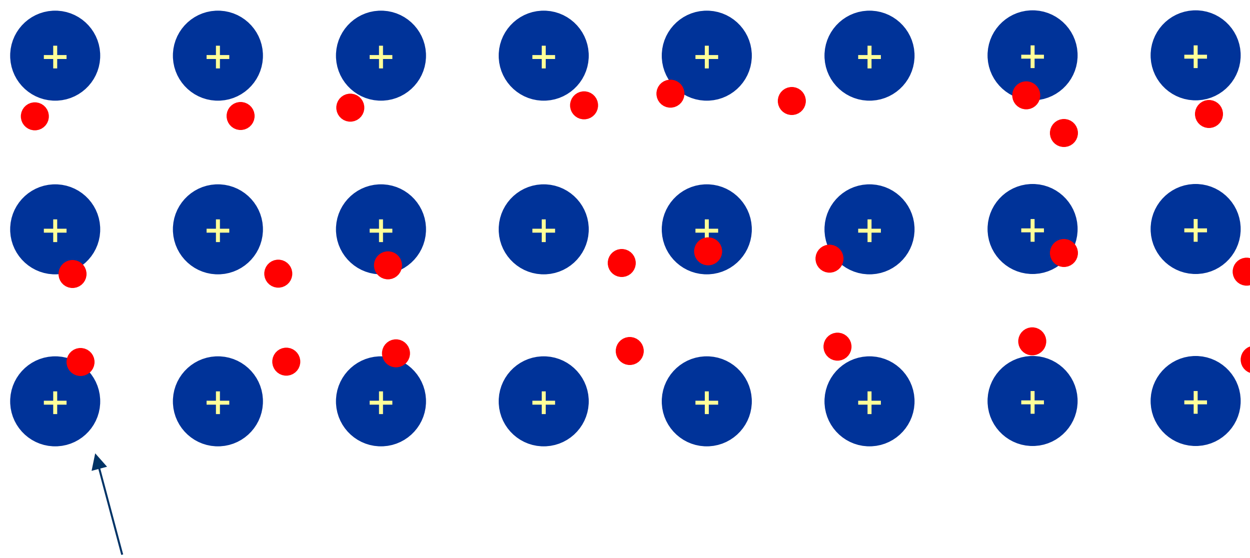




Electrons can gain energy from lattice (phonon) or photon to become "free"

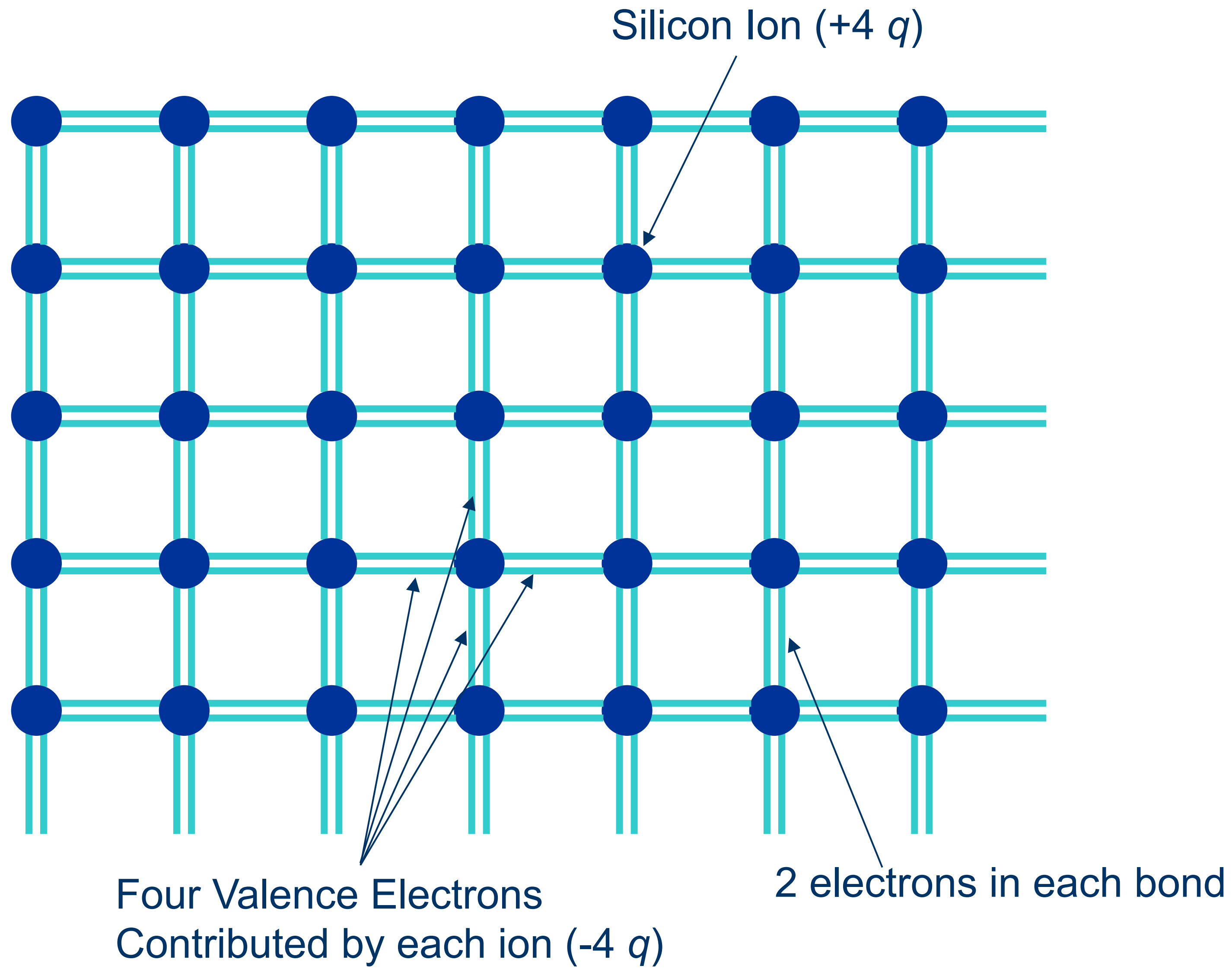
Model for Good Conductor

- The atoms are all ionized and a "sea" of electrons can wander about crystal:
- The electrons are the "glue" that holds the solid together
- Since they are "free", they respond to applied fields and give rise to conductions



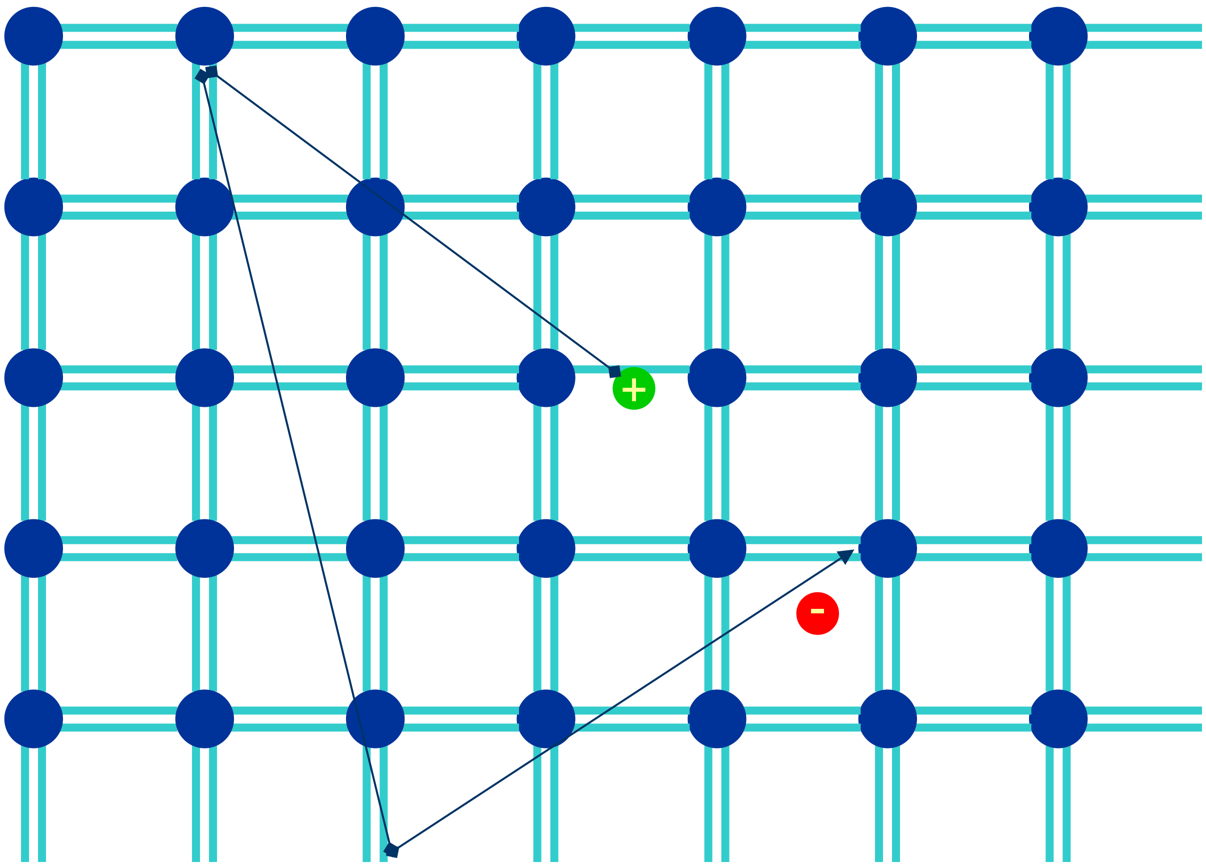
On time scale of electrons, lattice looks stationary…

Bond Model for Silicon (T=0K)

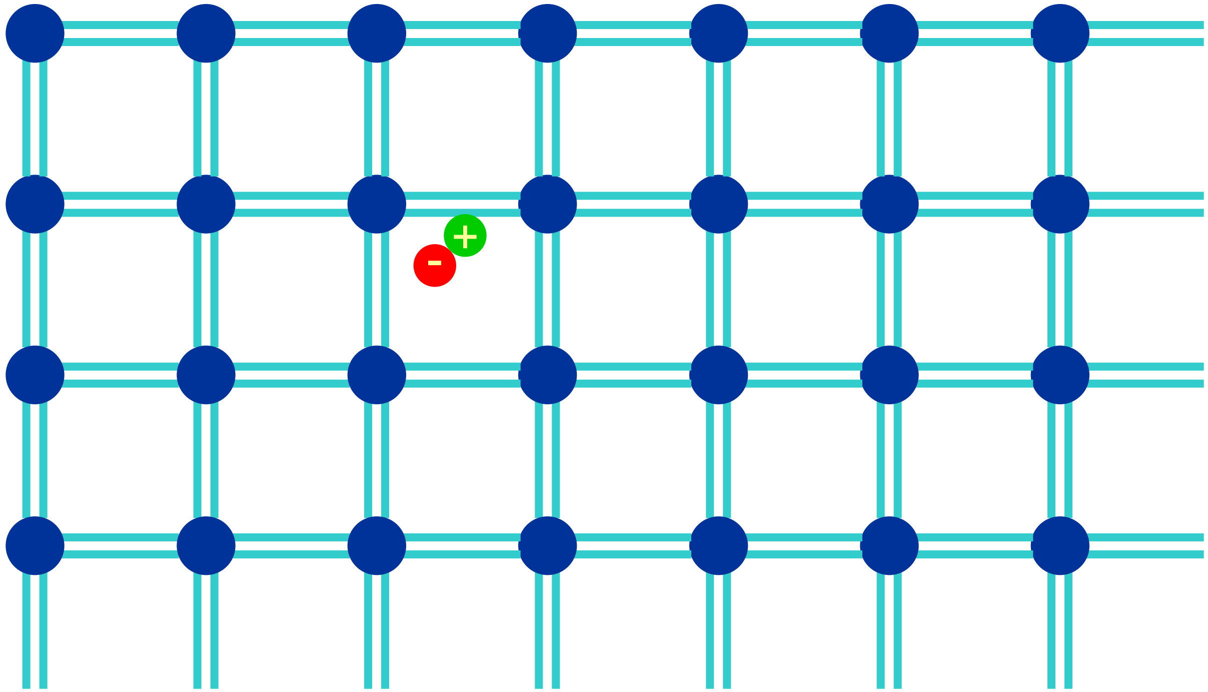


Bond Model for Silicon (T>0K)

- Some bond are broken: free electron
- Leave behind a positive ion or trap (a hole)



Holes?



- Notice that the vacancy (hole) left behind can be filled by a neighboring electron
- It looks like there is a positive charge traveling around!
- Treat holes as legitimate particles.

Yes, Holes!

- The hole represents the void after a bond is broken
- Since it is energetically favorable for nearby electrons to fill this void, the hole is quickly filled
- But this leaves a new void since it is more likely that a valence band electron fills the void (much larger density that conduction band electrons)
- The net motion of many electrons in the valence band can be equivalently represented as the motion of a hole

$$J_{vb} = \sum_{vb} (-q) v_i = \cancel{\sum_{Filled\ Band} (-q) v_i} - \sum_{Empty\ States} (-q) v_i$$

$$J_{vb} = - \sum_{Empty\ States} (-q) v_i = \sum_{Empty\ States} q v_i$$

More About Holes

- When a conduction band electron encounters a hole, the process is called *recombination*
- The electron and hole annihilate one another thus depleting the supply of carriers
- In thermal equilibrium, a generation process counterbalances to produce a steady stream of carriers

Intrinsic Carrier Concentration and Doping

Thermal Equilibrium (Pure Si)

- Balance between generation and recombination determines $n_o = p_o$
- Strong function of temperature: $T = 300\ ^{o}\mathrm{K}$

$$G = G_{th}(T) + \cancel{G_{opt}}$$

$$R = k(n \times p)$$

$$G = R$$

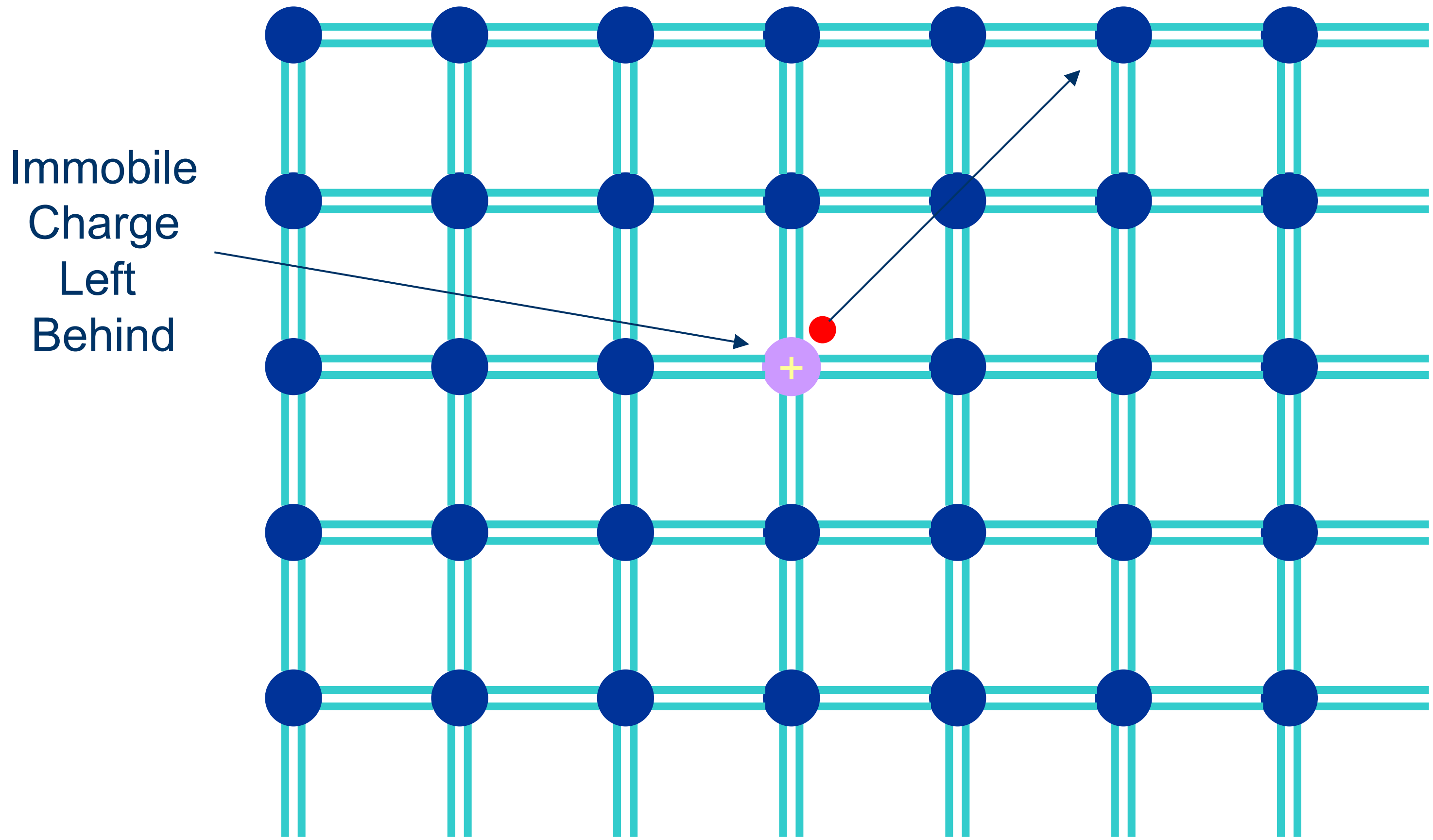
$$k(n \times p) = G_{th}(T)$$

$$n \times p = G_{th}(T)/k = n_i^{\,2}(T)$$

$$n_i(T) \cong 10^{10}\,\mathrm{cm}^{-3} \text{ at } 300\ \mathrm{K}$$

Doping with Group V Elements

- P, As (group 5): extra bonding electron … lost to crystal at room temperature



Donor Accounting

- Each ionized donor will contribute an extra "free" electron
- The material is charge neutral, so the total charge concentration must sum to zero:

$$\rho = \underbrace{-qn_0}_{\text{Free Electrons}} + \underbrace{qp_0}_{\text{Free Holes}} + \underbrace{qN_d}_{\text{Ions (Immobile)}} = 0$$

- By Mass-Action Law: $n \times p = n_i^2(T)$

$$-qn_0 + q\frac{n_i^2}{n_0} + qN_d = 0$$

$$-qn_0^2 + qn_i^2 + qN_d n_0 = 0$$

Donor Accounting (cont)

- Solve quadratic: $n_0^2 - N_d n_0 - n_i^2 = 0$

$$n_0 = \frac{N_d \pm \sqrt{N_d^2 + 4n_i^2}}{2}$$

- Only positive root is physically valid:

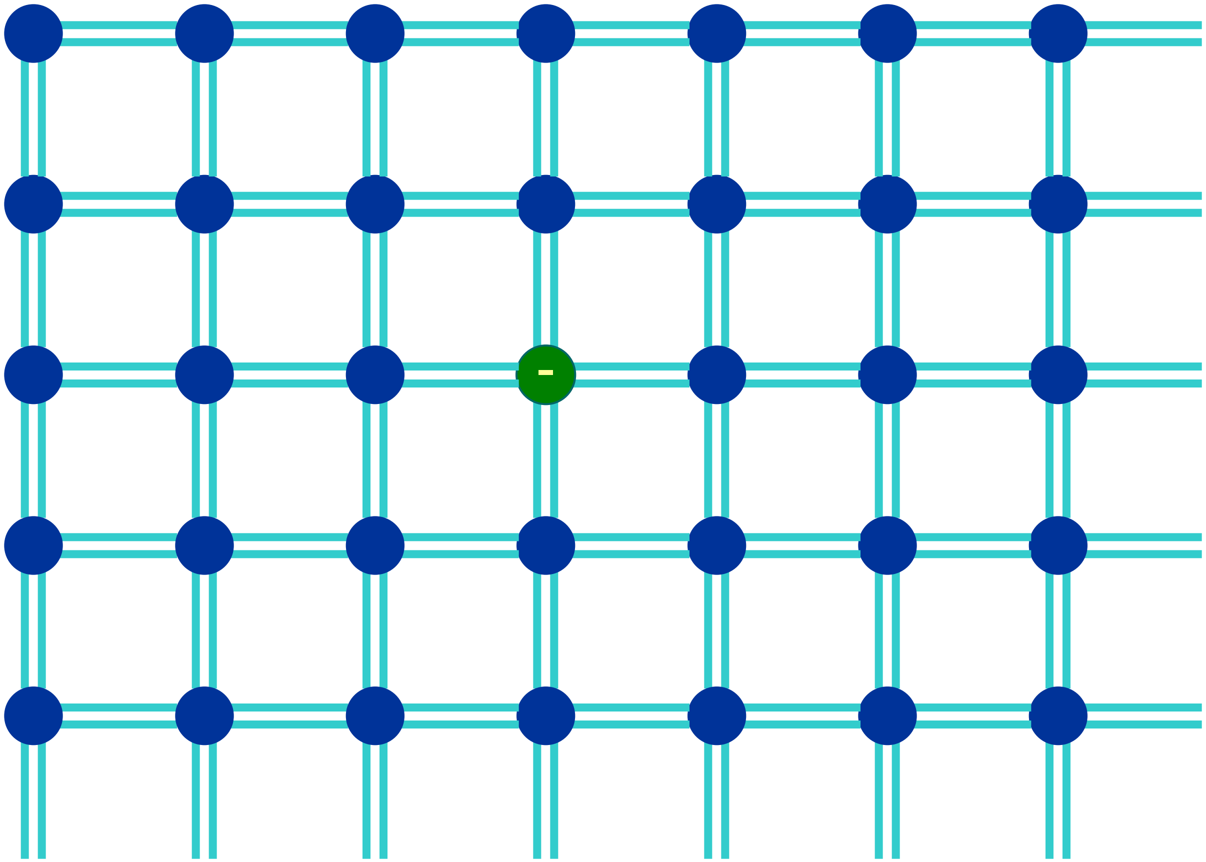
$$n_0 = \frac{N_d + \sqrt{N_d^2 + 4n_i^2}}{2}$$

- For most practical situations: $N_d >> n_i$

$$n_0 = \frac{N_d + N_d\sqrt{1 + 4\left(\frac{n_i}{N}\right)^2}}{2} \approx \frac{N_d}{2} + \frac{N_d}{2} = N_d$$

Doping with Group III Elements

- Boron: 3 bonding electrons → one bond is unsaturated
- Only free hole … negative ion is immobile!



Mass Action Law

- Balance between generation and recombination:

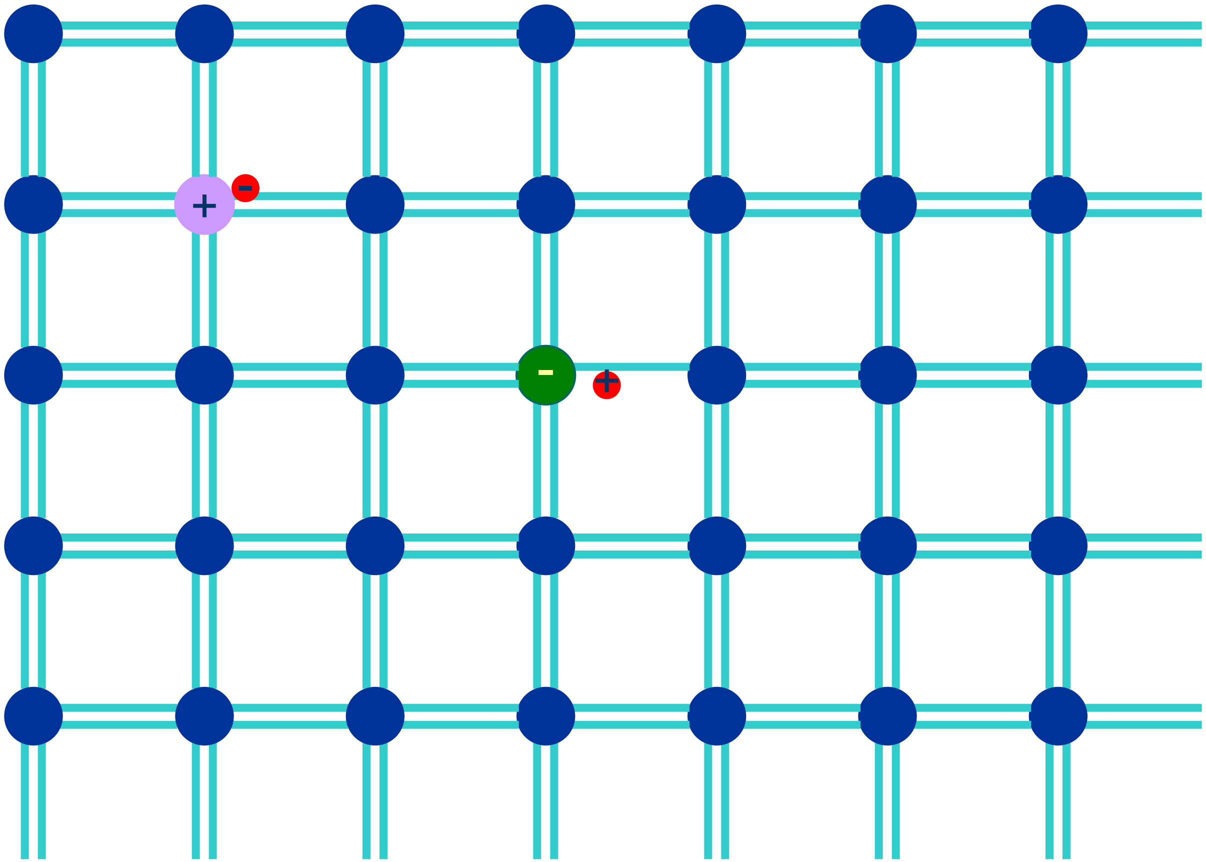
$$p_o \cdot n_o = n_i^2 \qquad (T = 300\,\mathrm{K},\ n_i = 10^{10}\,\mathrm{cm}^{-3})$$

- N-type case: $n_0 = N_d^+ \cong N_d$ $\qquad n_0 \cong \dfrac{n_i^2}{N_d}$

- P-type case: $p_0 = N_a^- \cong N_a$ $\qquad p_0 \cong \dfrac{n_i^2}{N_a}$

Compensation

- Dope with *both* donors and acceptors:
  - Create free electron and hole!



Compensation (cont.)

- More donors than acceptors: $N_d > N_a$

$$n_o = N_d - N_a >> n_i \qquad p_o = \frac{n_i^2}{N_d - N_a}$$

- More acceptors than donors: $N_a > N_d$

$$p_o = N_a - N_d >> n_i \qquad n_o = \frac{n_i^2}{N_a - N_d}$$

Drift Currents

Thermal Equilibrium

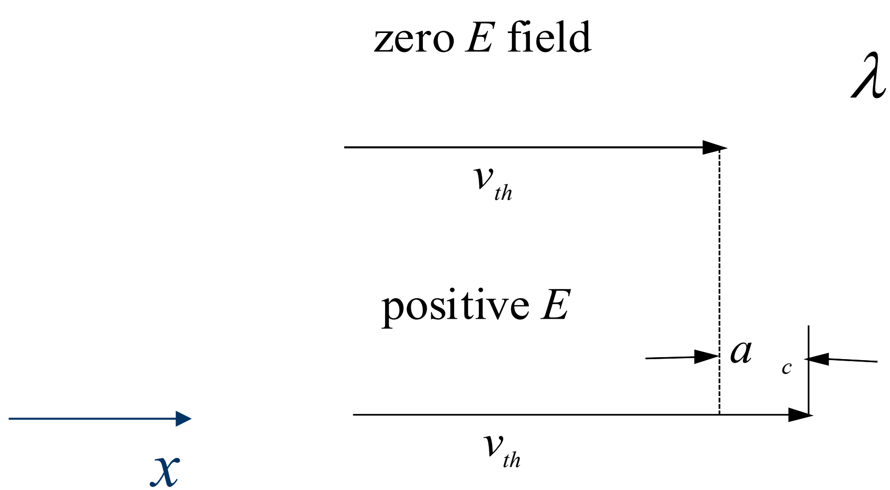
Rapid, random motion of holes and electrons at "thermal velocity" $v_{th} = 10^7$ cm/s with collisions every $\tau_c = 10^{-13}$ s.

$$\tfrac{1}{2} m_n^* v_{th}^2 = \tfrac{1}{2} kT$$

Apply an electric field E and charge carriers accelerate … for τ_c seconds

$$\lambda = v_{th}\tau_c$$

$$\lambda = 10^7 \text{cm} / s \times 10^{-13} s = 10^{-6} \text{cm}$$



(hole case)

Drift Velocity and Mobility

For holes:

$$v_{dr} = a \cdot \tau_c = \left(\frac{F_e}{m_p} \right) \tau_c = \left(\frac{qE}{m_p} \right) \tau_c = \left(\frac{q\tau_c}{m_p} \right) E$$

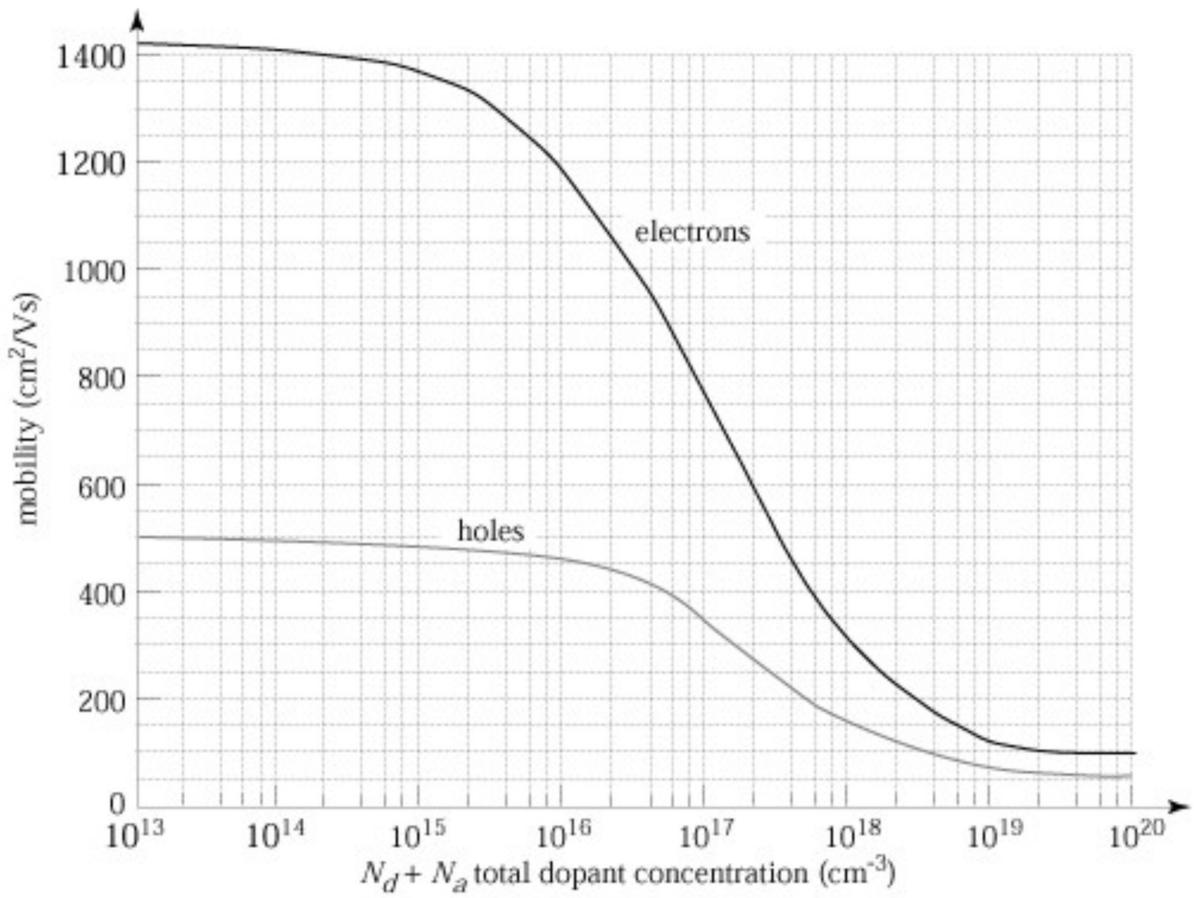
$$v_{dr} = \mu_p E$$

For electrons:

$$v_{dr} = a \cdot \tau_c = \left(\frac{F_e}{m_p} \right) \tau_c = \left(\frac{-qE}{m_p} \right) \tau_c = -\left(\frac{q\tau_c}{m_p} \right) E$$

$$v_{dr} = -\mu_n E$$

Mobility vs. Doping in Silicon at 300 °K

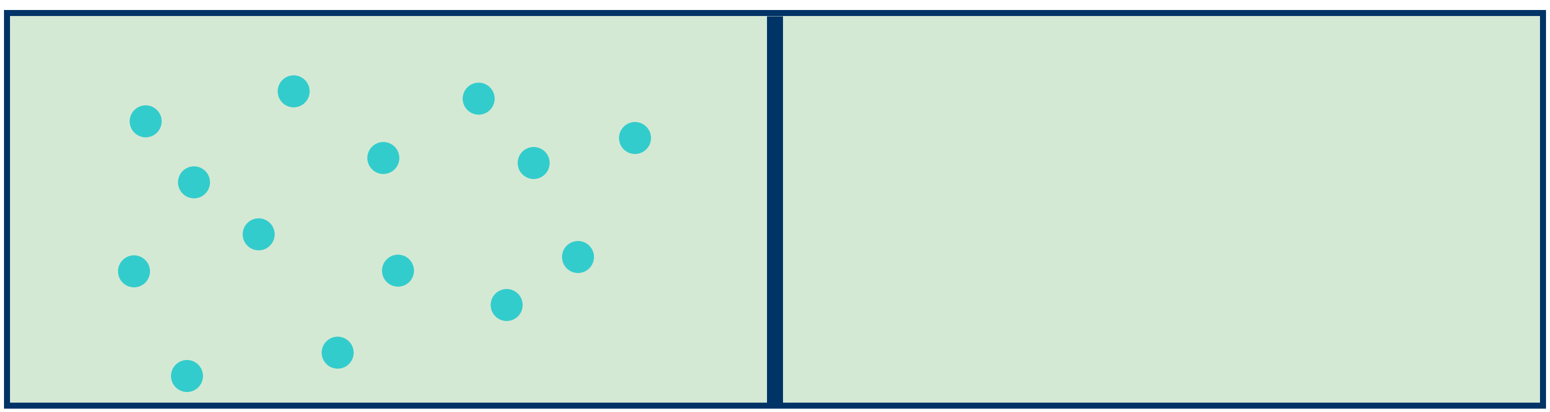


Typical values: $\mu_n = 1000$ $\mu_p = 400$

Diffusion Currents

Diffusion

- Diffusion occurs when there exists a concentration gradient
- In the figure below, imagine that we fill the left chamber with a gas at temperate T
- If we suddenly remove the divider, what happens?
- The gas will fill the entire volume of the new chamber. How does this occur?

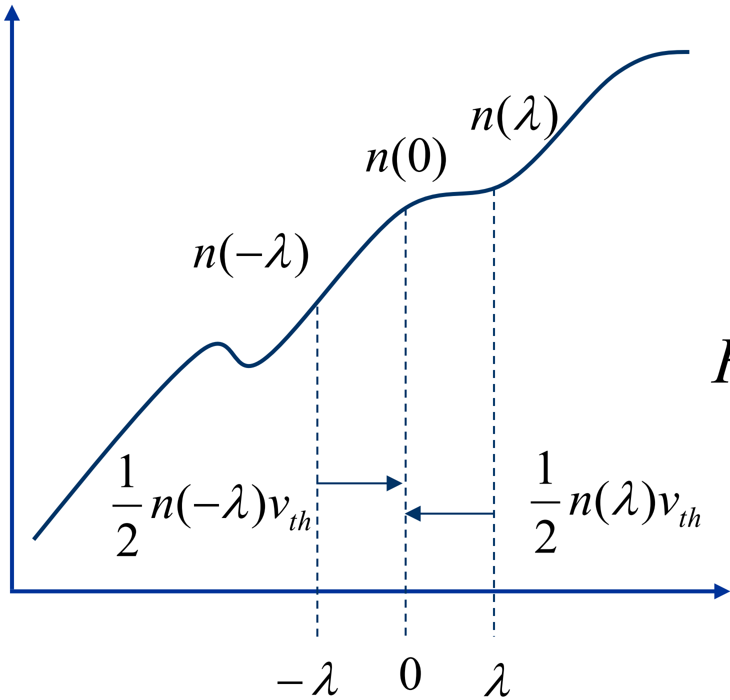


Diffusion (cont)

- The net motion of gas molecules to the right chamber was due to the concentration gradient
- If each particle moves on average left or right then eventually half will be in the right chamber
- If the molecules were charged (or electrons), then there would be a net current flow
- The diffusion current flows from high concentration to low concentration:

Diffusion Equations

- Assume that the mean free path is λ
- Find flux of carriers crossing x=0 plane



$$F = \frac{1}{2}v_{th}\left(n(-\lambda) - n(\lambda)\right)$$

$$F = \frac{1}{2}v_{th}\left(\left[n(0) - \lambda\frac{dn}{dx}\right] - \left[n(0) + \lambda\frac{dn}{dx}\right]\right)$$

$$F = -v_{th}\lambda\frac{dn}{dx}$$

$$J = -qF = qv_{th}\lambda\frac{dn}{dx}$$

Einstein Relation

- The thermal velocity is given by kT

$$\tfrac{1}{2} m_n^* v_{th}^2 = \tfrac{1}{2} kT$$

Mean Free Time

$$\lambda = v_{th} \tau_c$$

$$v_{th}\lambda = v_{th}^2 \tau_c = kT \frac{\tau_c}{m_n^*} = \frac{kT}{q} \frac{q\tau_c}{m_n^*}$$

$$J = q v_{th} \lambda \frac{dn}{dx} = q\left(\frac{kT}{q}\mu_n\right)\frac{dn}{dx}$$

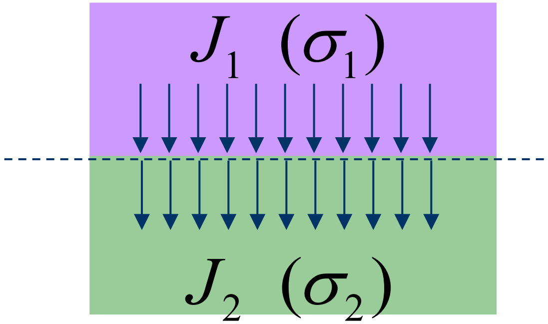
$$D_n = \left(\frac{kT}{q}\right)\mu_n$$

Total Current and Boundary Conditions

- When both drift and diffusion are present, the total current is given by the sum:

$$J = J_{drift} + J_{diff} = q\mu_n nE + qD_n \frac{dn}{dx}$$

- In resistors, the carrier is approximately uniform and the second term is nearly zero
- For currents flowing uniformly through an interface (no charge accumulation), the field is discontinous



$$J_1 = J_2$$

$$\sigma_1 E_1 = \sigma_2 E_2$$

$$\frac{E_1}{E_2} = \frac{\sigma_2}{\sigma_1}$$

Reference

- Edward Purcell, *Electricity and Magnetism*, Berkeley Physics Course Volume 2 (2nd Edition), pages 133-142.