Module 1-3: Non-Ideal Amps and Op-Amps

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Feedback Control

- Feedback is a universal way to design systems
  - Thermostat in your house
  - Cruise control / keep-in-lane technology (driver assist)
  - Walking / standing in humans

- Speed control example:
  - Set desired speed of car: v_{des}
  - Measure the current speed of car: v_{car}
  - Find the difference: $v_{diff} = v_{des} - v_{car}$ (this is also called the error signal)
  - Proportional scheme: Adjust car speed (accelerate) based on $K_p\, v_{diff}$
  - PID control with dynamics: Use proportional, integral, and derivative of v_{car} to control speed

Negative Feedback Block Diagram

- To find the transfer function, note that the error signal is a function of the input and output:

$$s_{err} = s_{in} - f \cdot s_{out}$$

$$s_{out} = G \cdot s_{err} = G \cdot (s_{in} - f s_{out})$$

Closed-Loop Transfer Function

- Solve :

$$s_{out}(1+Gf) = G \cdot s_{in}$$

$$G_{closed} = \frac{s_{out}}{s_{in}} = \frac{G}{1+Gf}$$

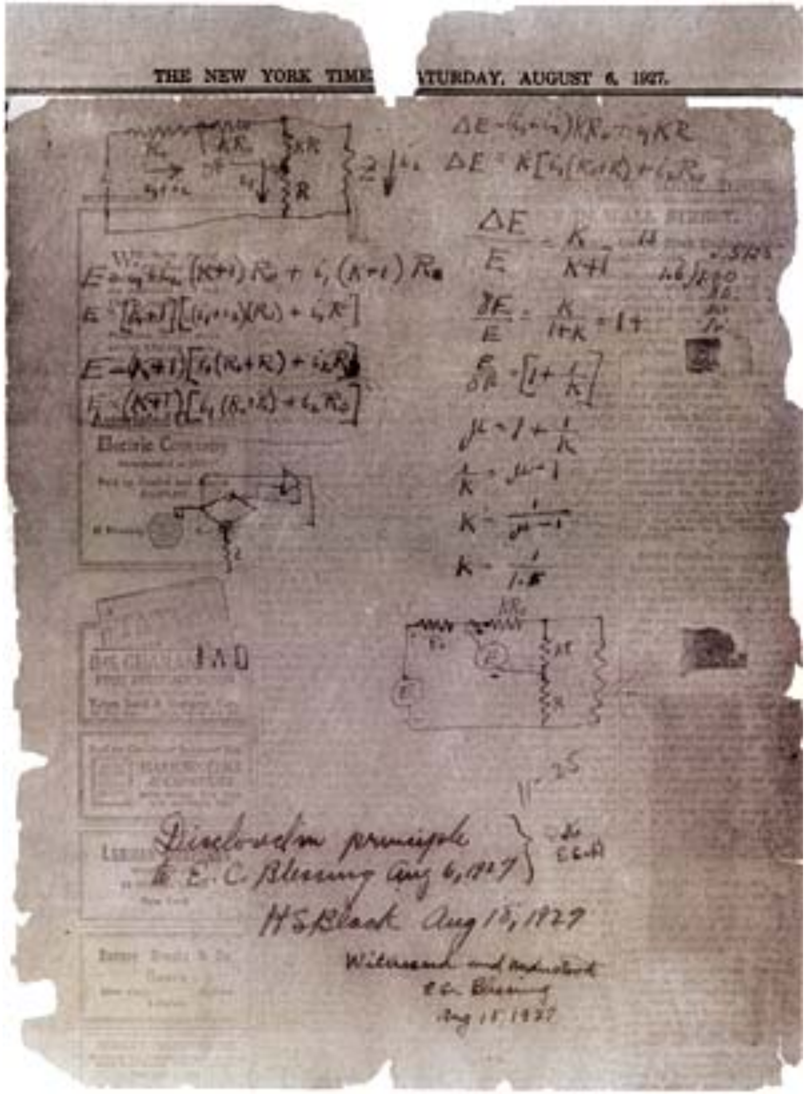
- For very large gain G, such that $Gf >> 1$, we have

$$G_{closed} \approx \frac{G}{Gf} = \frac{1}{f}$$

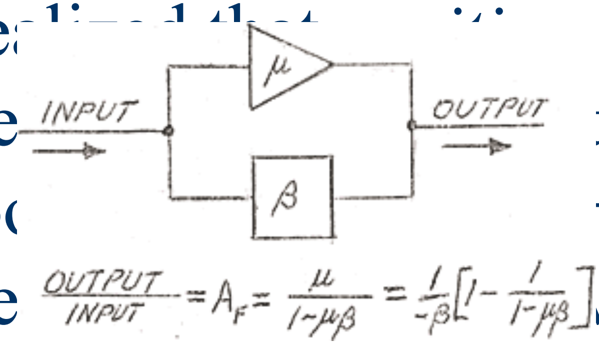
Electronic Feedback

- We have already seen electronic feedback in op-amps
- Example: Non-inverting amplifier
  - Resistor divider samples output voltage
  - Error signal formed at input of op-amp
  - Op-amp output is a gained version of the error signal
- Strangeness:
  - Op-amp gain is very large (ideally infinity)
  - Error signal is driven to zero
    - All practical op-amps have finite gain, so error signal is nearly zero

History



- Invention of electronic negative feedback
- Inventor Harold Black was on a ferry ride to Bell Labs (1927) thinking about how to increase the gain and improve linearity of of vacuum tube amplifiers.
- Rea fee o boc *ative* fee ve the linearity…



Precision Analog

- Feedback allows us to use a really “crappy” open-loop op-amp and get good performance
  - When gain is sufficiently high, the gain is determined by the feedback network, not the open-loop gain
  - Open loop gain can vary over temperature, process, it can age, it can be highly non-linear ... In the end, if it’s high enough, it does not play a role !

Benefits of Feedback

- Closed-loop gain depends on passive feedback network
  - Can set gain precisely
- Linearization
  - Op-amp transfer function is almost perfectly linear, despite using a very non-linear core amplifier
- Bandwidth enhancement (see next section)
- Interference rejection (loop can correct for unwanted signals that are injected into the signal path)

Loop Gain

- Recall that the closed-loop transfer function is given by

$$G_{closed} = \frac{G}{1+Gf} = \frac{G}{1+T}$$

- For a precise transfer function, the key to feedback is to realize sufficiently high loop gain $T = Gf \gg 1$

Noise Rejection

- Another benefit of high loop gain is interference rejection. Imagine an unwanted signal couples into the loop as show

$$s_{out} = Gs_{err} + s_{noise} = G(s_{in} - fs_{out}) + s_{noise}$$

- The noise is rejected like 1/(1+*T*). Any unwanted signal, including distortion, is rejected by the loop

$$s_{out} = \frac{G}{1+T} s_{in} + \frac{1}{1+T} s_{noise}$$

Positive Feedback

- Up to now we have been considering a *negative feedback* system whereby the output is *subtracted* from the input
- If we were to add the output to the input, the system would be a *positive feedback* system, which is also useful but not what we intended.
- Positive feedback systems tend to “rail out”, in other words they are regenerative

Stability

- Any real amplifier will introduce some phase shift when the input frequency increases. For example, a single-pole system has the following form
- As the frequency increases, the phase of the output signal lags the input, asymptotically up to 90°.
- When the system has more poles, the phase shift can reach 180°. What happens to

$$G(j\omega) = \frac{1}{1 + j\omega\tau}$$

$$\angle[G(j\omega)] = -\arctan(j\omega\tau)$$

Instability

- If the loop gain has a phase shift of 180°, then our negative feedback system appears like a positive feedback system at frequency $T = G(j\omega_x) f = -1$
- Since there is always noise and disturbances in the system at this frequency in the system, this noise is regenerated and potentially can cause problems if $|T| > 1$.
- The condition for stability is then to ensure that when loop gain is unity, the phase of T should be less than 180°
- The *phase margin* is a measure of stability. A good design should have 60° phase margin or more.

Oscillation

- The condition $T(j\omega) = -1$ is in fact how we build oscillators, which are inherently unstable
- If the circuit has $T = -1$ at a particular frequency, then the gain at that frequency is theoretically infinite
  - Poles are imaginary axis
- Any noise or disturbance can leads to a strong oscillation at this particular frequency
  - Take EECS 142 to understand this in more detail

Op-Amp Circuit as a Feedback System

Practical Op-Amps

- Linear Imperfections:
  - Finite open-loop gain ($A_0 < \infty$)
  - Finite input resistance ($R_i < \infty$)
  - Non-zero output resistance ($R_o > 0$)
  - Finite bandwidth / Gain-BW Trade-Off
- Other (non-linear) imperfections:
  - Slew rate limitations
  - Finite swing
  - Offset voltage
  - Input bias and offset currents
  - Noise and distortion

Simple Model of Amplifier

- Input capacitance and output capacitance are added
- Any amplifier has input capacitance due to transistors and packaging / board parasitics
- Output capacitance is usually dominated by the load
  - Driving cables or a board trace
  - Intrinsic capacitance of actuator

Transfer Function

- Using the concept of impedance, it's easy to derive the transfer function

Operational Transconductance Amp

- Also known as an "OTA"
  - If we "chop off" the output stage of an op-amp, we get an OTA
- An OTA is essentially a G_m amplifier. It has a current output, so if we want to drive a load resistor, we need an output stage (buffer)
- Many op-amps are internally constructed from an OTA + buffer

Op-Amp Model

- The following model closely resembles the insides of an op-amp.
- The input OTA stage drives a high Z node to generate a very large voltage gain.
- The output buffer then can drive a low impedance load and preserve the high voltage gain

Op-Amp Gain / Bandwidth

- The dominant frequency response of the op-amp is due to the time constant formed at the high-Z node

$$G = G_m R_o$$

$$\omega_{-3dB} = \frac{1}{R_o C_x}$$

- An interesting observation is that the gain-bandwidth product depends on G_m and C_x only

$$G \times \omega_{-3dB} = \frac{G_m}{C_x}$$

Preview: Driving Capacitive Loads

- In many situations, the load is capacitor rather than a resistor
- For such cases, we can directly use an OTA (rather than a full op-amp) and the gain / bandwidth product are now determined by the load capacitance

$$G = G_m R_o$$

$$\omega_{-3dB} = \frac{1}{R_o(C_x + C_L)}$$

OTA Power Consumption

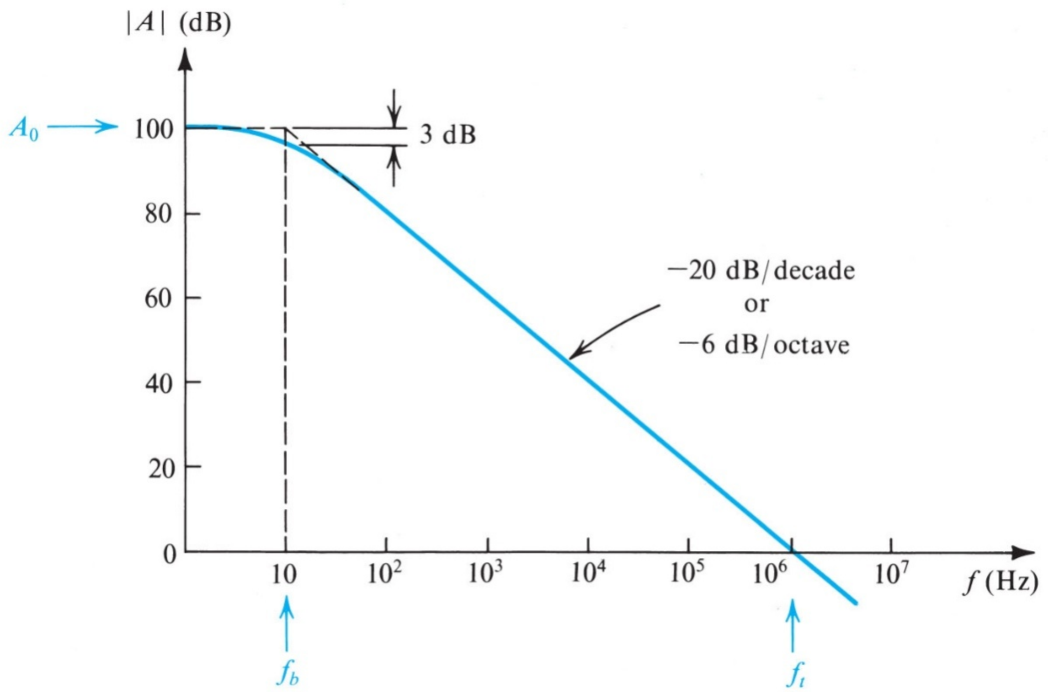
- For a fixed load, the current consumption of the OTA is fixed by the gain/bandwidth requirement, assuming load dominates

$$C_L \gg C_x$$

- G_m scales with current, so driving a larger capacitance requires more power

Gain/Bandwidth Trade-off

Open-Loop Frequency Response



$$A(j\omega) = \frac{A_0}{1 + j\omega / \omega_b}$$

A_0 : dc gain

ω_b : 3dB frequency

$\omega_t = A_0\omega_b$: unity-gain bandwidth

(or "gain-bandwidth product")

For high frequency, $\omega >> \omega_b$

$$A(j\omega) = \frac{\omega_t}{j\omega}$$

Single pole response with a dominant pole at ω_b

Bandwidth Extension

- Suppose the core amplifier is single pole with bandwidth:

$$G(j\omega) = \frac{G_0}{1 + j\omega\tau}$$

$$G_{fb}(j\omega) = \frac{G(j\omega)}{1 + G(j\omega)f} = \frac{\dfrac{G_0}{1 + j\omega\tau}}{1 + \dfrac{G_0}{1 + j\omega\tau} f}$$

- When used feedback, the overall transfer function is given by

$$G_{fb}(j\omega) = \frac{G_0}{1 + j\omega\tau + G_0 f}$$

$$G_{fb}(s) = \frac{\dfrac{G_0}{1 + G_0 f}}{1 + j\omega \dfrac{\tau}{1 + G_0 f}} = \frac{G_{closed-loop}(0)}{1 + j\omega \dfrac{\tau}{1 + T}}$$

Gain / Bandwidth Product in Feedback

- Even though the bandwidth expanded by (1+T), the gain drops by the same factor. So overall the gain-bandwidth (GBW) product is constant
- The GBW product depends only the the G_m of the op-amp and the C_x internal capacitance (or load in the case of an OTA)

$$GBW = G \times BW = \frac{G_0}{1+T} \times (1+T) \frac{1}{R_o C_x}$$

$$GBW = G_0 \times R_o C_x = G_m R_o \times \frac{1}{R_o C_x}$$

$$GBW = \frac{G_m}{C_x}$$

Unity Gain Frequency

- The GBW product is also known as the unity gain frequency.
- To see this, consider the frequency at which the gain drops to unity

$$|G| = \left| \frac{G_0}{1 + j\omega_u \tau} \right| = 1 \qquad \frac{G_0}{\sqrt{1 + \omega_u^2 \tau^2}} = 1$$

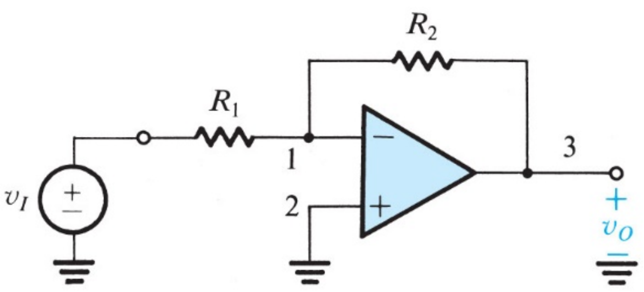
$$G_0^2 = 1 + \omega_u^2 \tau^2 \qquad \omega_u^2 = (G_0^2 - 1)/\tau^2 \approx G_0^2/\tau^2$$

$$\omega_u = G_0/\tau = \frac{G_m R_o}{C_x R_o} = \frac{G_m}{C_x}$$

Unity Gain Feedback Amplifier

- An amplifier that has a feedback factor f=1, such as a unity gain buffer, has the full GBW product frequency range

Closed-Loop Op Amp



Steps to find frequency response of closed-loop amplifiers:

1. Find the transfer function with finite open-loop gain. For example, for inverting amplifier:

$$G = \frac{v_o}{v_I} = \left(-\frac{R_2}{R_1}\right)\frac{1}{1+\dfrac{(1+R_2/R_1)}{A}}$$

2. Substitute A with $A(j\omega) = \dfrac{A_0}{1+j\omega/\omega_b}$

3. Simplify the expression

$$G(\omega) = \left(-\frac{R_2}{R_1}\right)\frac{1}{1+(1+R_2/R_1)\dfrac{1+j\omega/\omega_b}{A_0}}$$

$$= \left(-\frac{R_2}{R_1}\right)\frac{1}{1+\dfrac{(1+R_2/R_1)}{A_0}+\dfrac{j\omega}{\left(\dfrac{A_0\omega_b}{1+R_2/R_1}\right)}}$$

Frequency Response of Closed-Loop Inverting Amplifier Example

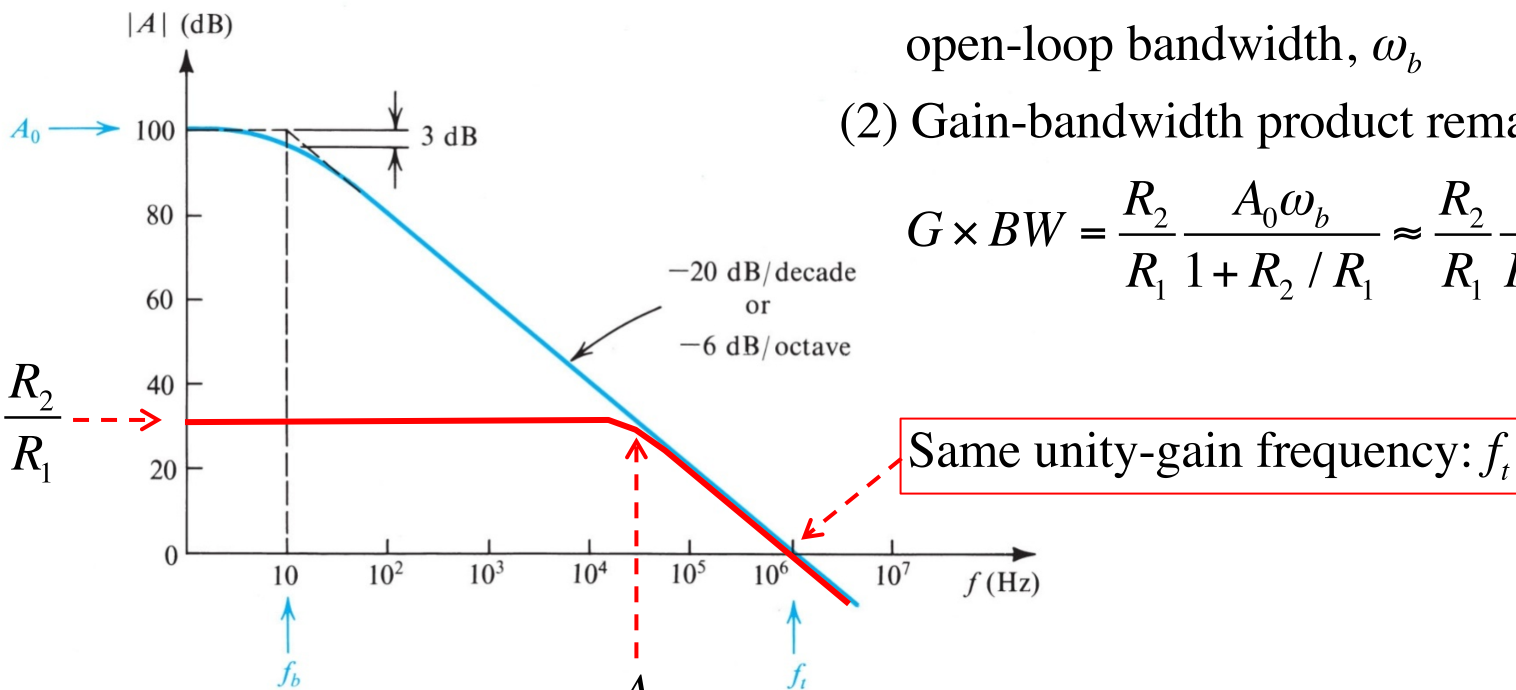
$$G(\omega) \approx \left(-\frac{R_2}{R_1}\right)\frac{1}{1+\dfrac{j\omega}{\omega_{3dB}}} \qquad \text{where } \omega_{3dB} = \frac{A_0\omega_b}{1+R_2/R_1}$$

Note:

(1) 3-dB frequency is higher than open-loop bandwidth, ω_b

(2) Gain-bandwidth product remains unchanged:

$$G \times BW = \frac{R_2}{R_1}\frac{A_0\omega_b}{1+R_2/R_1} \approx \frac{R_2}{R_1}\frac{A_0\omega_b}{R_2/R_1} = A_0\omega_b = \omega_t$$



$$f_{3dB} \approx \frac{A_0}{R_2/R_1} f_b$$

Non-Dominant Poles

- As we have seen, poles in the system tend to make an amplifier less stable. A single pole cannot do harm since it has a maximum phase shift of 90°
- A second pole in the system is not affected by feedback (prove this) and it will add phase shift as the frequency approaches this second pole
- For this reason, non-dominant poles should be at a much higher frequency than the unity-gain frequency

Positive Feedback

- Positive Feedback is also useful
- We can create a comparator circuit with *hysteresis*
- Also, as long as $T < 1$, we can get stable gain … instead of reducing the gain (negative feedback), positive feedback enhances the gain.
- In theory we can boost the gain to any desired level simply by making T close to unity:

$$T = 1 - \epsilon$$

- ε is a very small number
  - In practice if the gain varies over process / temperature / voltage, then the circuit can go stable and oscillate
  - Positive feedback also has a narrow-banding effect

Back to Circuit Model

- Here's the equivalent circuit for an amplifier with feedback

Circuit Interpretation

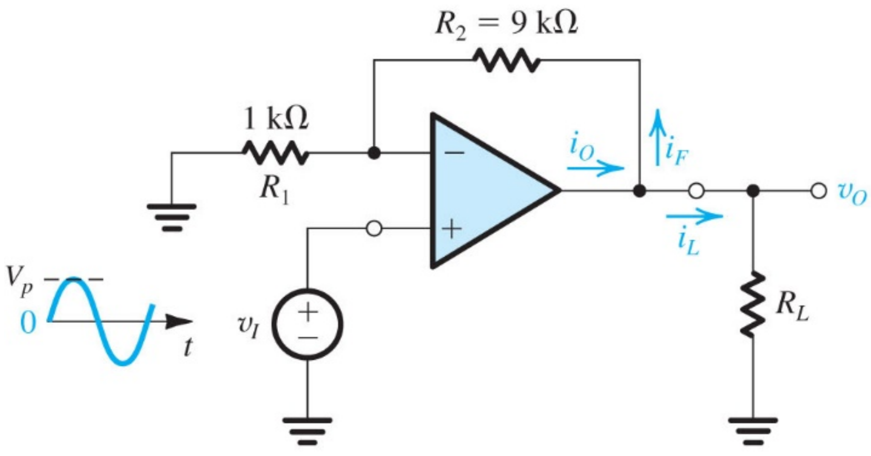
- Here we see the action of the feedback is to lower the impedance seen by the G_m by the loop gain, which expands the bandwidth by the same factor

(cont)

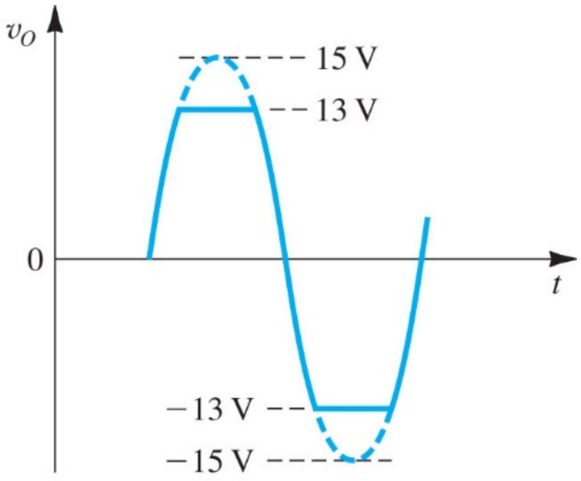
Op-Amp Non-Linearities

Output Saturation

- The output voltage swing is limited by
    1. Saturation voltage (usually a volt or two lower than power supply voltage)
    2. Maximum output current (in case of small load resistance)
- Output waveform appears to be "clipped" when either condition happens
- **Output power is limited**



(a)



(b)

Slew Rate

Amplifier output is limited by "slew rate": maxium rate of change possible at output

$$SR = \left.\frac{dv_o}{dt}\right|_{\max}$$

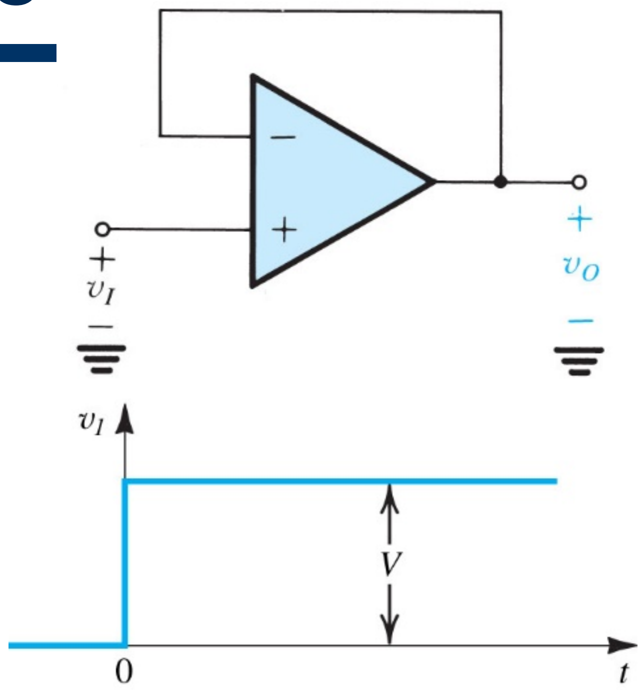
SR is specified in datasheet in V/μs.

Note

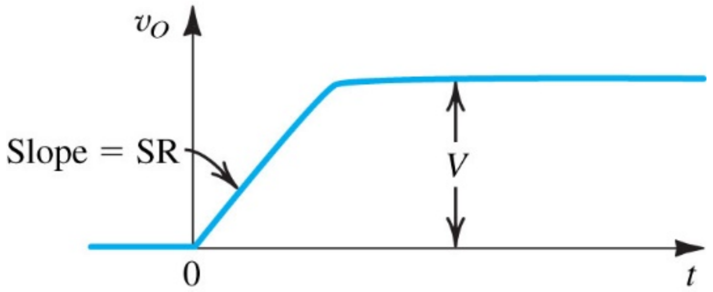
SR limit is different from bandwidth limit:

• Limited bandwidth is a linear phenomenon, it does not change the shape of input sinusoid

• SR limitation can cause nonlinear distortion to input sinusoidal signal



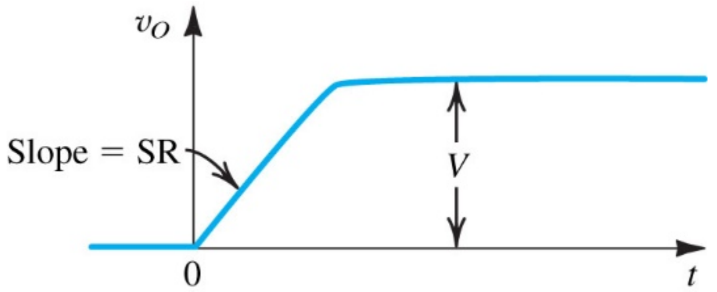
Output not able to follow input; Slope limited by SR



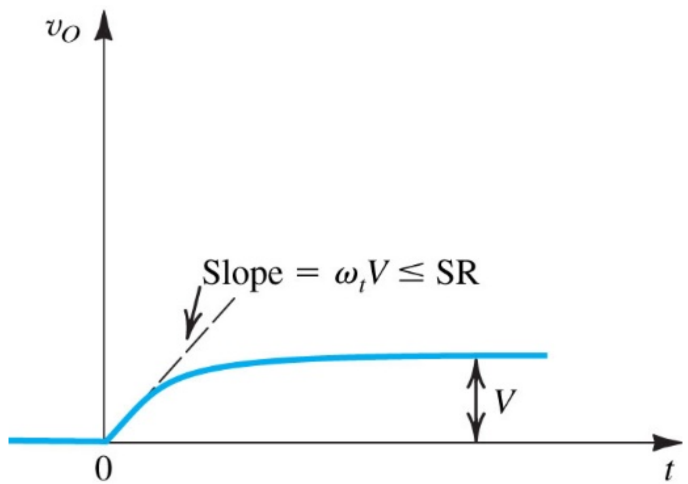
(c)

Slew Rate vs. Bandwidth Limits

For step function input waveform, both SR and bandwidth limits cause the output to rise with a finite slope, but there is an important difference:

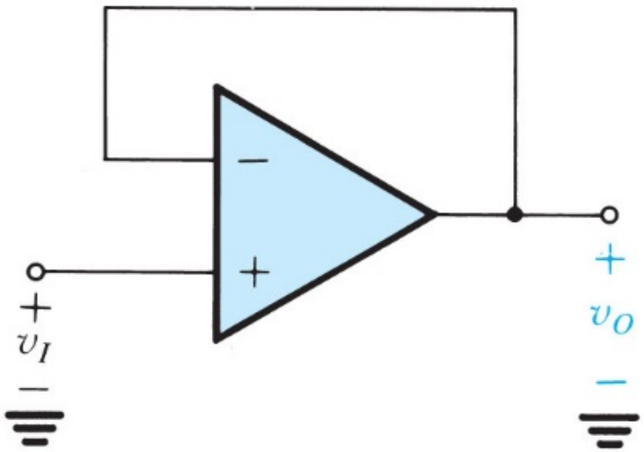


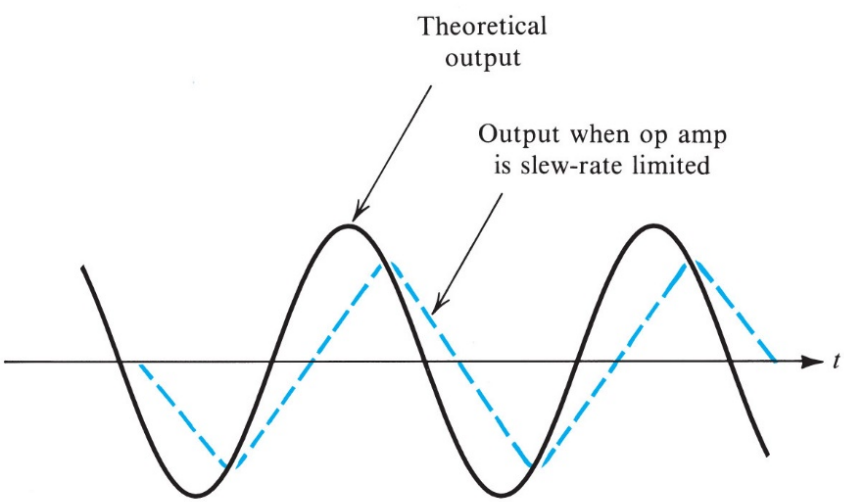
Slew rate limited output:
Slope = SR



Bandwidth limited output:
Slope = $\omega_t V <$ SR
(V is the steady state output voltage)

Full-Power Bandwidth





For sinusoidal input to unity-gain follower:

$$v_I = V_i \sin \omega t$$

Rate of change:

$$\frac{dv_I}{dt} = V_i \omega \cos \omega t \leq \text{ SR}$$

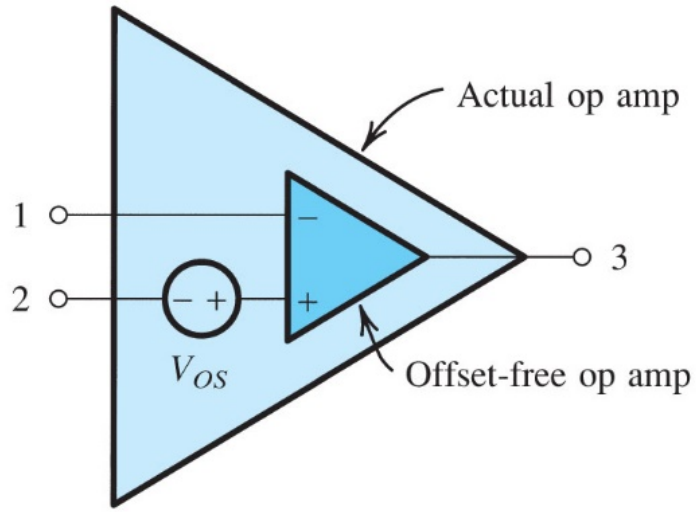
Full-power bandwidth:

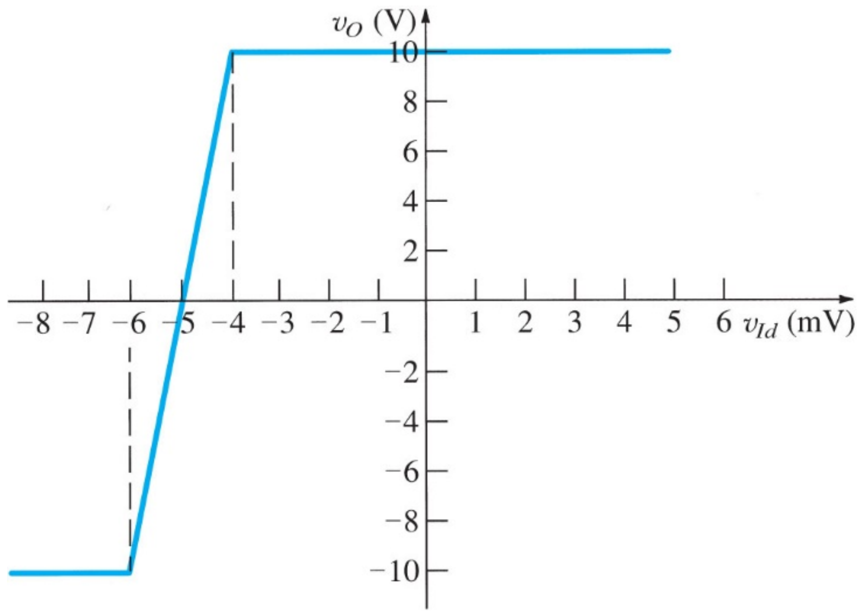
The frequency at which SR-limited distortion starts to occur for an output sinusoid with maximum rated output voltage, $V_{o\max}$,

$$\omega_M V_{o\max} = SR$$

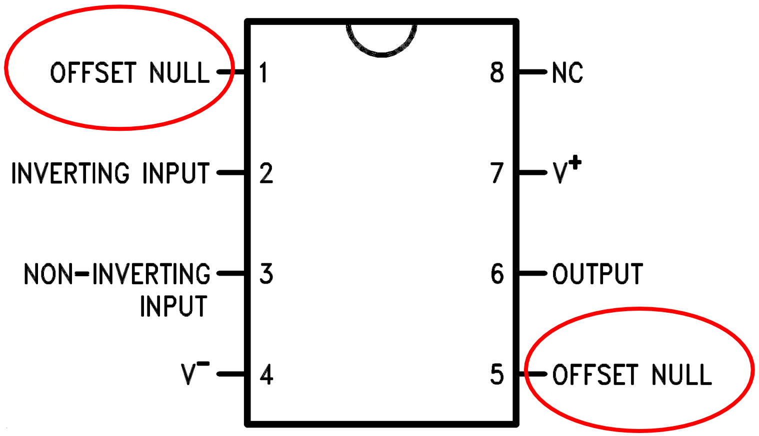
$$f_M = \frac{SR}{2\pi V_{o\max}}$$

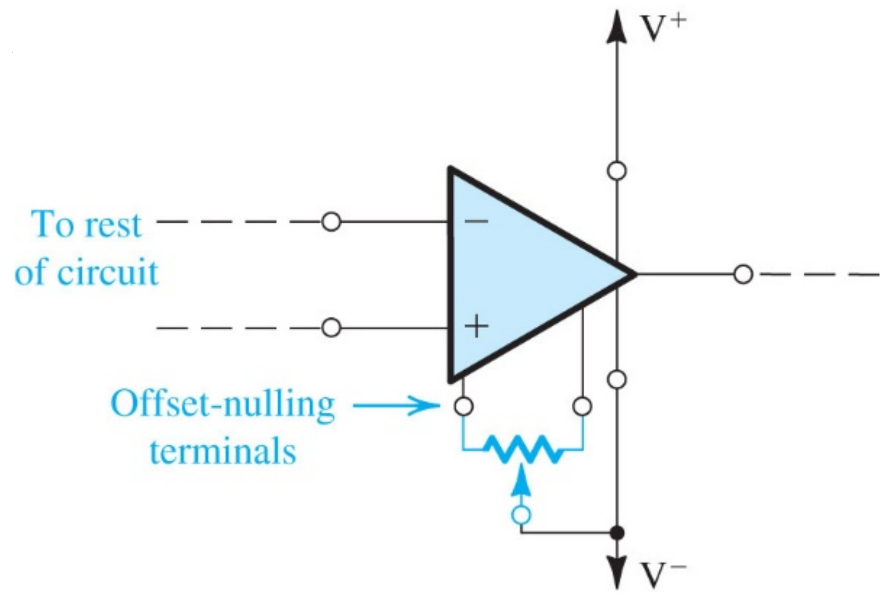
Offset Voltage

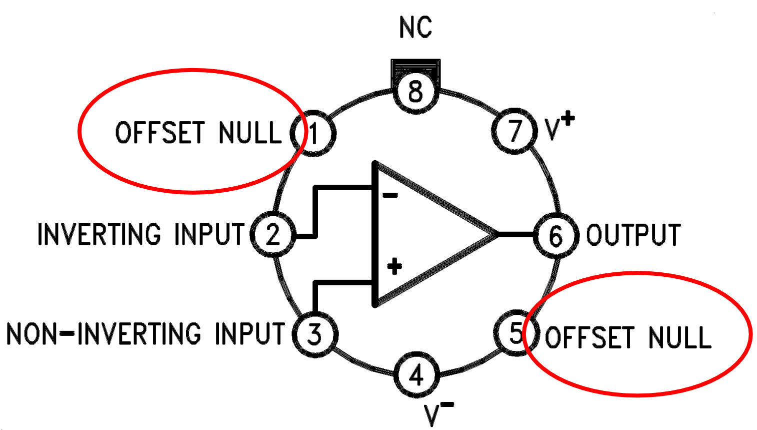




Trimming of Offset Voltage

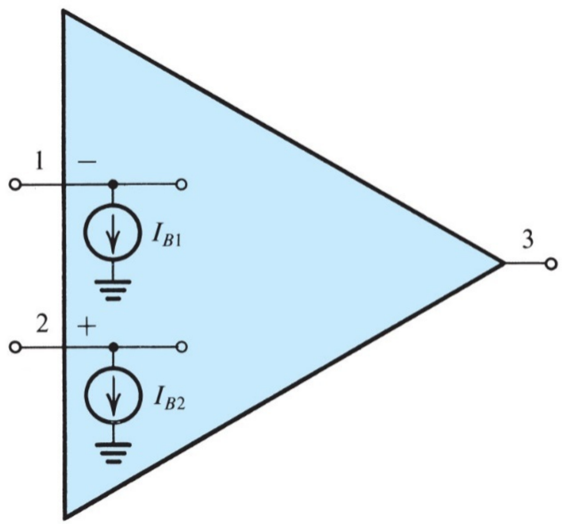






The output dc offset voltage of an op amp can be trimmed to zero by connecting a potentiometer to the two offset-nulling terminals. The wiper of the potentiometer is connected to the negative supply of the op amp.

Input Bias Currents and Offset Currents



The input terminals need to be supplied with bias currents, I_{B1} and I_{B2}, for Op Amp to function. (This will become clear towards the end of the semester).

Input bias current: $I_B = \dfrac{I_{B1} + I_{B2}}{2}$

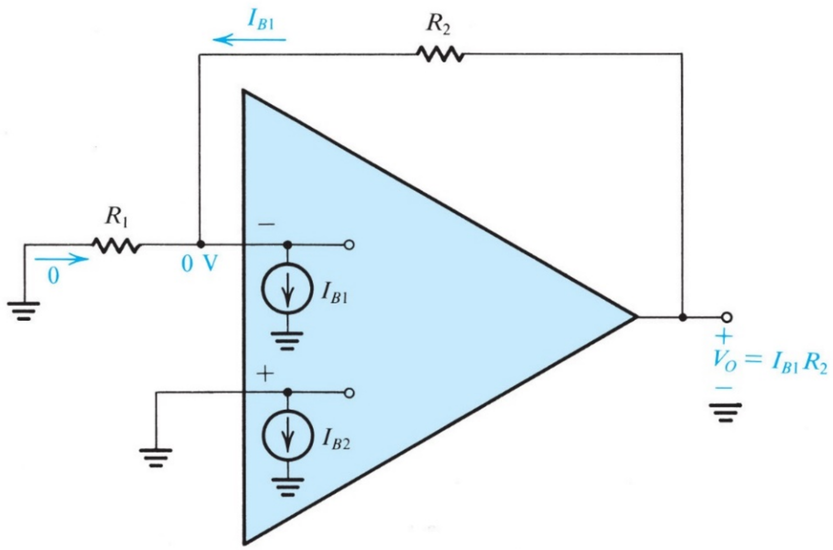
Input offset current: $I_{OS} = \left| I_{B1} - I_{B2} \right|$

Typical bipolar transistor Op amps:

$I_B \sim 100 \text{ nA}$

$I_{OS} \sim 10 \text{ nA}$

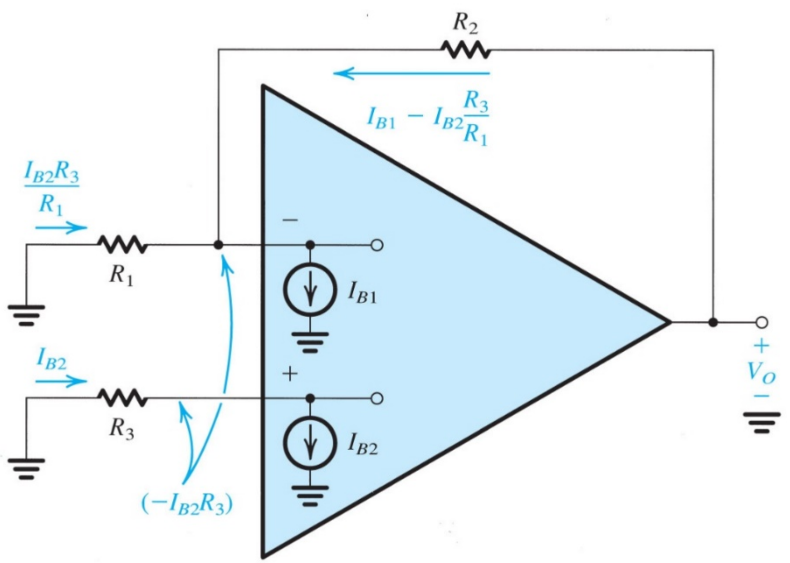
Effect of Input Bias Current



In the absence of input voltage, the output should be zero for ideal Op Amp. However, with non-zero I_B,

$$V_O = I_{B1} R_2 \approx I_B R_2$$

Reducing the Effect of Input Bias Currents



$$V_O = -I_{B2}R_3 + R_2\left(I_{B1} - \frac{I_{B2}R_3}{R_1}\right)$$

First approximate $I_{B1} = I_{B2} = I_B$

$$V_O = -I_B R_3 + I_B R_2 - \frac{I_B R_3}{R_1}R_2 = I_B\left(R_2 - R_3\left(1 + \frac{R_2}{R_1}\right)\right)$$

Choose $R_3 = \dfrac{R_2}{1+\dfrac{R_2}{R_1}}$, $V_O = 0$

Now consider $I_{B1} = I_B + \dfrac{I_{OS}}{2}$

and $I_{B2} = I_B - \dfrac{I_{OS}}{2}$

$$V_O = I_{OS}R_2$$

Op-Amp Noise

Op-Amp Distortion

Datasheet Examples

analog.com

Apply Filters to this Table | Reset Table | Hold Shift Key for secondary sorting

Part#	Vsupply span (min) (V)	Vsupply span (max) (V)	Iq/Amp (typ) (A)	Amps per Package	GBP (typ) (Hz)	Slew Rate (typ) (V/us)	Ibias (max) (A)	Vos (max) (V)	CMRR (min) (dB)	0.1 to 10 Hz Voltage Noise (typ) (V p-p)	VNoise Density (typ) (V/rtHz)	US Price 1000 to 4999 ($ US)
AD8099	5	12	15m	1	3.8G	470	13µ	500µ	-105	-	950p	$2.00
AD8003	4.5	10	9.5m	3	1.65G	3.8k	-	-	-48	-	1.8n	$2.92
ADA4895-1	3	10	3m	1	1.5G	943	6µ	350µ	-100	99n	1n	$1.89
ADA4895-2	3	10	3m	2	1.5G	943	6µ	350µ	-100	99n	1n	$3.21
AD8021	4.5	24	7.8m	1	1G	130	11.3µ	1m	-98	-	2.1n	$1.31
AD8001	6	12	5m	1	880M	1.2k	25µ	5.5m	-54	-	2n	$1.51
AD829	9	36	5.3m	1	750M	230	7µ	1m	-120	-	1.7n	$2.78
ADA4861-3	5	12	16.1m	3	730M	680	-	-	-56.5	-	3.2n	$0.96

Analog.com's website: Sort by GBP and then SR

Low Power Op-Amps

Part#	Vsupply span (min) (V)	Vsupply span (max) (V)	Iq/Amp (typ) (A)	Amps per Package	GBP (typ) (Hz)	Slew Rate (typ) (V/us)	Ibias (max) (A)	Vos (max) (V)	CMRR (min) (dB)	0.1 to 10 Hz Voltage Noise (typ) (V p-p)	VNoise Density (typ) (V/rtHz)	US Price 1000 to 4999 ($ US)
ADA4312-1	12	12	-	1	-	2.1k	-	-	-	-	-	$1.89
ADLD8403	11.75	13.2	-	2	-	-	-	-	-	-	-	$1.59
AD8398A	12	12	-	1	-	600	-	-	-	-	4.8n	$1.45
AD8390A	10	24	-	-	-	260	-	-	-	-	5n	$1.08
AD8398	12	12	-	1	-	820	-	-	-	-	2.85n	$2.30
AD8504	1.8	5	750n	4	7k	4m	10p	3m	-67	6μ	190n	$1.00
AD8502	1.8	5	750n	2	7k	4m	10p	3m	-67	6μ	190n	$0.70
AD8500	1.8	5	750n	1	7k	4m	10p	1m	-75	-	190n	$0.71
OP481	2.7	12	5μ	4	105k	28m	10n	1.5m	-65	10μ	85n	$3.65
OP281	2.7	12	5μ	2	105k	28m	10n	1.5m	-65	10μ	75n	$2.79
ADA4505-1	1.8	5	9μ	1	50k	6m	2p	3m	-90	2.95μ	65n	$0.41

Analog.com's website: Sort by Bias current and then GBP

Look at Datasheet

ANALOG DEVICES

Low Noise, High Speed Amplifier for 16-Bit Systems

AD8021

FEATURES

Low noise
- 2.1 nV/√Hz input voltage noise
- 2.1 pA/√Hz input current noise

Custom compensation
- Constant bandwidth from G = −1 to G = −10

High speed
- 200 MHz (G = −1)
- 190 MHz (G = −10)

Low power
- 34 mW or 6.7 mA typical for 5 V supply

Output disable feature, 1.3 mA

Low distortion
- −93 dBc second harmonic, f_C = 1 MHz
- −108 dBc third harmonic, f_C = 1 MHz

DC precision
- 1 mV maximum input offset voltage
- 0.5 μV/°C input offset voltage drift

CONNECTION DIAGRAM

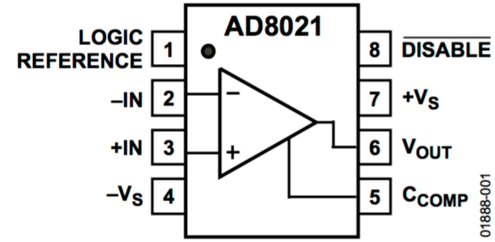


Figure 1. SOIC-8 (R-8) and MSOP-8 (RM-8)

The AD8021 allows the user to choose the gain bandwidth product that best suits the application. With a single capacitor, the user can compensate the AD8021 for the desired gain with little trade-off in bandwidth. The AD8021 is a well-behaved amplifier that settles to 0.01% in 23 ns for a 1 V step. It has a fast overload recovery of 50 ns.

The AD8021 is stable over temperature with low input offset