Mixers

The Mixer is a critical component in communication circuits. It translates information content to a new frequency.
Why use a mixer (transmit side)?

1) Translate information to a frequency appropriate for transmission

Example: Antennas smaller and more efficient at high frequencies

2) Spectrum sharing: Move information into separate channels in order to share spectrum and allow simultaneous use

3) Interference resilience

![Geographic map of cell sites]

UC Berkeley EECS 242

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Why use mixer in the receiver?

$Q \sim \frac{\omega_o}{2\Delta\omega}$  
Q of filter

Bandpass filter at $\omega_o$ requires a high-Q for narrowband signals

$\Delta f \sim 200$ kHz (GSM)

$f_o \sim 1$ GHz

$Q = \frac{10^9}{2 \times 200 \times 10^6} = \frac{1000}{0.4} = 2500$  
High Q
Mixers in Receivers (cont)

High Q $\Rightarrow$ Insertion Loss

Filter center frequency must change to select a given channel $\Rightarrow$ tunable filter difficult to implement

Mixing has big advantage! Translate information down to a fixed (intermediate frequency) or IF.

1 GHz $\Rightarrow$ 10 MHz: 100x decrease in Q required

Don’t need a tunable filter

Superheterodyne receiver architecture

High Q channel filter

Issue: Mixer has high noise factor
Mixers Specifications

- **Conversion Gain**: Ratio of voltage (power) at output frequency to input voltage (power) at input frequency
  - Downconversion: RF power / IF power
  - Up-conversion: IF power / RF power
- **Noise Figure**
  - DSB versus SSB
- **Linearity**
- **Image Rejection**
- **LO Feedthrough**
  - Input
  - Output
- **RF Feedthrough**
We know that any non-linear circuit acts like a mixer
Squarer Example

\[ y = \frac{2AB}{2} \left\{ \cos(\omega_1 + \omega_2) t + \cos(\omega_1 - \omega_2) t \right\} \]

DC & second harmonic
Desired mixing

Product component:

What we would prefer:

\[ v_{IF} = v_{LO} \cdot v_{RF} \cos(\omega_1 \pm \omega_2) \]
\[ v_{RF} = v_{RF} \cos \omega_1 t \]

A true quadrant multiplier with good dynamic range is difficult to fabricate
LTV Mixer

\[ f_1 + f_2 \rightarrow \text{LTI} \rightarrow f_1 + f_2 \quad \text{No new frequencies} \]

\[ f_1 + f_2 \rightarrow \text{LTV} \rightarrow \text{New tones in output} \]

Example: Suppose the resistance of an element is modulated harmonically

\[ v_o(\text{IF}) = i_{in} \cdot R(t) \]

\[ = I_o \cos(\omega_{RF}t) \cdot R_o \cos(\omega_{LO}t) \]

\[ = \frac{I_o R_o}{2} \{ \cos(\omega_{RF} + \omega_{LO})t + \cos(\omega_{RF} - \omega_{LO})t \} \]
Time Varying Systems

In general, any periodically time varying system can achieve frequency translation

\[ v(t) = p(t)v_i(t) \quad p(t + T) = p(t) \]

\[ = \sum_{n=-\infty}^{\infty} c_n e^{j\omega_0 nt} v_i(t) \]

\[ c_n = \frac{1}{T} \int_0^T p(t) e^{-j\omega_0 nt} dt \quad v_i(t) = A(t)\cos \omega_1 t = A(t) \left( \frac{e^{j\omega_1 t} + e^{-j\omega_1 t}}{2} \right) \]

\[ v_o(t) = A(t) \sum_{-\infty}^{\infty} c_n \frac{e^{j(\omega_0 nt + \omega_1 t)} + e^{j(\omega_0 nt - \omega_1 t)}}{2} \]

consider n=1 plus n=-1
Desired Mixing Product

\[ v_o(t) = \frac{c_1}{2} e^{j(\omega_o t - \omega_1 t)} + \frac{c_{-1}}{2} e^{-j(\omega_o t + \omega_1 t)} \]

\[ = c_1 \cos(\omega_o t - \omega_1 t) \]

Output contains desired signal (plus a lot of other signals) → filter out undesired components
Convolution in Frequency

Ideal multiplier mixer:

\[ y(t) = p(t)x(t) \]

\[ Y(f) = X(f) * P(f) \]

\[ P(f) = \sum_{n=-\infty}^{\infty} c_n \delta(f - nf_{LO}) \]

\[ Y(f) = \int\sum_{n=-\infty}^{\infty} c_n \delta(\sigma - nf_{LO})X(f - \sigma) \, d\sigma \]

\[ = \sum_{n=-\infty}^{\infty} c_n \left( \int_{-\infty}^{\infty} \delta(\sigma - nf_{LO})X(f - \sigma) \, d\sigma \right) \]

\[ = \sum_{n=-\infty}^{\infty} c_n X(f - nf_{LO}) \]
Convolution in Frequency (cont)

Translated spectrum peaks:

\[ f - nf_{LO} = f_{RF} \]

\[ f = f_{RF} + nf_{LO} \]

Input spectrum is translated into multiple “sidebands” or “image” frequencies

⇒ Also, the output at a particular frequency originates from multiple input frequency bands
How Low can you LO?

Take the simplest mixer:

\[
\cos \omega_{LO} t \quad x(t) \quad \times \quad \cos(\omega_{LO} - \omega_{RF}) \quad \text{output IF}
\]

Side note:

Which LO frequency to pick? LO1 or LO2?

\[
f_{LO} = f_{LO} + \frac{n\Delta f}{N}
\]

Channel spacing

No. of channels

Tuning range: \( \frac{\Delta f}{f_{LO}} \Rightarrow f_{LO} \) larger implies smaller tuning range
Question: Why filter before mixer in spectrum analyzer?
Answer: Image rejection

Receiver architecture is getting complicated…
Origin of Image Problem

If we could multiply by a complex exponential, then image problem goes away…

\[ e^{j\theta_{LO}} \cos(\omega_{RF} t) = e^{j(\theta_{LO} + \theta_{RF})} + e^{+j(\theta_{LO} - \theta_{RF})} \]

\[ e^{j\theta_{RF}} + e^{-j\theta_{RF}} \]

\[ \theta_{RF} = \theta_{LO} - \theta_{IF} \]  
High side injection

\[ \theta_{IM} = \theta_{LO} + \theta_{IF} \]  
(Low side injection) Image Freq.
Average response of LTI system:

\[ y_1(t) = H_1[x(t)] = \int_{-\infty}^{\infty} h_1(t)x(t-\tau) d\tau \]

\[ \overline{y_1(t)} = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} y_1(t) dt \]

\[ = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \left(\int_{-\infty}^{\infty} h_1(\tau)x(t-\tau) d\tau\right) dt \]

\[ = \int_{-\infty}^{\infty} \left(\lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x(t-\tau) dt\right) h_1(\tau) d\tau \]

\[ \overline{x(t)} \]
Average Value Property

\[ y_1(t) = x(t) \int_{-\infty}^{\infty} h_1(t) \, dt \]

\[ H_1(j\omega) = \int_{-\infty}^{\infty} h_1(t) e^{-j\omega t} \, dt \]

\[ y_1(t) = x(t) H_1(0) \]

“DC gain”
Recall the definition for the autocorrelation function

\[ \phi_{xx}(t) = \overline{x(t)x(t+\tau)} \]

\[ = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x(t)x(t+\tau) \, dt \]
Autocorrelation Function

\[
y_1^2(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h_1(\tau_1) h_2(\tau_2) \phi_{xx}(\tau_1 - \tau_2) d\tau_1 d\tau_2
\]

\[
\phi_{xx}(j\omega) = \int_{-\infty}^{\infty} \phi_{xx}(\tau) e^{-j\omega\tau} d\tau
\]

\[
\phi_{xx}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \phi_{xx}(j\omega) e^{j\omega\tau} d\omega
\]

\(\phi_{xx}(j\omega)\) is a real and even function of \(\omega\)

since \(\phi_{xx}(\tau)\) is a real and even function of \(\tau\)
Autocorrelation Function (2)

\[
y_1^2(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h_1(\tau_1) h_1(\tau_2) \frac{1}{2\pi} \int_{-\infty}^{\infty} \phi_{xx}(j\omega) e^{j\omega(\tau_1 - \tau_2)} d\omega d\tau_1 d\tau_2
\]

\[
= \frac{1}{2\pi} \int_{-\infty}^{\infty} \phi_{xx}(j\omega) \int_{-\infty}^{\infty} h_1(\tau_1) h_1(\tau_2) e^{j\omega(\tau_1 - \tau_2)} d\tau_1 d\tau_2
\]

\[
= \frac{1}{2\pi} \int_{-\infty}^{\infty} \phi_{xx}(j\omega) \left( \int_{-\infty}^{\infty} h_1(\tau_1) e^{j\omega \tau_1} d\tau_1 \right) \left( \int_{-\infty}^{\infty} h_1(\tau_2) e^{-j\omega \tau_2} d\tau_2 \right) d\omega
\]

\[
H_1^*(j\omega) = \left( \int_{-\infty}^{\infty} h_1(\tau) e^{-j\omega \tau} d\tau \right)^*
\]

\[
y_1^2(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \phi_{xx}(j\omega) H_1(j\omega) H_1^*(j\omega) d\omega
\]

\[
= \frac{1}{2\pi} \int_{-\infty}^{\infty} \phi_{xx}(j\omega) H_1(j\omega)^2 d\omega
\]
Consider $x(t)$ as a voltage waveform with total average power $\overline{x^2(t)}$. Let’s measure the power in $x(t)$ in the band $0<\omega<\omega_1$.

The average power in the frequency range $0<\omega<\omega_1$ is now

$$\overline{y_1^2(t)} = \frac{1}{2\pi} \int_{-\infty}^{\omega_1} \phi_{xx}(j\omega) H_1(j\omega)^2 d\omega$$

$$= \frac{1}{2\pi} \int_{-\omega_1}^{\omega_1} \phi_{xx}(j\omega) d\omega$$

W/radian

$$= \int_{-f_1}^{f_1} \phi_{xx}(j2\pi f) df$$

W/Hz
Average Power in $X(t)$ (2)

\[ \frac{f_1}{0} = 2 \int_{0}^{f_1} \phi_{xx} (j2\pi f) df \]

Generalize: To measure the power in any frequency range apply an ideal bandpass filter with passband $\omega_1 < \omega < \omega_2$

\[ y_1^2 (t) = 2 \int_{f_1}^{f_2} \phi_{xx} (j2\pi f) df \]

The interpretation of $\phi_{xx}$ as the “power spectral density” (PSD) is clear.
A spectrum analyzer measures the PSD of a signal

**Poor man’s spectrum analyzer:**

- **VCO**
- **Sweep generation**
- **Sharp filter**
- **Wide dynamic range mixer**
- **Phase noise**
- **Linear wide tuning range**
- **CRT**
EECS 242: Current Commutating Active Mixers

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Balanced Mixer

- An unbalanced mixer has a transfer function:

\[ y(t) = x(t) \times s(t) = (1 + A(t) \cos(\omega_{RF} t)) \times \begin{cases} 0 \\ 0 \end{cases} \]

which contains both RF, LO, and IF

- For a single balanced mixer, the LO signal is “balanced” (bipolar) so we have

\[ y(t) = x(t) \times s(t) = (1 + A(t) \cos(\omega_{RF} t)) \times \begin{cases} +1 \\ -1 \end{cases} \]

- As a result, the output contacts LO but no RF component

- For a double balanced mixer, the LO and RF are balanced so there is no LO or RF leakage

\[ y(t) = x(t) \times s(t) = A(t) \cos(\omega_{RF} t) \times \begin{cases} +1 \\ -1 \end{cases} \]
Consider the simplest ideal multiplying mixer:

- What’s the noise figure for the conversion process?
- Input noise power due to source is $kTB$ where $B$ is the bandwidth of the input signal
- Input signal has power $P_s$ at either the lower or upper sideband

$$SNR_i = \frac{P_s}{kTB}$$
Noise in Ideal Mixers

- At the IF frequency, we have the down-converted signal \( G \cdot P_s \) and down-converted noise from two sidebands, LO - IF and LO + IF.

\[
SNR_o = \frac{G \cdot P_s}{(G' + G'')kT}B
\]

For ideal mixer, \( G = G' = G'' \)

\[
F = \frac{SNR_i}{SNR_o} = \frac{P_s}{kT} \frac{2kTB}{P_s} = 2
\]

\( NF = 3 \text{dB} \)

For a real mixer, noise from multiple sidebands can fold into IF frequency & degrade NF.
Noise in CMOS Current Commutating Mixer

(After Terrovitis, JSSC)

\[ I_{o1} = I_1 - I_2 = F(V_{LO}(t), I_B + i_s) \]

Assume \( i_s \) is small relative to \( I_B \) and perform Taylor series expansion

\[ I_{o1} \approx F(V_{LO}(t), I_B) + \frac{\delta F}{\delta I_B} (V_{LO}(t), I_B) \cdot i_s + ... \]

\[ I_{o1} = P_o(t) + P_1(t) \cdot i_s \]
Noise in Current Commutating Mixers

\[ p_1(t) = \frac{g_{m1}(t) - g_{m2}(t)}{g_{m1}(t) + g_{m2}(t)} = \frac{i_1 - i_2}{i_s} \]

Note that with good device matching

\[ p_1(t) = -p_1\left(t + \frac{T_o}{2}\right) \]

Expand \( p_1(t) \) into a Fourier series:

\[ p_{1,2k} = \frac{1}{T_{LO}} \int_0^{T_{LO}} p_1(t)e^{-j2\pi 2kt/T_{LO}} dt = \int_0^{T_{LO}/2} + \int_{T_{LO}/2}^{T_{LO}} = 0 \]

Only odd coefficients of \( p_{1,n} \) non-zero
Assume LO signal strong so that current (RF) is alternatively sent to either $M_2$ or $M_3$. This is equivalent to multiplying $i_{RF}$ by $\pm 1$.

$$v_{IF} \approx \text{sign}(V_{LO}) g_m R_L v_{RF} = g(t) g_m R_L v_{RF}$$

Period waveform with period $= T_{LO}$
Current Commutating Mixer (2)

\[ g(t) = \text{square wave} = \frac{4}{\pi} (\cos \omega_{LO}t - \cos 3\omega_{LO}t + ...) \]

Let \[ v_{RF} = A \cos \omega_{RF}t \]

\[ LPF(v_{IF}) = \frac{4}{\pi} \frac{1}{2} \cos (\omega_{RF} - \omega_{LO})t \cdot g_mR_L \cdot A \]

\[ A_v = \frac{\bar{v}_{IF}}{A} = \frac{2}{\pi} g_mR_L \quad \text{gain} \]

LO-RF isolation good, but LO signal appears in output (just a diff pair amp).
Strong LO might desensitize (limit) IF stage (even after filtering).
Double Balanced Mixer

- LO signal is rejected up to matching constraints
- Differential output removes even order non-linearities
- Linearity is improved: Half of signal is processed by each side
- Noise higher than single balanced mixer since no cancellation occurs
Common Gate Input Stage
Gilbert Micromixer

- The LNA output is often single-ended. A good balanced RF signal is required to minimize the feedthrough to the output. LC bridge circuits can be used, but the bandwidth is limited. A transformer is a good choice for this, but bulky and bandwidth is still limited.

- A broadband single-ended to differential conversion stage is used to generate highly balanced signals. Gm stage is Class AB.
Active and Passive Balun
**Bleeding the Switching Core**

- Large currents are good for the gm stage (noise, conversion gain), but require large devices in the switching core → hard to switch due to capacitance or requires a large LO (large Vgs-Vt)

- A current source can be used to feed the Gm stage with extra current.

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Current Re-Use Gm Stage

\[ I_0 = I_{o1} - I_{o2} \]
Single, Dual, and Back Gate

[Diagram of circuits for single, dual, and back gate configurations]
Rudell CMOS Mixer

- Gain programmed using current through M16 (set by resistance of triode region devices M9/M10)
- Common mode feedback to set output point
- Cascode improves isolation (LO to RF)
Passive Mixers/Sampling
Triode Region Mixer
Improved Linearity

To improve $M_1$, apply local series feedback

$$Z_s = j\omega L$$

Provide input matching and feedback

$\Rightarrow$ No DC headroom sacrificed
Recap: CMOS Mixer Operation

\[ I_{o1} = I_1 - I_2 = F(V_{LO}(t), I_B + i_s) \]

\[ \approx F(V_{LO}(t), I_B) + \frac{\partial F}{\partial I_B}(V_{LO}(t), I_B) \cdot i_s + ... \]

\[ = P_o(t) + P_1(t) \cdot i_s + ... \]

\[ P_1(t) = \frac{g_{m1}(t) - g_{m2}(t)}{g_{m1}(t) + g_{m2}(t)} \quad \text{Periodic} \]

Fourier Series expansion

\[ P_{1,2k} \equiv 0 \quad P_1(t) = -P_1 \left( t + \frac{T_{LO}}{2} \right) \]
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