EE 42/100
Lecture 7: Amplifiers

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Prof. Ali M. Niknejad

University of California, Berkeley
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An amplifier is a multi-terminal element. It usually has an input terminal, an output terminal, both referenced to a common reference (ground), which is supplied through another terminal, labeled $V_{\text{ref}}$, $\text{GND}$.

There is also a positive supply terminal, labeled $V_{\text{pos}}$, $V_{\text{supply}}$, or $V_{CC}$, or $V_{DD}$. In some cases, the amplifier has a negative supply terminal, labeled as $V_{\text{neg}}$, $V_{SS}$ or $V_{EE}$. The supplies are also known as the rails of an amplifier.

Inside, the amplifier is constructed with various devices such as resistors, capacitors, and transistors. Sometimes inductors and transformers also appear inside, especially in high frequency amplifiers.
The symbol for the amplifier is a triangle because the amplifier nominally amplifies only in one direction. We call this behavior “unilateral” gain, because a signal at the input is amplified and appears at the output.

The supplies are connect to batteries or supply voltages. If a negative supply is not needed, the negative terminal of the battery, usually ground, is connected there. The currents drawn from the supplies is usually a DC current.

The input signal is applied to the input terminal. It is usually connected to a voltage or current source. It can be a DC or an AC signal.

The output is usually connected to a load, which is usually modeled by a resistor. The load resistance can model a transducer (speakers, or a screen), an analog-to-digital converter, or another amplifier (cascade amplifiers). The output signal is a faithful reproduction of the input.
**KCL/Current Directions**

- Since energy is usually delivered to the load, we like to think of the output current $i_{out}$ exiting the amplifier. Also, since power flows into the amplifier from the supplies and the input, we draw the currents appropriately.

- Recall that KCL applies to any closed surface. If we construct such a surface around an amplifier, we have

  $$i_{in} - i_{out} + I_{pos} + I_{neg} + i_{gnd} = 0$$

- Oftentimes, we only draw the input/output terminals of the amplifier, and so KCL is not satisfied. This is just a convenience to simplify the drawing.
Power Gain

- Energy conservation requires that the power flowing out from the supplies is absorbed by the amplifier and load

\[ P_{in} + P_{pos} + P_{neg} = P_{load} \]

\[ i_{in} v_{in} + I_{pos} V_{sup} + (-I_{neg} \cdot (-V_{neg})) = i_{load} v_{load} \]

- If we only examine the input/output relation, then we see that it’s possible that

\[ P_{load} > P_{in} \]

- which means the amplifier has power gain.
**Amplifier Packaging**

- Depending on the frequency of operation, different packaging options are available for the amplifier. Often a DIP (dual in-line package) is used to package integrated circuit amplifiers. Inside the package an integrated circuit die is wire-bonded to the package leads.

- For larger number of pins, pin grid array (PGA) or plastic quad flat packages (PQFP) are used. Ball grid array (BGA) can be surface mounted using solder balls.
Amplifier Modules

- High frequency amplifiers and precision amplifiers are often packaged using a metal housing to limit the amount of interference in the package.
- Often co-axial cables are used to deliver the input and output signals. Coaxial cables use a ground shield that is effective against noise pickup.
- High power amplifiers have heat sinks to keep the temperature cool.
**Ideal Voltage Amplifiers**

- An ideal voltage amplifier, the input signal is a pure voltage, and the amplifier faithfully “copies” the input to the output while increasing the magnitude of the voltage (regardless of the load)

\[ v_{out} = A_v v_{in} \]

- where \( A_v \) is the voltage gain of the amplifier. The voltage gain is usually specified using a dB scale

\[ A_v [\text{dB}] = 20 \cdot \log(A_v) \]

- so a gain of 100 is 40dB.

- The input current flowing into the amplifier is ideally zero

\[ i_{in} \approx 0 \]

- which means that the *input resistance of the amplifier* is infinite. Note that the power flow into the input is therefore zero, which means the amplifier has infinite power gain.

![Ideal Voltage Amplifier Circuit Diagram]

\[ V_o = A_v V_s \]
**Ideal Current Amplifiers**

- An ideal current amplifier, the input signal is a pure current, and the amplifier faithfully “copies” the input to the output while increasing the magnitude of the current (regardless of the load)

\[ i_{out} = A_i i_{in} \]

- where \( A_i \) is the current gain of the amplifier. The current gain is usually specified using a dB scale

\[ A_i [\text{dB}] = 20 \cdot \log(A_i) \]

- so a gain of 1000 is 60dB.

- The input voltage into the amplifier is ideally zero

\[ v_{in} \approx 0 \]

- which means that the input resistance of the amplifier is zero. Note that the power flow into the input is therefore zero, which means the amplifier has infinite power gain.
**CURRENT AMP**

- **Maximize** $i_{in} \Rightarrow R_{in}$ very small
- **Maximize** $i_{out} \Rightarrow R_L$ also small

$R_{in} \ll R_s$
$R_L \ll R_{out}$
Ideal Transconductance Amplifiers

• An ideal transconductance amplifier, the input signal is a pure voltage, while the output is a current. The amplifier faithfully “copies” the input to the output (regardless of the load)

\[ i_{out} = Gv_{in} \]

• where \( G \) is the transconductance gain of the amplifier. The transconductance gain is usually specified using a dBS scale

\[ G[\text{dBS}] = 20 \cdot \log(G/1S) \]

• so a gain of \( 10^4 S \) is 80dBS.

• The input current into the amplifier is ideally zero

\[ i_{in} \approx 0 \]

• which means that the input resistance of the amplifier is infinite. Note that the power flow into the input is therefore zero, which means the amplifier has infinite power gain.
**Ideal Transresistance Amplifiers**

- An ideal transresistance amplifier, the input signal is a pure current, while the output is a voltage. The amplifier faithfully "copies" the input to the output (regardless of the load)

\[ v_{out} = R \cdot i_{in} \]

- where \( R \) is the transresistance gain of the amplifier. The transresistance gain is usually specified using a dBΩ scale

\[ R[\text{dBΩ}] = 20 \cdot \log(R/1Ω) \]

- so a gain of \( 10^6Ω \) is 120dBΩ.

- The voltage current into the amplifier is ideally zero

\[ v_{in} \approx 0 \]

- which means that the *input resistance of the amplifier* is zero. Note that the power flow into the input is therefore zero, which means the amplifier has infinite power gain.
A real voltage amplifier has three important imperfections. First, the input current is non-zero, which means it has finite power gain. This input current also leads to a loading effect, because the input of the amplifier is now smaller than the source voltage is the source has any resistance.

Likewise, the output of the amplifier will vary if the loading at the output varies. This means that the output can only faithfully amplify a range of load resistances.

The output voltage is also limited in range. Typically it cannot exceed the positive/negative supply rails.
**Input Resistance**

- The input current can be modeled by an input resistance. A test source is connected to the input terminals and the current flow at the input terminal is modeled by a Thevenin equivalent resistance

\[ R_{in} = \frac{v_{in}}{i_{in}} \]

- When a voltage supply with source resistance \( R_s \) is connected at the input, the actual input voltage is reduced by the voltage divider formula

\[ v_{in} = \frac{R_{in}}{R_{in} + R_s} v_s \]

- which means the performance is the most ideal with \( R_{in} \rightarrow \infty \Omega \) or in practice, \( R_{in} \gg R_s \).
Output Resistance

- Since the output voltage varies depending on the load connected at the output, the output terminal have an equivalent Thevenin resistance at the output terminals.

- To find this resistance, we can take the ratio of the open circuit voltage to the short circuit current, or connect a voltage (current) source and monitor the output current (voltage)

\[ R_{out} = \frac{v_{out}}{i_{out}} \]

- When a load is connected to the output, the actual load voltage is reduced by the “loading” effect

\[ v_L = \frac{R_L}{R_L + R_{out}} v_{th} \]

- An ideal voltage amplifier has \( R_L \ll R_{out} \).
**Equivalent Circuit Model**

- The complete equivalent circuit model for a voltage amplifier is shown above. The input and output resistance model the fact that the amplifier is subject to gain reduction due to the loading effects of a source/load resistance.
- The dependent source models the gain from the input to the output, but it’s ideal because a real amplifier can only provide gain over a limited range (determined by the power supply voltages).
$i_x = \frac{V_x}{R_{in}}$

$\Rightarrow R + h = R_{in}$
EX

**MICROPHONE**: \( V_s \sim 1 \text{mV} \)
\( R_s \sim 1 \text{M}\Omega \)

**DESIRE** \( V_o = 1 \text{V} \)

\( A_v = 1000 \text{ V/V} \)
\( R_i = 100 \Omega \)
\( R_o = 100 \Omega \)

**ACTUAL GAIN?**
\[ V_{in} = \frac{100\Omega}{100\Omega + 1\text{M}\Omega} \quad V_s \approx \frac{100\Omega}{1\text{M}\Omega} \quad V_s = 10^2 \times 10^{-6} \quad V_s \]

\[ = 10^{-4} \quad V_s \]

\[ V_o = \frac{10\Omega}{10\Omega + 100\Omega} \quad A_V V_{in} \approx 0.1 \times 10^{-3} \quad V_s \]

\[ \approx 10^{-5} \quad V_s \]

\[ A_{eff} = \frac{V_o}{V_s} = 10^{-2} \]
$V_o = A_v \cdot V_s$

IDEAL SOURCE
EX optical receiver

\[ A = \frac{V_o}{i_{in}} = R_T \]

\[ R_{in} \ll R_S \]
\[ R_{out} \ll R_L \]
Example: Amplifying Weak Signals

- Suppose that we wish to amplify a weak signal, on the order of 1mV, to a range appropriate for digitization. Suppose the signal originates from a sensor (such as an ECG electrode) with higher source resistance, say 1 MΩ.

- The digitizer is an analog-to-digital converter (ADC), which takes an analog input voltage and produces a digital representation of the voltage, which can be stored using digital memory.

- The ADC usually can only amplify signals larger than a certain range. In this examples, the range of the ADC is given by $1 \text{ mV} < V_{in} < 1 \text{ V}$. Also, the ADC has an input resistance of 1 kΩ.

- We select the following amplifier to perform this job

$$A_v = 20 \text{ dB}, \ R_{in} = 1 \text{ k} \Omega, \ R_{out} = 1 \text{ k} \Omega$$
**Effective Gain**

\[ A_{eff} = \frac{R_{in}}{R_{in} + R_s} A_v \frac{R_{out}}{R_{out} + R_L} \]

- Because of the loading effects, the actual gain of the amplifier is going to be lower. By how much?

- In this example, \( R_s \) is much larger than the input resistance \( R_{in} \) of the amplifier, which results in significant loading.

\[ A_{eff} = \frac{10^3}{10^6 + 10^3} \cdot 10 \cdot \frac{10^3}{10^3 + 10^3} \]

\[ \approx \frac{10^3}{10^6} \cdot 10 \cdot \frac{1}{2} = 5 \times 10^{-3} \]

- The gain is now in less than 1, which is exactly opposite of what we’re trying to do. To solve this problem, we need a buffer (next lecture) or an amplifier with a higher input resistance.

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\[ v_s \sim 1 \text{ mV} \]

\[ v_x \]

\[ 1 \text{ k\Omega} \]

\[ 1 \text{ k\Omega} \]

\[ + \]

\[ G_v v_x \]

\[ R_L \]

\[ 1 \text{ M\Omega} \]

\[ v_s = 1 \text{ M\Omega} \]
When several ideal amplifiers are placed in cascade, the gain increases, as you might expect

\[ A_v = A_{v,1} \cdot A_{v,2} \cdots \]

When real amplifiers are cascaded, we have to take loading effects into account. For instance, for the cascade of the two amplifiers from the previous example, we can derive a new amplifier equivalent circuit model

\[ R_{in} = R_{out} = 1 \, k\Omega \]

but

\[ A_v = \frac{1}{2} A_{v,1} A_{v,2} = 50 \]
AMPLIFIER

$A_v = 10$
$R_{in} = 1 \text{M}\Omega$
$R_{out} = 10 \text{M}\Omega$

\[ R_{in} \cdot A_v V_s = \frac{R_{in}}{R_{in} + R_{out}} \cdot A_v V_s \]

\[ A_v \cdot \frac{1}{A_v \cdot 10} \]

\[ V_o = ? \]
Dynamic Range of Amplifiers

- Because of the limited output range of the amplifier, the input-output curves saturate (usually at a voltage lower or equal to the supply rails).
- This means that the input voltage cannot exceed a certain range before the amplifier behavior becomes non-linear.
The maximum input signal that we can apply to the amplifier is thus related to the supply voltage and the gain of the amplifier:

\[ v_{in,\text{max}} \leq \frac{V_{\text{sup}}}{G_v} \]

Similarly, the smallest input has to be larger than:

\[ v_{in,\text{min}} \geq \frac{-V_{\text{sup}}}{G_v} \]

For a high gain amplifier, this is a very limited input range. Unless the supply voltage can be increased (limited by technology), the output dynamic range is fixed.
When the input signal goes beyond the linear range, the output can exhibit clipping, which is very non-linear.
**Distortion**

- When the input range even approaches the linear input range, the amplifier behavior becomes much more non-linear. All real amplifiers are non-linear and the input / output relationship can be modeled by a Taylor series (for small signals)

\[ v_{out} = a_1 v_{in} + a_2 v_{in}^2 + \cdots \]

- This non-linearity produces *distortion* in the signal, which can change the signal enough to make the system operation poor or unacceptable. Audio applications are a commonly known example.
Gain Reduction

- Note too that the effective gain of the system reduces as the input exceeds the input range of the amplifier. That's because the output saturates at $V_{sup}$ whereas the input keeps increasing.

\[ G_{eff} = \frac{V_{sup}}{v_{in}} \leq G_v \]
**Noise**

- Even though the subject of noise is beyond the scope of this course, it's important to realize that all real amplifiers produce noise at their output. We must exercise care when amplifying weak signals. If we choose the wrong amplifier, the output noise can be larger than the output signal!