MIXERS (STEPHEN MAAS)

A MIXER IS A FREQUENCY TRANSLATION DEVICE:

\[ RF_1 \rightarrow \times \rightarrow RF_2 \text{ (IF) } \]

\[ \text{LO} \]

\[ RF_1 = A_1 \cos (\omega_1 t) \]
\[ \text{LO} = A_3 \cos (\omega_{LO} t) \]
\[ RF_2 = RF_1 \otimes \text{LO} \]
\[ = A_2 \cos (\omega_2 t) \]

\[ \omega_2 = \omega_1 \pm \omega_{LO} \]

THIS IS NOT A LINEAR TIME-INVARIANT CIRCUIT!

A MULTIPLIER ACTS LIKE AN IDEAL MIXER:

\[ V_m = A_1 A_2 \cos \omega_1 t \cdot \cos \omega_2 t \times G \]
\[ = A_1 A_2 x G \left\{ \cos (\omega_1 + \omega_2) t + \cos (\omega_1 - \omega_2) t \right\} \]

A TIME-VARYING CIRCUIT ALSO ACTS LIKE A MIXER
\[ V_0 = G \cdot V_i \]

**FOR FIXED \( G \), CIRCUIT IS LINEAR**

\[ V_i = A_1 \cos \omega_1 t \]

\[ V_i = G \cdot V_i = A_1 A_2 \cos \omega_1 t \cos \omega_2 t \]

**\( G \) IS ALSO A MIXER**

A **LINEAR TIME-VARYING CIRCUIT CAN ACT LIKE A MIXER.**

**TWO FAMILIES OF MIXERS:**

- **NON-LINEAR**
  - **"MULTIPLYING" MIXER**
  
  \[ y = f(x) = a_1 x + a_2 x^2 + \ldots \]

  \[ y_3 = f(x_1 + x_2) = a_1 (x_1 + x_2) + a_2 (x_1 x_2)^2 + \ldots \]

  \[ \underbrace{a_1 (x_1 + x_2)} + \underbrace{a_2 (x_1^2 + x_2^2 + 2 x_1 x_2)} + \ldots \]

  **ANY DEVICE WITH SQ LAW CAN MIX**

- **TIME-VARYING**
  - **"SWITCHING" MIXERS**
PERIODICALLY TIME-VARYING MIXER

\[ U(t) = p(t) \ V_i(t) \]

\[ p(t+T) = p(t) \]

\[ = \sum_{n=-\infty}^{\infty} c_n e^{j\omega_0 t} \ V_i(t) \]

\[ c_n = \frac{1}{T} \int_{t}^{t+T} p(t) e^{-j\omega_0 t} \ dt \]

\[ V_i(t) = A(t) \cos \omega_1 t = A(t) \left( e^{j\omega_1 t} + e^{-j\omega_1 t} \right) \]

\[ U_0(t) = A(t) \sum_{n=-\infty}^{\infty} c_n e^{j\left(\omega_0 t + \omega_1 t\right)} + e^{j\left(\omega_0 t - \omega_1 t\right)} \]

\[ c_1 = c_{-1} \]

\[ = c_1 \cos \left(\omega_0 t - \omega_1 t\right) \]

\[ \text{DESIRED SIGNAL ... FILTER OUT UNWANTED SIGNALS} \]
MULTIPLICATION IN TIME IS CONVOLUTION IN FREQUENCY:

\[ y(t) = p(t) \times x(t) \]

\[ Y(f) = X(f) \ast P(f) \]

\[ P(f) = \sum_{n=-\infty}^{\infty} c_n \delta(f - nf_{20}) \]

\[ Y(f) = \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} c_n \delta(f - nf_{20}) X(f - \sigma) d\sigma \]

\[ = \sum_{n=-\infty}^{\infty} \left( \int_{-\infty}^{\infty} \delta(f - nf_{20}) X(f - \sigma) d\sigma \right) \]

\[ = \sum_{n=-\infty}^{\infty} c_n X(f - nf_{20}) \]

TRANSLATE SPECTRUM

\[ f = f_{RF} + nf_{20} \]
MIXER CHARACTERISTICS:

- CONVERSION GAIN (OR LOSS)
- NOISE FIGURE
- BANDWIDTH
- DISTORTION
- SPURIOUS RESPONSE
  \[ f_{IF} = m f_{RF} + n f_{LO} \quad (m,n) \]
- FEED THROUGH
- DC POWER
- LO/RF/IF ISOLATION (REJECTION)

SPURIOUS SIGNALS:

CONSIDER A DIODE MIXER

\[ g(t) = \left. \frac{dI}{dV} \right|_{V=V_{LO}} = \frac{9I_0}{KT} e^{\frac{qV_{LO}}{kT}} \]

\[ \approx \frac{9}{KT} I_0(V_{LO}(t)) \]

\[ I(V) = I_0 \left( e^{\frac{qV}{kT}} - 1 \right) \]

\[ g(t) = \sum_{k=-\infty}^{\infty} g_k e^{jkw_p t} \quad w_p: \text{LO REFERENCE} \]

\[ i(t) = \sum_{k=-\infty}^{\infty} i_k e^{j(kw_p + w_s) t} = g(t) V_s e^{j\omega_0 t} \]

**BUT THE VOLTAGE WAVEFORM WILL ALSO HAVE**
**FREQ COMPONENTS AT ALL MIXING PRODUCTS**

**LET** \[ \omega_0 = |\omega_s - \omega_p| \]
\[ i(t) = \sum_{k=-\infty}^{\infty} I_k e^{j(k\omega_p + \omega_0)t} \]
\[ u(t) = \sum_{k=-\infty}^{\infty} V_k e^{j(k\omega_p + \omega_0)t} \]
\[ g(t) = \sum_{k=-\infty}^{\infty} G_k e^{j k \omega_p t} \]

\[ i(t) = u(t) g(t) \]

\[ \sum_{k=-N}^{N} i_k e^{j(\omega_0 + k\omega_p)t} = \sum_{n} \sum_{m} G_m V_n e^{j(\omega_0 + (n+m)\omega_p)t} \]

\[
\begin{pmatrix}
    I_{-N} \\
    I_{-N+1} \\
    \vdots \\
    I_{0} \\
    I_{1} \\
    \vdots \\
    I_{N}
\end{pmatrix}
\begin{pmatrix}
    G_0 & G_1 & G_2 & \cdots & G_{2N} \\
    G_1 & G_0 & & & \cdots \\
    G_2 & G_0 & & \ddots & \\
    \vdots & \vdots & \ddots & \vdots & \ddots \\
    \vdots & \vdots & \ddots & G_1 & G_0 \\
    G_{2N} & G_0 & \cdots & G_1 & G_0
\end{pmatrix}
\begin{pmatrix}
    I_{-N} \\
    I_{-N+1} \\
    \vdots \\
    I_{0} \\
    I_{1} \\
    \vdots \\
    I_{N}
\end{pmatrix}
\]

\[ \omega_n = n\omega_p + \omega_0 \]
\[ \omega_{-n} = -\omega_n \]

\[ V(\omega) \]
\[ I(\omega) \]

\[ \omega_0 \]
\[ -\omega_1 \]
\[ \omega_p \]
\[ \omega_1 \]
\[ -\omega_2 \]
\[ 2\omega_p \]
\[ \omega_2 \]
\[ \omega \]
SINGLE DIODE MIXER

Ideal "LC" is a short at FRQ far away from resonance. B/c of this all ports isolated: no LO feedthru to IF, no IF feedthru to IF, no LO feedthru to RF.

Since diode is short circuited at all freq except RF, LO, IF

\[
\begin{pmatrix}
I_{IF} \\
I_{RF}
\end{pmatrix} =
\begin{pmatrix}
Y_{11} & Y_{12} \\
Y_{21} & Y_{22}
\end{pmatrix}
\begin{pmatrix}
V_{IF} \\
V_{RF}
\end{pmatrix}
\]

All other freq products shorted.

Diode can be chosen to provide match at all ports.
PRACTICAL IMPLEMENTATION:

RING COUPLES POWER FROM LO TO RF LINE. SINCE DIODE IS MATCHED, RF PORT DOES NOT SEE A REFLECTION. RF IS ISOLATED FROM LO DUE TO NARROW RESONANT RING.

BALANCED DIODE MIXERS

USE BALUN OR HYBRIDS

RECALL:

\[ S_{90} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & -j & 1 \\ 0 & 0 & 1 & -j \\ -j & 1 & 0 & 0 \\ 1 & -j & 0 & 0 \end{pmatrix} \]

\[ S_{180} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \\ 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{pmatrix} \]
Transformers as hybrids\[8\] Lumped Element Hybrid

\[2\omega \approx 20\%\] 180\(^\circ\) Hybrid

\[\omega L = \frac{1}{\omega C} = \sqrt{2} \cdot Z_0\]
T-Line Baluns

\[ Z_0 = \sqrt{2Z_2} \]

\[ Z_{oo} = \frac{1}{2} Z_0 \quad Z_{oo} = 10 \, Z_{oo} \quad \text{for good performance} \]

Convenient Ground Return
Singly Balanced Diode Mixers

- Better RF \& LO isolation
- Rejection of spurious signals
- Rejection of AM noise from LO

Transformer realization with 180° hybrid
DOUBLY BALANCED MIXERS

- Reject all even order spurious responses

- IF, RF, LO all isolated

\[ Voltage \ (V_{IF}) = S(+) \cdot V_{RF}(+) \]
MOS REALIZATION

MULTIPLY RF BY ±1

FULLY BALANCED: VERY SIMPLE MIXER
- $S(f)$ has no even harmonics
- $S(f)$ has no DC component: if contains no RF
- Mixing with even components of RF cannot occur
- If current excited transformers
  (in even mode + no IF voltage at secondaries)

MICROWAVE REALIZATION:

IF band can overlap w/ RF/LO bands