EECS 142

Integrated Circuits for Communication

Lecture 8: Distortion Metrics

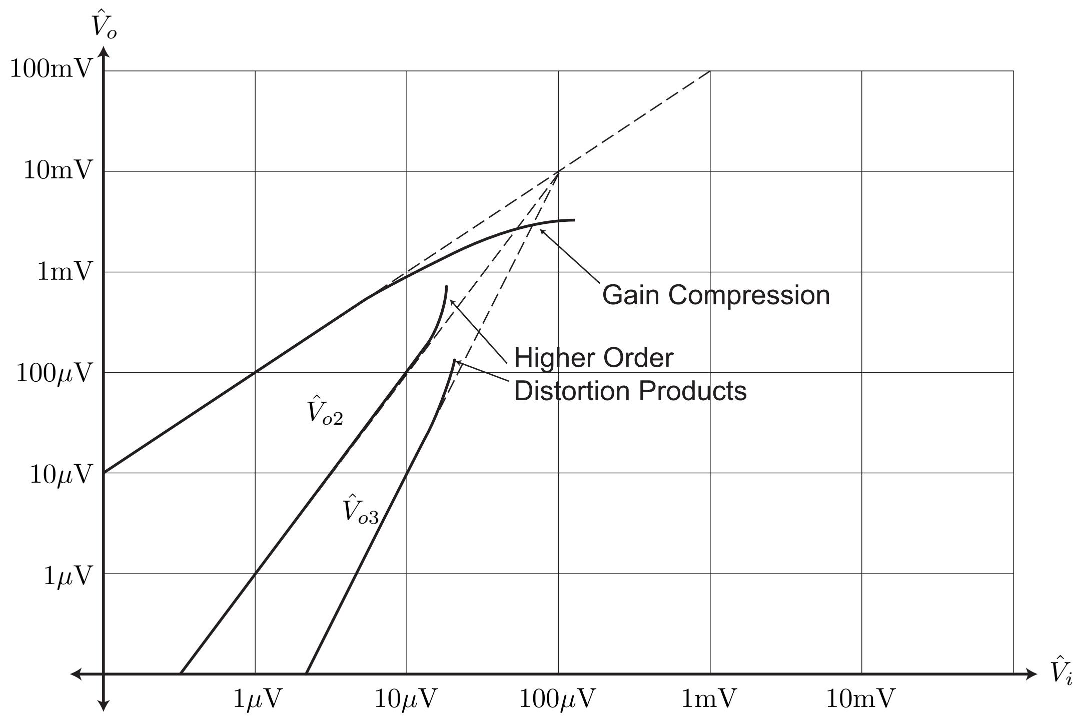
Prof. Ali M. Niknejad

University of California, Berkeley

Output Waveform

- In general, then, the output waveform is a Fourier series

$$v_o = \hat{V}_{o1} \cos \omega_1 t + \hat{V}_{o2} \cos 2\omega_1 t + \hat{V}_{o3} \cos 3\omega_1 t + \ldots$$



Fractional Harmonic Distortion

- The fractional second-harmonic distortion is a commonly cited metric

$$HD_2 = \frac{\text{ampl of second harmonic}}{\text{ampl of fund}}$$

- If we assume that the square power dominates the second-harmonic

$$HD_2 = \frac{a_2 \frac{S_1^2}{2}}{a_1 S_1}$$

or

$$HD_2 = \frac{1}{2} \frac{a_2}{a_1} S_1$$

Third Harmonic Distortion

- The fractional third harmonic distortion is given by

$$HD_3 = \frac{\text{ampl of third harmonic}}{\text{ampl of fund}}$$

- If we assume that the cubic power dominates the third harmonic

$$HD_3 = \frac{a_3 \frac{S_1^2}{4}}{a_1 S_1}$$

or

$$HD_3 = \frac{1}{4}\frac{a_3}{a_1}S_1^2$$

Output Referred Harmonic Distortion

- In terms of the output signal S_{om}, if we again neglect gain expansion/compression, we have $S_{om} = a_1 S_1$

$$HD_2 = \frac{1}{2}\frac{a_2}{a_1^2} S_{om}$$

$$HD_3 = \frac{1}{4}\frac{a_3}{a_1^3} S_{om}^2$$

- On a dB scale, the second harmonic increases linearly with a slope of one in terms of the output power whereas the thrid harmonic increases with a slope of 2.

Signal Power

- Recall that a general memoryless non-linear system will produce an output that can be written in the following form

$$v_o(t) = \hat{V}_{o1} \cos \omega_1 t + \hat{V}_{o2} \cos 2\omega_1 t + \hat{V}_{o3} \cos 3\omega_1 t + \dots$$

- By Parseval's theorem, we know the total power in the signal is related to the power in the harmonics

$$\int_T v^2(t) dt = \int_T \sum_j \hat{V}_{oj} \cos(j\omega_1 t) \sum_k \hat{V}_{ok} \cos(k\omega_1 t) dt$$

$$= \sum_j \sum_k \int_T \hat{V}_{oj} \cos(j\omega_1 t) \hat{V}_{ok} \cos(k\omega_1 t) dt$$

Power in Distortion

- By the orthogonality of the harmonics, we obtain Parseval's Them

$$\int_T v^2(t)dt = \sum_j \sum_k \tfrac{1}{2}\delta_{jk}\hat{V}_{oj}\hat{V}_{ok} = \tfrac{1}{2}\sum_j \hat{V}_{oj}^2$$

- The power in the distortion relative to the fundamental power is therefore given by

$$\frac{\text{Power in Distortion}}{\text{Power in Fundamental}} = \frac{V_{o2}^2}{V_{o1}^2} + \frac{V_{o3}^2}{V_{o1}^2} + \cdots$$

$$= HD_2^2 + HD_3^2 + HD_4^2 + \cdots$$

Total Harmonic Distortion

- We define the *Total Harmonic Distortion* (THD) by the following expression

$$THD = \sqrt{HD_2^2 + HD_3^2 + \cdots}$$

- Based on the particular application, we specify the maximum tolerable THD
- Telephone audio can be pretty distorted ($THD < 10\%$)
- High quality audio is very sensitive ($THD < 1\%$ to $THD < .001\%$)
- Video is also pretty forgiving, $THD < 5\%$ for most applications
- Analog Repeaters $< .001\%$. RF Amplifiers $< 0.1\%$

Intermodulation Distortion

- So far we have characterized a non-linear system for a single tone. What if we apply two tones

$$S_i = S_1 \cos\omega_1 t + S_2 \cos\omega_2 t$$

$$S_o = a_1 S_i + a_2 S_i^2 + a_3 S_i^3 + \cdots$$

$$= a_1 S_1 \cos\omega_1 t + a_1 S_2 \cos\omega_2 t + a_3 (S_i)^3 + \cdots$$

- The second power term gives

$$a_2 S_1^2 \cos^2\omega_1 t + a_2 S_2^2 \cos^2\omega_2 t + 2a_2 S_1 S_2 \cos\omega_1 t \cos\omega_2 t$$

$$= a_2 \frac{S_1^2}{2}(\cos 2\omega_1 t + 1) + a_2 \frac{S_2^2}{2}(\cos 2\omega_2 t + 1) +$$
$$a_2 S_1 S_2 \left(\cos(\omega_1 + \omega_2)t - \cos(\omega_1 - \omega_2)t\right)$$

Second Order Intermodulation

- The last term $\cos(\omega_1 \pm \omega_2)t$ is the second-order intermodulation term
- The intermodulation distortion IM_2 is defined when the two input signals have equal amplitude $S_i = S_1 = S_2$

$$IM_2 = \frac{\text{Amp of Intermod}}{\text{Amp of Fund}} = \frac{a_2}{a_1} S_i$$

- Note the relation between IM_2 and HD_2

$$IM_2 = 2HD_2 = HD_2 + 6\text{dB}$$

Practical Effects of IM_2

- This term produces distortion at a lower frequency $\omega_1 - \omega_2$ and at a higher frequency $\omega_1 + \omega_2$
- Example: Say the receiver bandwidth is from $800\mathrm{MHz} - 2.4\mathrm{GHz}$ and two unwanted interfering signals appear at $800\mathrm{MHz}$ and $900\mathrm{MHz}$.
- Then we see that the second-order distortion will produce distortion at $100\mathrm{MHz}$ and $1.7\mathrm{GHz}$. Since $1.7\mathrm{GHz}$ is in the receiver band, signals at this frequency will be corrupted by the distortion.
- A weak signal in this band can be “swamped” by the distortion.
- Apparently, a “narrowband” system does not suffer from IM_2? Or does it ?

Low-IF Receiver

- In a low-IF or direct conversion receiver, the signal is down-converted to a low intermediate frequency f_{IF}
- Since $\omega_1 - \omega_2$ can potentially produce distortion at low frequency, IM_2 is very important in such systems
- Example: A narrowband system has a receiver bandwidth of 1.9GHz - 2.0GHz. A sharp input filter eliminates any interference outside of this band. The IF frequency is 1MHz
- Imagine two interfering signals appear at $f_1 = 1.910\text{GHz}$ and $f_2 = 1.911\text{GHz}$. Notice that $f_2 - f_1 = f_{IF}$
- Thus the output of the amplifier/mixer generate distortion at the IF frequency, potentially disrupting the communication.

Cubic IM

- Now let's consider the output of the cubic term

$$a_3 s_i^3 = a_3 (S_1 \cos\omega_1 t + S_2 \cos\omega_2 t)^3$$

- Let's first notice that the first and last term in the expansion are the same as the cubic distortion with a single input

$$\frac{a_3 S_{1,2}^3}{4} (\cos 3\omega_{1,2} t + 3\cos\omega_{1,2} t)$$

- The cross terms look like

$$\binom{3}{2} a_3 S_1 S_2^2 \cos\omega_1 t \cos^2\omega_2 t$$

Third Order IM

- Which can be simplified to

$$3\cos\omega_1 t\cos^2\omega_2 t = \frac{3}{2}\cos\omega_1 t(1+\cos 2\omega_2 t) =$$

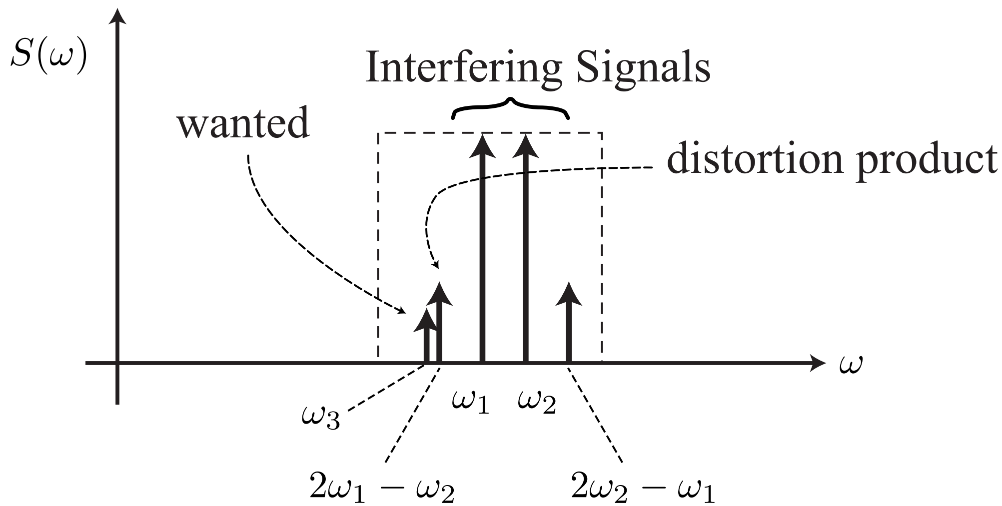
$$= \frac{3}{2}\cos\omega_1 t + \frac{3}{4}\cos(2\omega_2 \pm \omega_1)$$

- The interesting term is the intermodulation at $2\omega_2 \pm \omega_1$
- By symmetry, then, we also generate a term like

$$a_3 S_1^2 S_2 \frac{3}{4}\cos(2\omega_1 \pm \omega_2)$$

- Notice that if $\omega_1 \approx \omega_2$, then the intermodulation $2\omega_2 - \omega_1 \approx \omega_1$

Inband IM3 Distortion



- Now we see that even if the system is narrowband, the output of an amplifier can contain in band intermodulation due to IM_3.
- This is in contrast to IM_2 where the frequency of the intermodulation was at a lower and higher frequency. The IM_3 frequency can fall in-band for two in-band interferer

Definition of IM_3

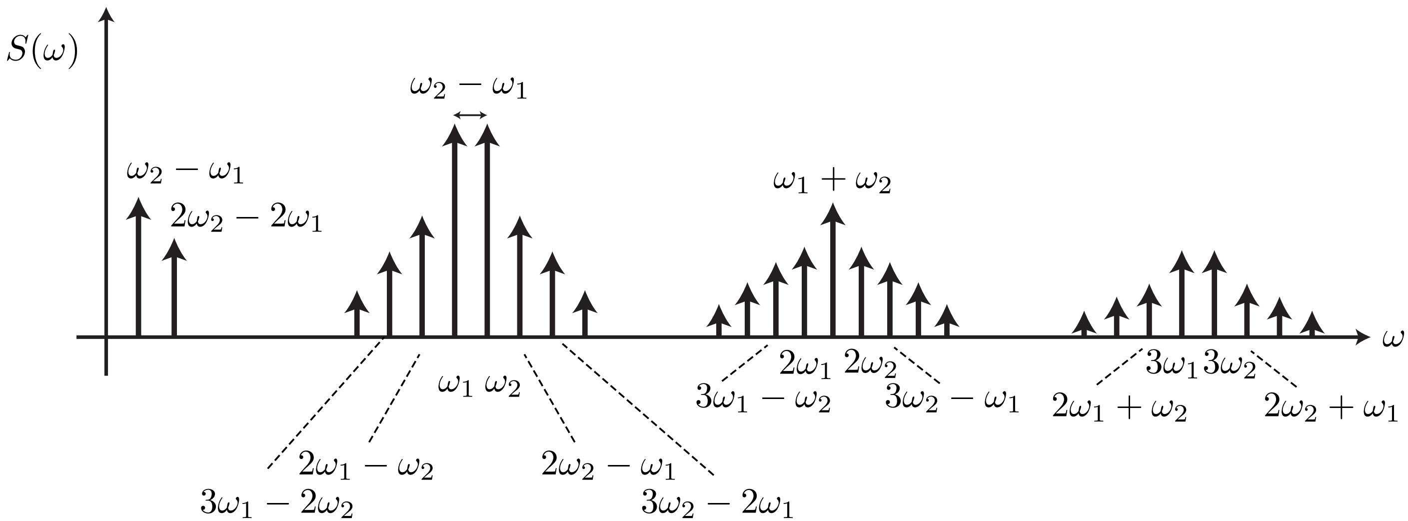
- We define IM_3 in a similar manner for $S_i = S_1 = S_2$

$$IM_3 = \frac{\text{Amp of Third Intermod}}{\text{Amp of Fund}} = \frac{3}{4}\frac{a_3}{a_1}S_i^2$$

- Note the relation between IM_3 and HD_3

$$IM_3 = 3HD_3 = HD_3 + 10\text{dB}$$

Complete Two-Tone Response



- We have so far identified the harmonics and IM_2 and IM_3 products
- A more detailed analysis shows that an order n non-linearity can produce intermodulation at frequencies $j\omega_1 \pm k\omega_2$ where $j + k = n$
- All tones are spaced by the difference $\omega_2 - \omega_1$

Distortion of AM Signals

- Consider a simple AM signal (modulated by a single tone)

$$s(t) = S_2(1 + m\cos\omega_m t)\cos\omega_2 t$$

- where the modulation index $m \le 1$. This can be written as

$$s(t) = S_2\cos\omega_2 t + \frac{m}{2}\cos(\omega_2 - \omega_m)t + \frac{m}{2}\cos(\omega_2 + \omega_m)t$$

- The first term is the RF carrier and the last terms are the modulation sidebands

Cross Modulation

- Cross modulation occurs in AM systems (e.g. video cable tuners)
- The modulation of a large AM signal transfers to another carrier going thru the same amp

$$S_i = \underbrace{S_1 \cos \omega_1 t}_{\text{wanted}} + \underbrace{S_2 (1 + m \cos \omega_m t) \cos \omega_2 t}_{\text{interferer}}$$

- CM occurs when the output contains a term like

$$K(1 + \delta \cos \omega_m t) \cos \omega_1 t$$

- Where δ is called the transferred modulation index

Cross Modulation (cont)

- For $S_o = a_1 S_i + a_2 S_i^2 + a_3 S_i^3 + \cdots$, the term $a_2 S_i^2$ does not produce any CM
- The term $a_3 S_i^3 = \cdots + 3a_3 S_1 \cos\omega_1 t \left(S_2(1 + m\cos\omega_m t)\cos\omega_2 t\right)^2$ is expanded to

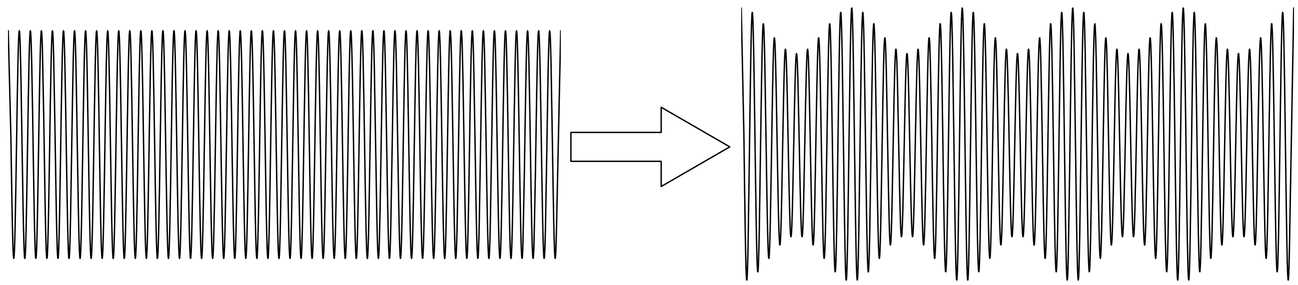
$$= \cdots + 3a_3 S_1 S_2^2 \cos\omega_1 t(1 + 2m\cos\omega_m t + m^2\cos^2\omega_m t)\times$$

$$\tfrac{1}{2}(1 + \cos 2\omega_2 t)$$

- Grouping terms we have in the output

$$S_o = \cdots + a_1 S_1 (1 + 3\frac{a_3}{a_1} S_2^2 m \cos\omega_m t)\cos\omega_1 t$$

CM Definition



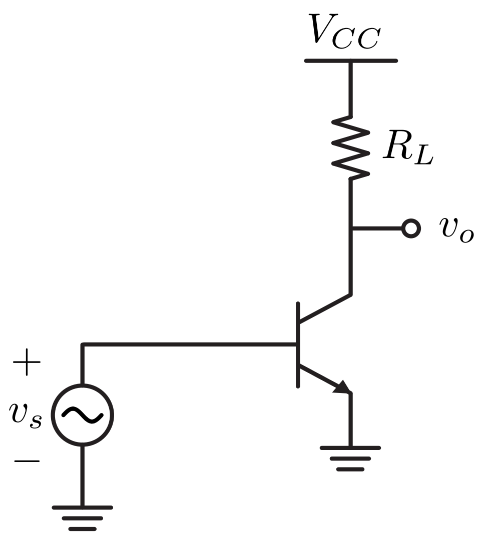
$$CM = \frac{\text{Transferred Modulation Index}}{\text{Incoming Modulation Index}}$$

$$CM = 3\frac{a_3}{a_1}S_2^2 = 4IM_3$$

$$= IM_3(\text{dB}) + 12\text{dB}$$

$$= 12HD_3 = HD_3(\text{dB}) + 22\text{dB}$$

Distortion of BJT Amplifiers



- Consider the CE BJT amplifier shown. The biasing is omitted for clarity.

- The output voltage is simply

$$V_o = V_{CC} - I_C R_C$$

- Therefore the distortion is generated by I_C alone. Recall that

$$I_C = I_S e^{qV_{BE}/kT}$$

BJT CE Distortion (cont)

- Now assume the input $V_{BE} = v_i + V_Q$, where V_Q is the bias point. The current is therefore given by

$$I_C = \underbrace{I_S e^{\frac{V_Q}{V_T}}}_{I_Q} e^{\frac{v_i}{V_T}}$$

- Using a Taylor expansion for the exponential

$$e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \cdots$$

$$I_C = I_Q(1 + \frac{v_i}{V_T} + \frac{1}{2}\left(\frac{v_i}{V_T}\right)^2 + \frac{1}{6}\left(\frac{v_i}{V_T}\right)^3 + \cdots)$$

BJT CE Distortion (cont)

- Define the output signal $i_c = I_C - I_Q$

$$i_c = \frac{I_Q}{V_T} v_i + \frac{1}{2}\left(\frac{q}{kT}\right)^2 I_Q v_i^2 + \frac{1}{6}\left(\frac{q}{kT}\right)^3 I_Q v_i^3 + \cdots$$

- Compare to $S_o = a_1 S_i + a_2 S_i^2 + a_3 S_i^3 + \cdots$

$$a_1 = \frac{qI_Q}{kT} = g_m$$

$$a_2 = \frac{1}{2}\left(\frac{q}{kT}\right)^2 I_Q$$

$$a_3 = \frac{1}{6}\left(\frac{q}{kT}\right)^3 I_Q$$

Example: BJT HD2

- For any BJT (Si, SiGe, Ge, GaAs), we have the following result

$$HD_2 = \frac{1}{4}\frac{q\hat{v}_i}{kT}$$

- where $\hat{v}_i$ is the peak value of the input sine voltage
- For $\hat{v}_i = 10\text{mV}$, $HD_2 = 0.1 = 10\%$
- We can also express the distortion as a function of the output current swing $\hat{i_c}$

$$HD_2 = \frac{1}{2}\frac{a_2}{a_1^2}S_{om} = \frac{1}{4}\frac{\hat{i_c}}{I_Q}$$

- For $\frac{\hat{i_c}}{I_Q} = 0.4$, $HD_2 = 10\%$

Example: BJT IM3

- Let's see the maximum allowed signal for $IM_3 \leq 1\%$

$$IM_3 = \frac{3}{4}\frac{a_3}{a_1}S_1^2 = \frac{1}{8}\left(\frac{q\hat{v}_i}{kT}\right)^2$$

- Solve $\hat{v}_i = 7.3\text{mV}$. That's a pretty small voltage. For practical applications we'd like to improve the linearity of this amplifier.