EECS 142

Integrated Circuits for Communication

Lecture 7: Distortion Analysis

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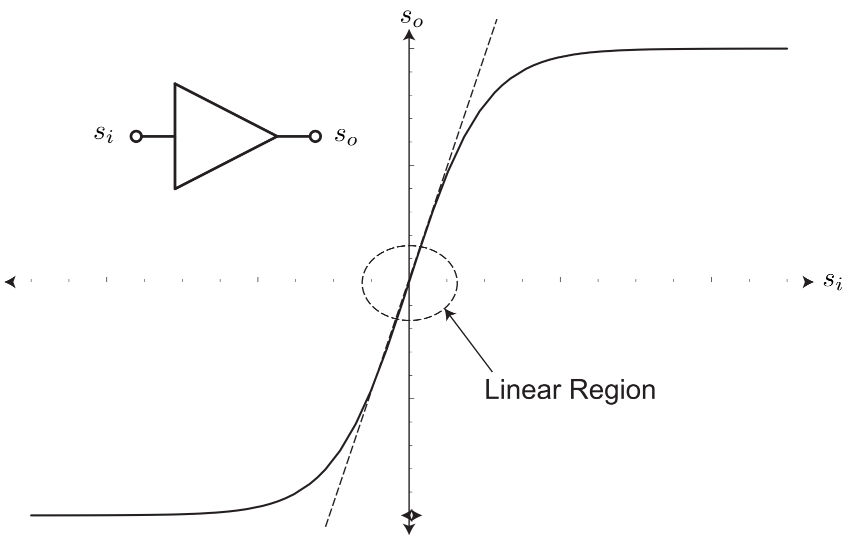
Introduction to Distortion



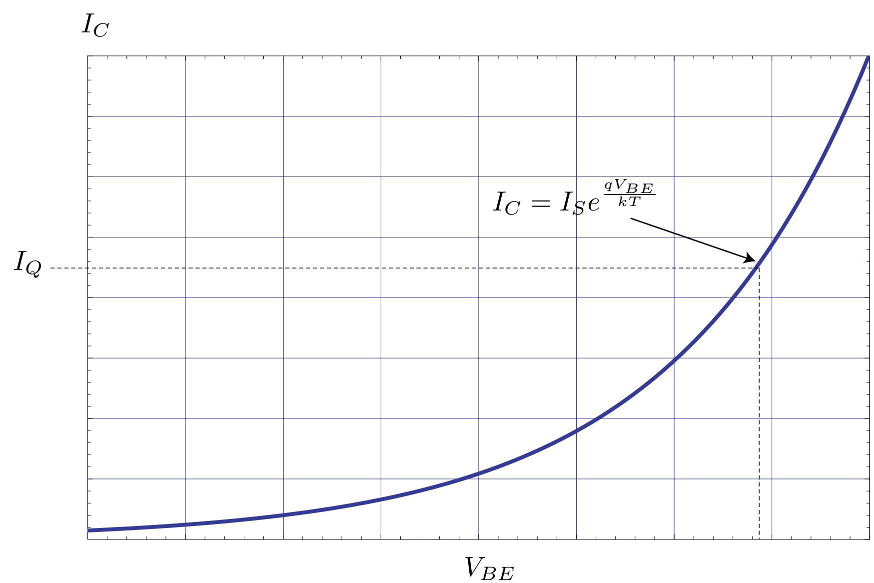
- Up to now we have treated amplifiers as small-signal linear circuits. Since transistors are non-linear, this assumption is only valid for extremeley small signals.
- Consider a class of memoryless non-linear amplifiers. In other words, let's neglect energy storage elements.
- This is the same as saying the output is an instantaneous function of the input. Thus the amplifier has no memory.

Distortion Analaysis Assumptions

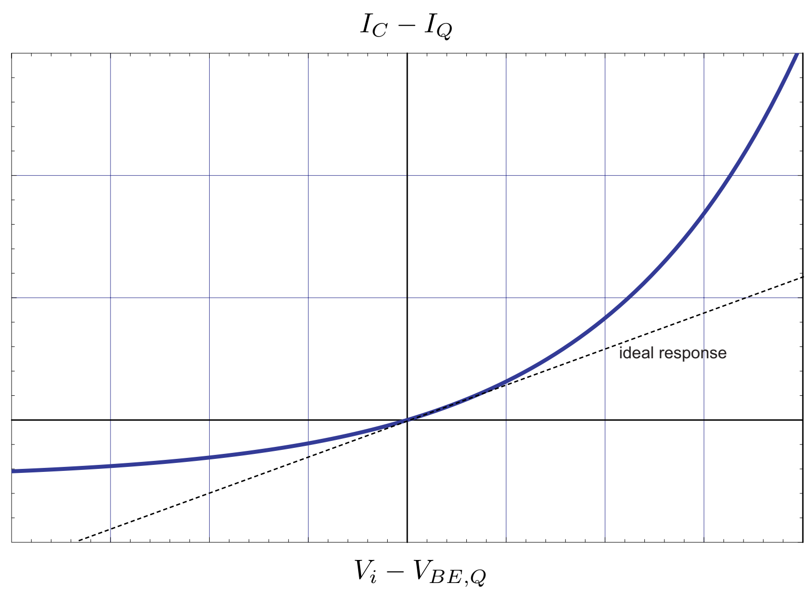
- We also assume the input/output description is sufficiently smooth and continuous as to be accurately described by a power series

$$s_o = a_1 s_i + a_2 s_i^2 + a_3 s_i^3 + \ldots$$

For instance, for a BJT (Si, SiGe, GaAs) operated in forward-active region, the collector current is a smooth function of the voltage V_{BE}

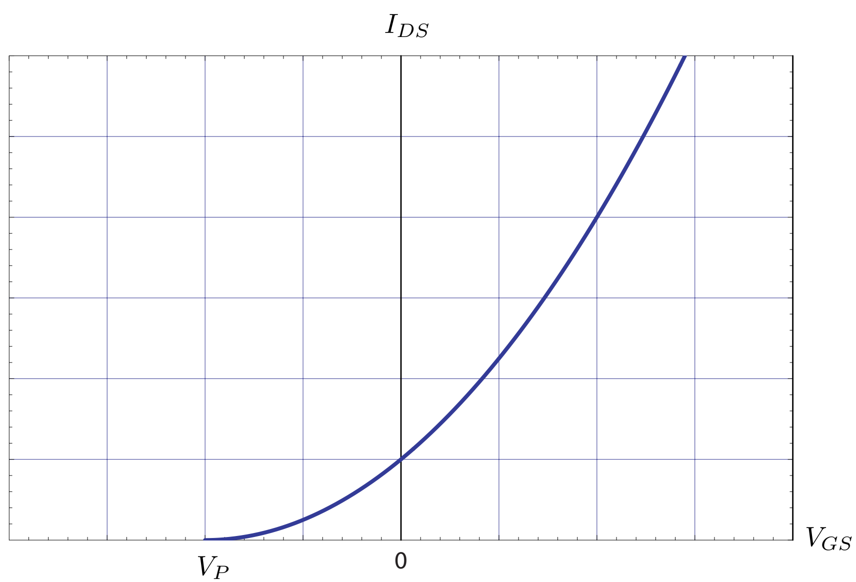


BJT Distortion



- We shift the origin by eliminating the DC signals, $i_o = I_C - I_Q$. The input signal is then applied around the DC level $V_{BE,Q}$.
- Note that an ideal amplifier has a perfectly linear line.

JFET Distortion

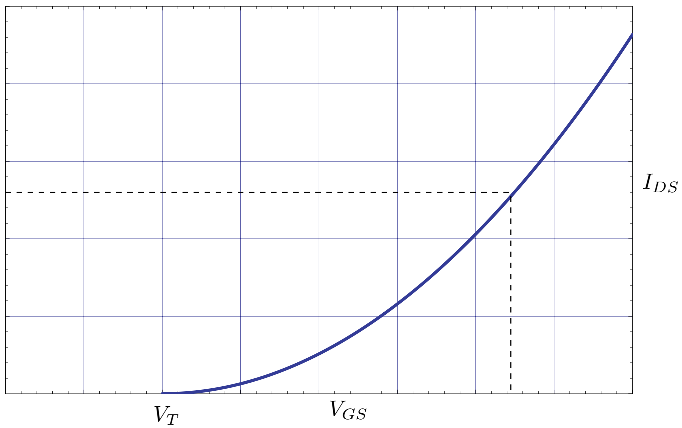


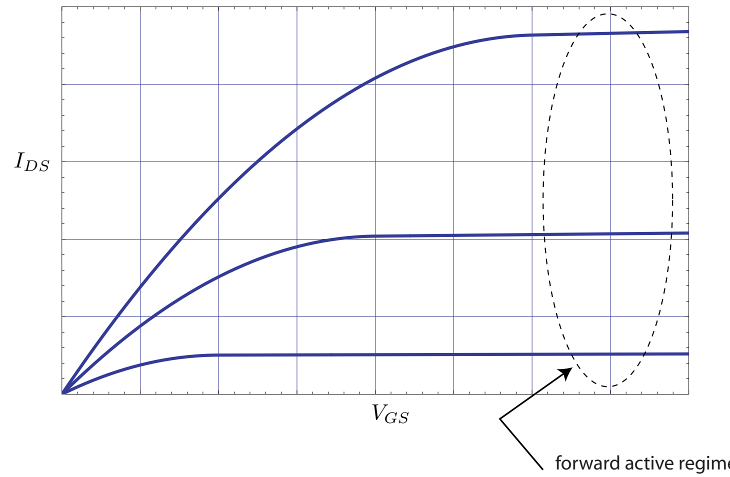
- JFETs are more common devices in RF circuits. The I-V relation is also approximately square law

$$I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$$

- The gate current (junction leakage) is typically very

MOSFET Distortion





- The long-channel device also follows the square law relation (neglecting bulk charge effects)

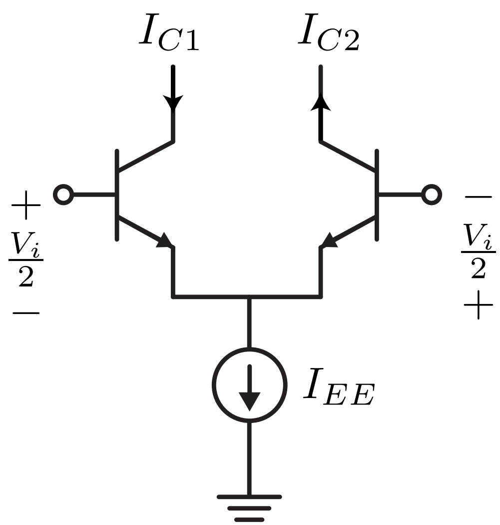
$$I_D = \frac{1}{2}\mu C_{ox}\frac{W}{L}\left(V_{GS} - V_t\right)^2\left(1 + \lambda V_{DS}\right)$$

- This is assuming the device does not leave the forward active (saturation) regime.

MOSFET Model

- Note that the device operation near threshold is not captured by our simple square-law equation
- The I-V curve of a MOSFET in moderate and weak inversion is easy to describe in a “piece-meal” fashion, but difficult to capture with a single equation.
- Short-channel devices are even more difficult due to velocity saturation and drain induced barrier lowering.

Differential Pair



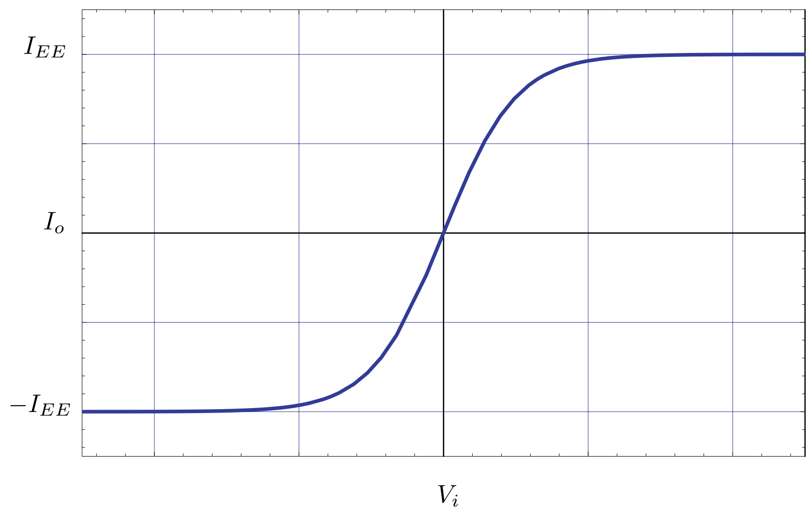
The differential pair is an important analog and RF building block.

- For a BJT diff pair, we have $V_i = V_{BE1} - V_{BE2}$

$$I_{C1,2} = I_S e^{\frac{qV_{BE1,2}}{kT}}$$

- The sum of the collector currents are equal to the current source $I_{C1} + I_{C2} = I_{EE}$

BJT Diff Pair



- The ideal BJT diff pair I-V relationship (neglecting base and emitter resistance) is give by

$$I_o = I_{C1} - I_{C2} = \alpha I_{EE} \tanh \frac{qV_i}{2kT}$$

- Notice that the output current saturates for large input voltages

Power Series Relation

- For a general circuit, let's represent this behavior with a power series

$$s_o = a_1 s_i + a_2 s_i^2 + a_3 s_i^3 + \ldots$$

- a_1 is the small signal gain
- The coefficients a_1,a_2,a_3,$\ldots$ are independent of the input signal s_i *but* they depend on bias, temperature, and other factors.

Harmonic Distortion

- Assume we drive the amplifier with a time harmonic signal at frequency ω_1

$$s_i = S_1 \cos \omega_1 t$$

- A linear amplifier would output $s_o = a_1 S_1 \cos \omega_1 t$ whereas our amplifier generates

$$s_o = a_1 S_1 \cos \omega_1 t + a_2 S_1^2 \cos^2 \omega_1 t + a_3 S_1^3 \cos^3 \omega_1 t + \dots$$

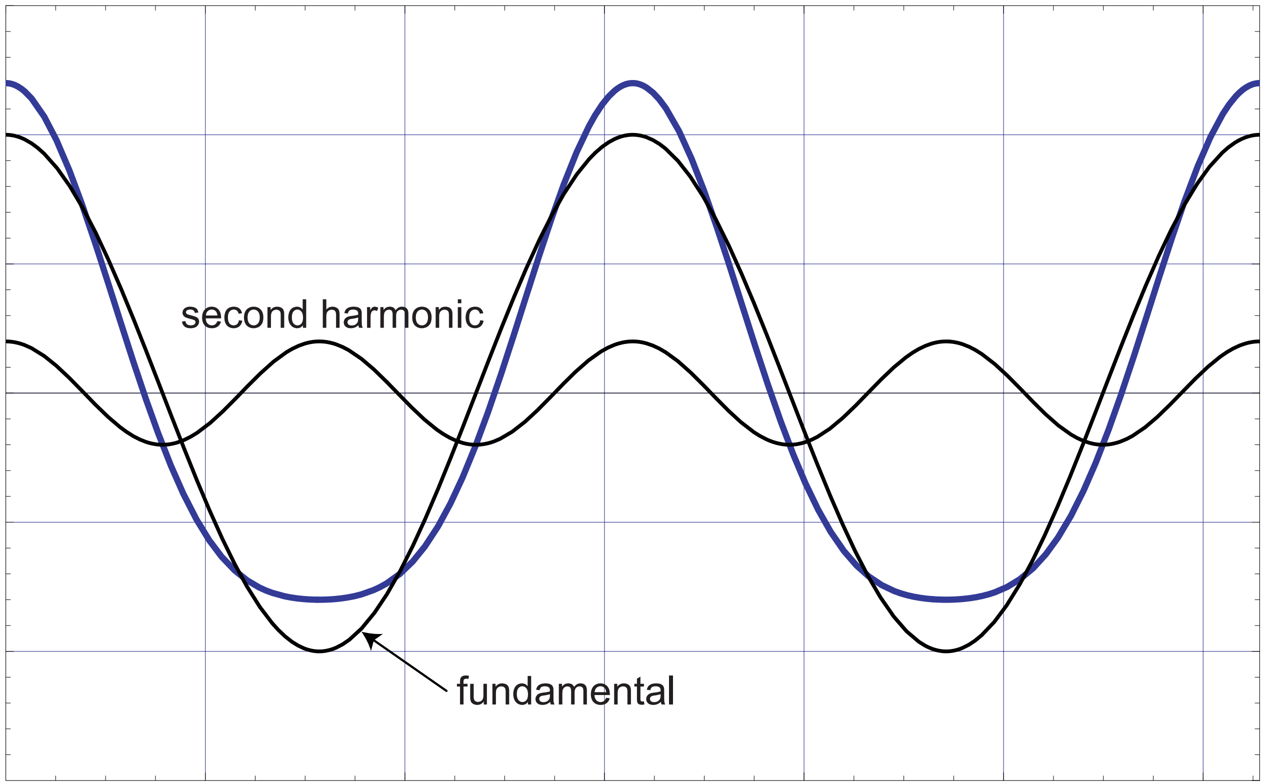
or

$$s_o = a_1 S_1 \cos \omega_1 t + \frac{a_2 S_1^2}{2}(1 + \cos 2\omega_1 t) + \\ \frac{a_3 S_1^3}{4}(\cos 3\omega_1 t + 3 \cos \omega_1 t) + \dots$$

Harmonic Distortion (cont)

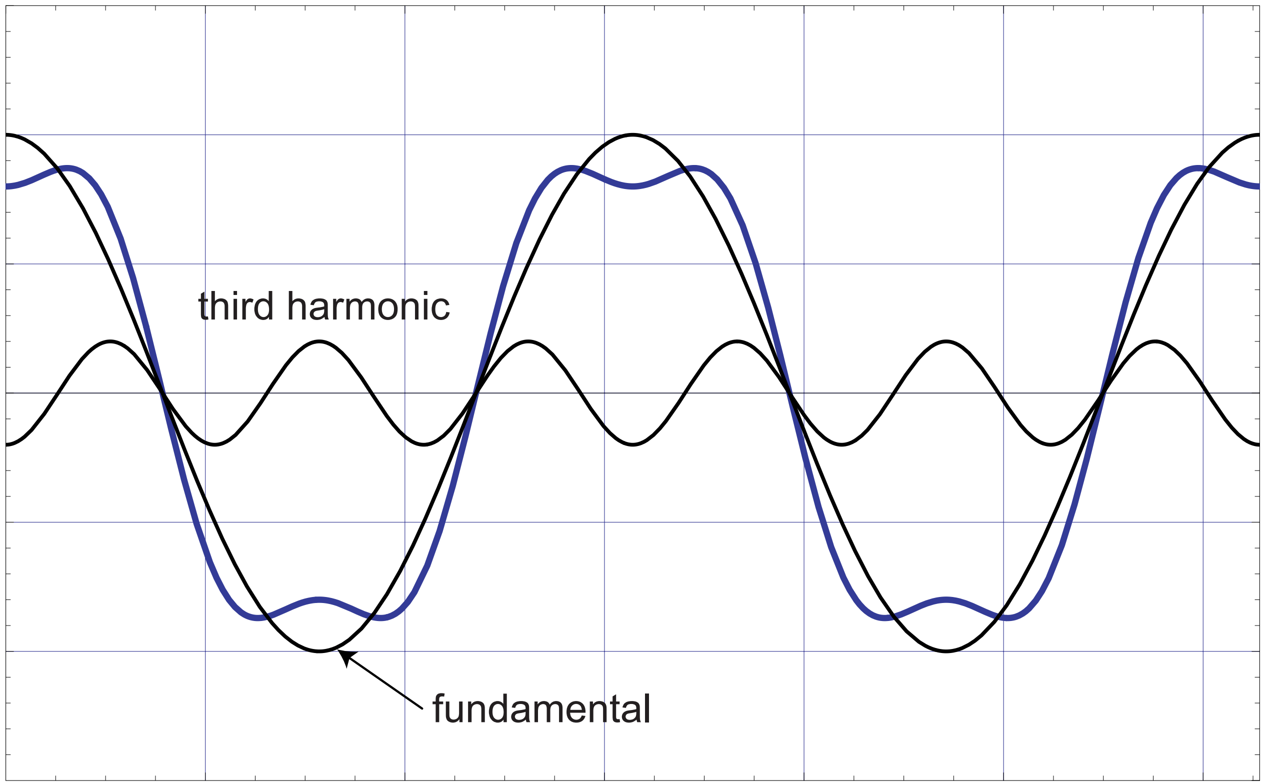
- The term $a_1 s_1 \cos \omega_1 t$ is the wanted signal.
- Higher harmonics are also generated. These are unwanted and thus called "distortion" terms. We already see that the second-harmonic $\cos 2\omega_1 t$ and third harmonic $\cos 3\omega_1 t$ are generated.
- Also the second order non-linearity produces a DC shift of $\frac{1}{2} a_2 S_1^2$.
- The third order generates both third order distortion and more fundamental. The sign of a_1 and a_3 determine whether the distortion product $a_3 S_1^3 \frac{3}{4} \cos \omega_1 t$ adds or subtracts from the fundamental.
- If the signal adds, we say there is gain expansion. If it subtracts, we say there is gain compression.

Second Harmonic Distortion Waveforms



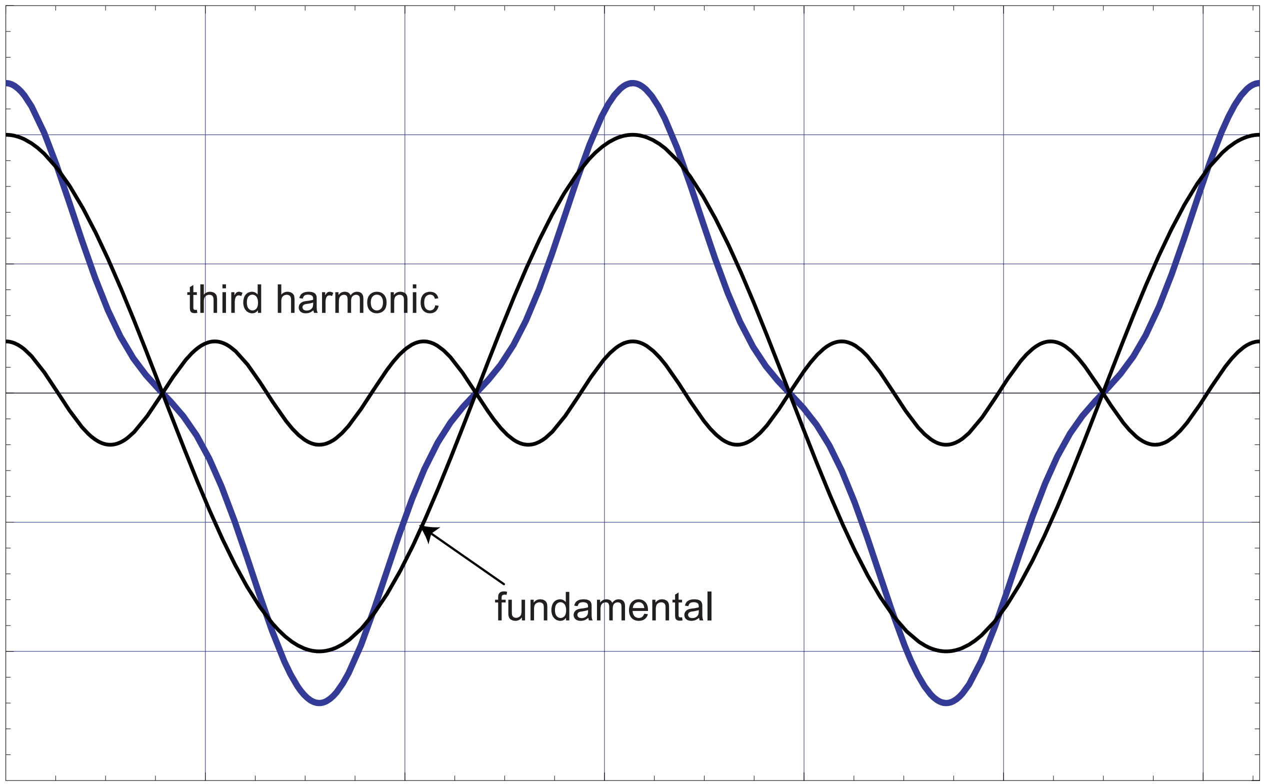
- The figure above demonstrates the waveform distortion due to second harmonic only.

Third Harmonic Distortion Waveform



- The above figure shows the effects of the third harmonic, where we assume the third harmonic is in phase with the fundamental.

Third Harmonic Waveform (cont)



- The above figure shows the effects of the third harmonic, where we assume the third harmonic is out of phase with the fundamental.

General Distortion Term

- Consider the term $\cos^n\theta = \frac{1}{2^n}\left(e^{j\theta} + e^{-j\theta}\right)^n$. Using the Binomial formula, we can expand to

$$= \frac{1}{2^n}\sum_{k=0}^{n}\binom{n}{k} e^{jk\theta} e^{-j(n-k)\theta}$$

- For $n = 3$

$$= \frac{1}{8}\left(\binom{3}{0}e^{-j3\theta} + \binom{3}{1}e^{j\theta}e^{-j2\theta} + \binom{3}{2}e^{j2\theta}e^{-j\theta} + \binom{3}{3}e^{j3\theta}\right)$$

$$= \frac{1}{8}\left(e^{-j3\theta} + e^{j3\theta}\right) + \frac{1}{8}3\left(e^{j\theta} + e^{-j\theta}\right) = \frac{1}{4}\cos 3\theta + \frac{3}{4}\cos\theta$$

General Distortion Term (cont)

- We can already see that for an odd power, we will see a nice pairing up of positive and negative powers of exponentials
- For the even case, the middle term is the unpaired DC term

$$\binom{2k}{k} e^{jk\theta} e^{-jk\theta} = \binom{2k}{k}$$

- So only even powers in the transfer function can shift the DC operation point.
- The general term in the binomial expansion of $(x + x^{-1})^n$ is given by

$$\binom{n}{k} x^{n-k} x^{-k} = \binom{n}{k} x^{n-2k}$$

General Distortion Term (cont)

- The term $\binom{n}{k}x^{n-2k}$ generates every other harmonic.
- If n is even, then only even harmonics are generated. If n is odd, likewise, only odd harmonics are generated.
- Recall that an "odd" function $f(-x) = -f(x)$ (anti-symmetric) has an odd power series expansion

$$f(x) = a_1 x + a_3 x^3 + a_5 x^5 + \ldots$$

- Whereas an even function, $g(-x) = g(x)$, has an even power series expansion

$$g(x) = a_0 + a_2 x^2 + a_4 x^4 + \ldots$$