EECS 142

Integrated Circuits for Communication

Lecture 4: Two-Port Circuits and Power Gain

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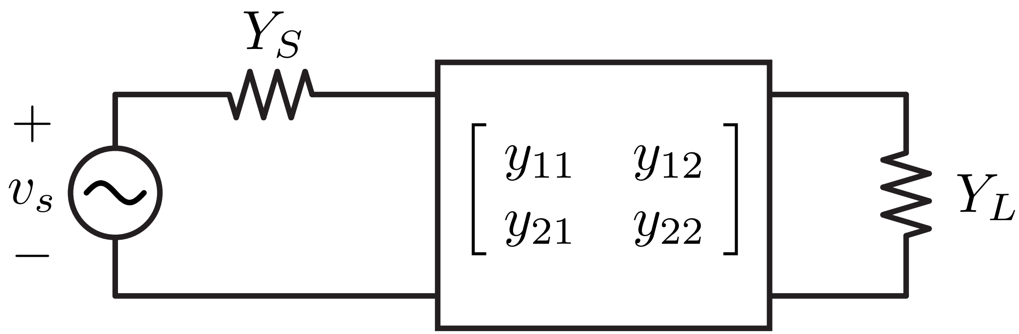
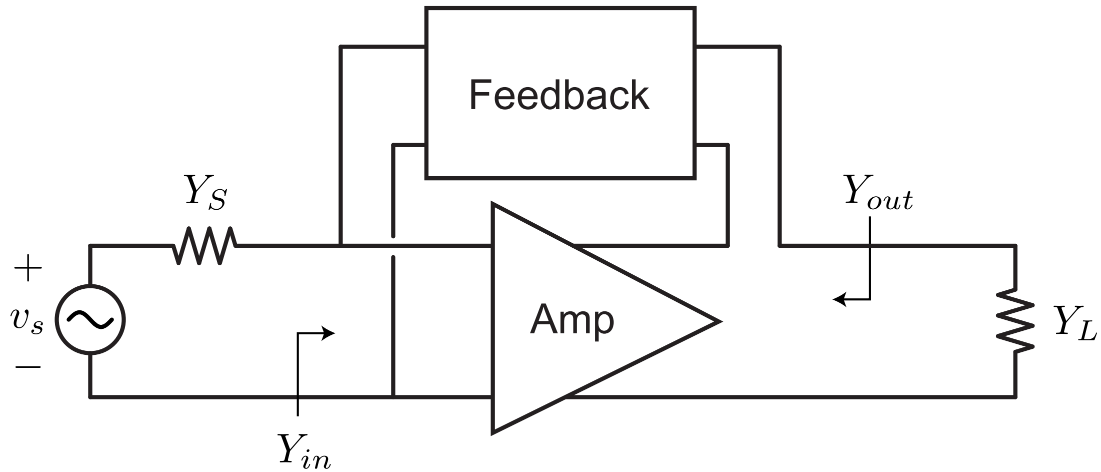
A Generic Amplifier



- Consider the generic two-port amplifier shown above. Note that any two-port linear and time-invariant circuit can be described in this way.
- We can use any two-port parameter set, including admittance parameters Y, impedance parameters Z, or hybrid or inverse-hybrid parameters H or G.

Choosing Two-Port Parameters



- The choice of parameter set is usually determined by convenience. For instance, if shunt feedback is applied, Y parameters are most convenient, whereas series feedback favors Z parameters. Other combinations of shunt/series can be easily described by H or G.
- $ABCD$ parameters are useful for cascading two-ports.

Y Parameters

- We'll primarily use the Y parameters

$$\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

- But in fact the choice depends largely on convenience. Often the form of feedback determines the best choice.
- All 2-port parameters are equivalent. Many of the results that we derive carry in terms of Y-parameters can be applied to other two-port parameters (input impedance, output impedance, gain, etc).

Admittance Parameters

- Notice that y_{11} is the short circuit input admittance

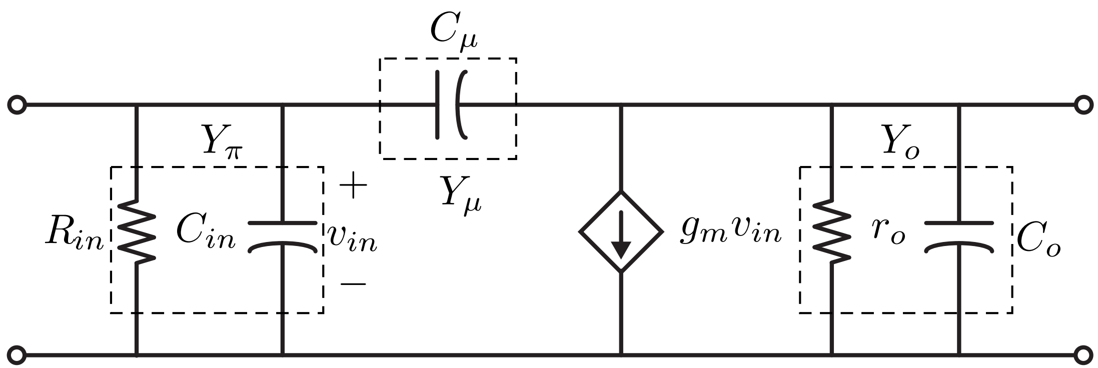
$$y_{11} = \left. \frac{i_1}{v_1} \right|_{v_2=0}$$

- The same can be said of y_{22}. The forward transconductance is described by y_{21}

$$y_{21} = \left. \frac{i_2}{v_1} \right|_{v_2=0}$$

- whereas the reverse transconductance is described by y_{12}.
- If a two-port amplifier is unilateral, then $y_{12} = 0$

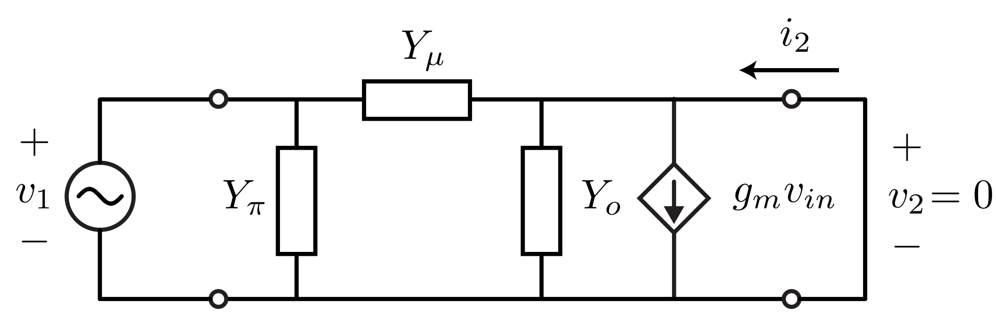
Hybrid-Π Admittance Parameters



- Let's compute the Y parameters for the common hybrid-Π model

$$y_{11} = y_\pi + y_\mu$$

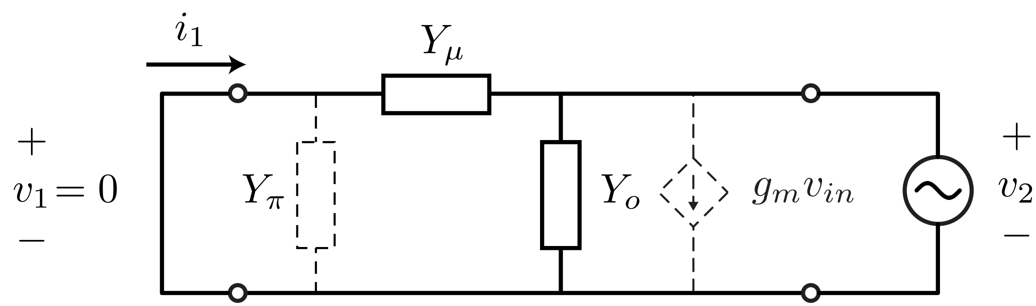
$$y_{21} = g_m - y_\mu$$



Admittance Parameters (cont)

$$y_{22} = y_o + y_\mu$$

$$y_{12} = -y_\mu$$



- Note that the hybrid-π model is unilateral if $y_\mu = sC_\mu = 0$. Therefore it's unilateral at DC.
- A good amplifier has a high ratio $\frac{y_{21}}{y_{12}}$ because we expect the forward transconductance to dominate the behavior

Why Use Two-Port Parameters?

- The parameters are generic and independent of the details of the amplifier $\rightarrow$ can be a single transistor or a multi-stage amplifier
- High frequency transistors are more easily described by two-port parameters (due to distributed input gate resistance and induced channel resistance)
- Feedback amplifiers can often be decomposed into an equivalent two-port unilateral amplifier and a two-port feedback section
- We can make some very general conclusions about the "optimal" power gain of a two-port, allowing us to define some useful metrics

Voltage Gain and Input Admittance

- Since $i_2 = -v_2 Y_L$, we can write

$$(y_{22} + Y_L)v_2 = -y_{21}v_1$$

- Which leads to the "internal" two-port gain

$$A_v = \frac{v_2}{v_1} = \frac{-y_{21}}{y_{22} + Y_L}$$

- Check low freq limit for a hybrid-Π: $A_v = -g_m Z_o || Z_L$ ✓
- The input admittance is easily calculated from the voltage gain

$$Y_{in} = \frac{i_1}{v_1} = y_{11} + y_{12}\frac{v_2}{v_1}$$

$$Y_{in} = y_{11} - \frac{y_{12}y_{21}}{y_{22} + Y_L}$$

Output Admittance

- By symmetry we can write down the output admittance by inspection

$$Y_{out} = y_{22} - \frac{y_{12}y_{21}}{y_{11} + Y_S}$$

- Note that for a unilateral amplifier $y_{12} = 0$ implies that

$$Y_{in} = y_{11}$$

$$Y_{out} = y_{22}$$

- The input and output impedance are de-coupled!

External Voltage Gain

- The gain from the voltage source to the output can be derived by a simple voltage divider equation

$$A_v' = \frac{v_2}{v_s} = \frac{v_2}{v_1}\frac{v_1}{v_s} = A_v \frac{Y_S}{Y_{in} + Y_S} = \frac{-Y_S y_{21}}{(y_{22} + Y_L)(Y_S + Y_{in})}$$

- If we substitute and simplify the above equation we have

$$A_v' = \frac{-Y_S y_{21}}{(Y_S + y_{11})(Y_L + y_{22}) - y_{12}y_{21}}$$

- Verify that this makes sense at low frequency for hybrid-Π:

$$A_v'(DC) = \frac{-Y_S y_{21}}{(Y_S + y_{11})(Y_L + y_{22})} = \frac{Z_{in}}{Z_{in} + Z_S} \times -g_m R_L || r_o$$

Feedback Amplifiers and Y-Params

- Note that in an ideal feedback system, the amplifier is unilateral and the closed loop gain is given by $\frac{y}{x} = \frac{A}{1+Af}$
- We found last lecture that the voltage gain of a general two-port driven with source admittance Y_S is given by

$$A_v' = \frac{-Y_S y_{21}}{(Y_S + y_{11})(Y_L + y_{22}) - y_{12}y_{21}}$$

- If we unilaterize the two-port by arbitrarily setting $y_{12} = 0$, we have an "open" loop forward gain of

$$A_{vu} = A_v'\big|_{y_{12}=0} = \frac{-Y_S y_{21}}{(Y_S + y_{11})(Y_L + y_{22})}$$

Identification of Loop Gain

- Re-writing the gain A'_v by dividing numerator and denominator by the factor $(Y_S + y_{11})(Y_L + y_{22})$ we have

$$A'_v = \frac{\frac{-Y_S y_{21}}{(Y_S+y_{11})(Y_L+y_{22})}}{1 - \frac{y_{12}y_{21}}{(Y_S+y_{11})(Y_L+y_{22})}}$$

- We can now see that the “closed” loop gain with $y_{12} \neq 0$ is given by

$$A'_v = \frac{A_{vu}}{1+T}$$

- where T is identified as the loop gain

$$T = A_{vu} f = \frac{-y_{12}y_{21}}{(Y_S + y_{11})(Y_L + y_{22})}$$

The Feedback Factor and Loop Gain

- Using the last equation also allows us to identify the feedback factor

$$f = \frac{Y_{12}}{Y_S}$$

- If we include the loading by the source Y_S, the input admittance of the amplifier is given by

$$Y_{in} = Y_S + y_{11} - \frac{y_{12}y_{21}}{Y_L + y_{22}}$$

- Note that this can be re-written as

$$Y_{in} = (Y_S + y_{11}) \left(1 - \frac{y_{12}y_{21}}{(Y_S + y_{11})(Y_L + y_{22})}\right)$$

Feedback and Input/Output Admittance

- The last equation can be re-written as

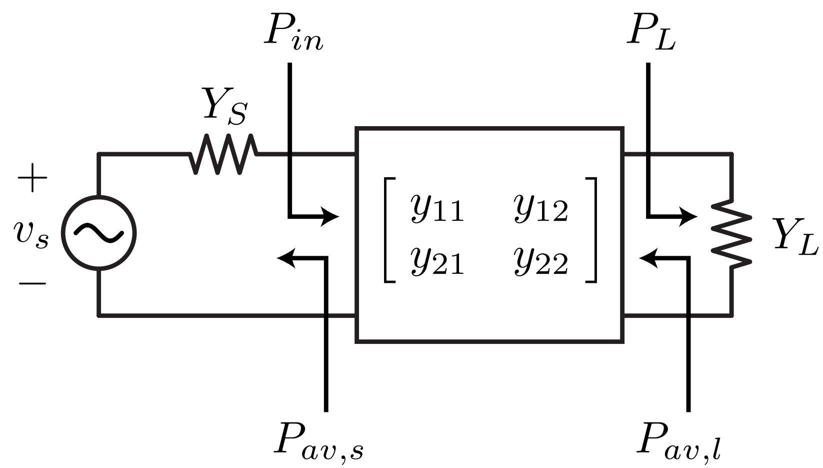
$$Y_{in} = (Y_S + y_{11})(1 + T)$$

- Since $Y_S + y_{11}$ is the input admittance of a unilateral amplifier, we can interpret the action of the feedback as raising the input admittance by a factor of $1 + T$.
- Likewise, the same analysis yields

$$Y_{out} = (Y_L + y_{22})(1 + T)$$

- It's interesting to note that the same equations are valid for series feedback using Z parameters, in which case the action of the feedback is to boost the input and output impedance.

Power Gain



- We can define power gain in many different ways. The *power gain* G_p is defined as follows

$$G_p = \frac{P_L}{P_{in}} = f(Y_L, Y_{ij}) \neq f(Y_S)$$

- We note that this power gain is a function of the load admittance Y_L and the two-port parameters Y_{ij}.

Power Gain (cont)

- The *available power gain* is defined as follows

$$G_a = \frac{P_{av,L}}{P_{av,S}} = f(Y_S, Y_{ij}) \neq f(Y_L)$$

- The available power from the two-port is denoted $P_{av,L}$ whereas the power available from the source is $P_{av,S}$.
- Finally, the *transducer gain* is defined by

$$G_T = \frac{P_L}{P_{av,S}} = f(Y_L, Y_S, Y_{ij})$$

- This is a measure of the efficacy of the two-port as it compares the power at the load to a simple conjugate match.

Derivation of Power Gain

- The power gain is readily calculated from the input admittance and voltage gain

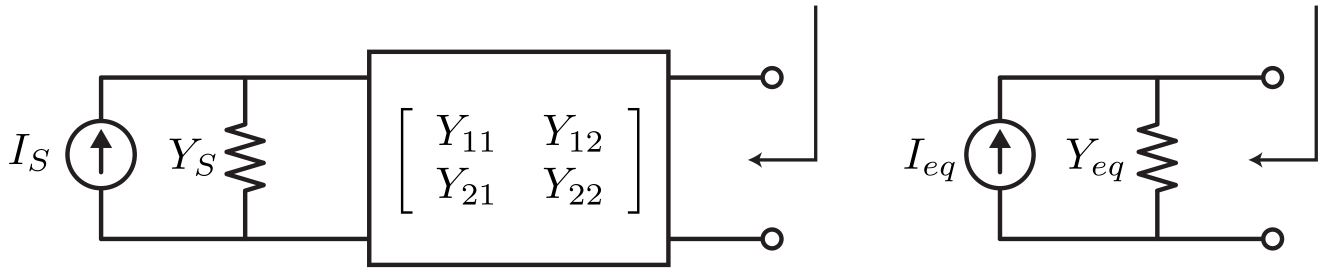
$$P_{in} = \frac{|V_1|^2}{2}\Re(Y_{in})$$

$$P_L = \frac{|V_2|^2}{2}\Re(Y_L)$$

$$G_p = \left|\frac{V_2}{V_1}\right|^2 \frac{\Re(Y_L)}{\Re(Y_{in})}$$

$$G_p = \frac{|Y_{21}|^2}{|Y_L + Y_{22}|^2}\frac{\Re(Y_L)}{\Re(Y_{in})}$$

Derivation of Available Gain



- To derive the available power gain, consider a Norton equivalent for the two-port where

$$I_{eq} = I_2 = Y_{21} V_1 = \frac{Y_{21}}{Y_{11} + Y_S} I_S$$

- The Norton equivalent admittance is simply the output admittance of the two-port

$$Y_{eq} = Y_{22} - \frac{Y_{21} Y_{12}}{Y_{11} + Y_S}$$

Available Gain (cont)

- The available power at the source and load are given by

$$P_{av,S} = \frac{|I_S|^2}{8\Re(Y_S)} \qquad P_{av,L} = \frac{|I_{eq}|^2}{8\Re(Y_{eq})}$$

$$G_a = \frac{|Y_{21}|^2}{|Y_{11} + Y_S|^2} \frac{\Re(Y_S)}{\Re(Y_{eq})}$$

Transducer Gain Derivation

- The transducer gain is given by

$$G_T = \frac{P_L}{P_{av,S}} = \frac{\frac{1}{2}\Re(Y_L)|V_2|^2}{\frac{|I_S|^2}{8\Re(Y_S)}} = 4\Re(Y_L)\Re(Y_S)\left|\frac{V_2}{I_S}\right|^2$$

- We need to find the output voltage in terms of the source current. Using the voltage gain we have and input admittance we have

$$\left|\frac{V_2}{V_1}\right| = \left|\frac{Y_{21}}{Y_L + Y_{22}}\right|$$

$$I_S = V(Y_S + Y_{in})$$

$$\left|\frac{V_2}{I_S}\right| = \left|\frac{Y_{21}}{Y_L + Y_{22}}\right| \frac{1}{|Y_S + Y_{in}|}$$

Transducer Gain (cont)

$$|Y_S + Y_{in}| = \left| Y_S + Y_{11} - \frac{Y_{12}Y_{21}}{Y_L + Y_{22}} \right|$$

- We can now express the output voltage as a function of source current as

$$\left| \frac{V_2}{I_S} \right|^2 = \frac{|Y_{21}|^2}{|(Y_S + Y_{11})(Y_L + Y_{22}) - Y_{12}Y_{21}|^2}$$

- And thus the transducer gain

$$G_T = \frac{4\Re(Y_L)\Re(Y_S)|Y_{21}|^2}{|(Y_S + Y_{11})(Y_L + Y_{22}) - Y_{12}Y_{21}|^2}$$

Comparison of Power Gains

- It's interesting to note that *all* of the gain expression we have derived are in the exact same form for the impedance, hybrid, and inverse hybrid matrices.
- In general, $P_L \leq P_{av,L}$, with equality for a matched load. Thus we can say that

$$G_T \leq G_a$$

- The maximum transducer gain as a function of the load impedance thus occurs when the load is conjugately matched to the two-port output impedance

$$G_{T,max,L} = \frac{P_L(Y_L = Y_{out}^*)}{P_{av,S}} = G_a$$

Comparison of Power Gains (cont)

- Likewise, since $P_{in} \leq P_{av,S}$, again with equality when the the two-port is conjugately matched to the source, we have

$$G_T \leq G_p$$

- The transducer gain is maximized with respect to the source when

$$G_{T,max,S} = G_T(Y_{in} = Y_S^*) = G_p$$