EECS 142

Integrated Circuits for Communication

Lecture 3: High-Speed Amplifiers and Tuned Amplifiers

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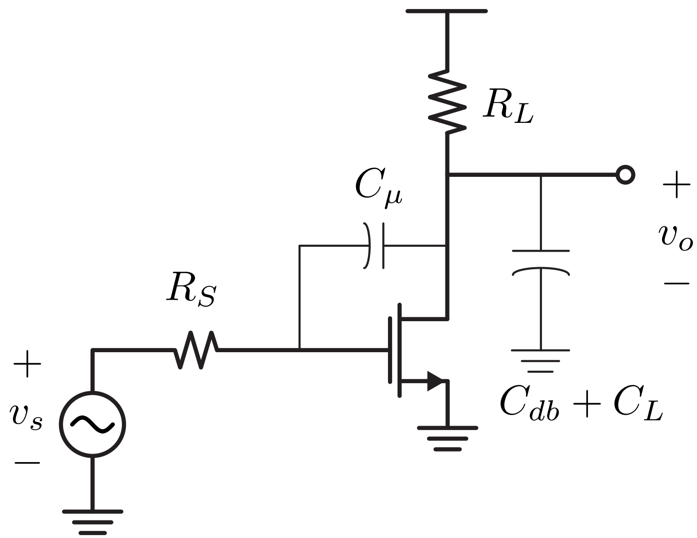
CS/CE Amplifier Bandwidth



- Due to Miller multiplication, the input cap is usually the dominant pole

$$\omega_0^{-1} \approx R_s(C_{in} + |A_v|C_\mu)$$

$$\omega_0^{-1} = R_s C_{in}(1 + \mu|A_v|) \approx R_s C_{in} \mu |A_v|$$

CS/CE Amplifier Gain-Bandwidth

- Assuming the voltage gain is given by the low-frequency value of $g_m R_L$, we have

$$\omega_0^{-1} = R_s C_{in} \mu g_m R_L = (g_m R_s)(g_m R_L) \frac{C_{in}}{g_m} \mu$$

$$\omega_0^{-1} = |A_v|^2 \frac{R_s}{R_L} \omega_T^{-1} \mu$$

- The amplifier has a bandwidth reduction factor of A_v^2

$$\omega_0 \times |A_v|^2 = \omega_T \times \left(\frac{R_L}{R_s} \right) \times \frac{1}{\mu}$$

Bandwidth Example

- Say we need a gain of 60 dB ($A_v = 1000$) and $\frac{R_L}{R_s} = 2$. The technology has a capacitance ratio of $\mu = 0.2$:

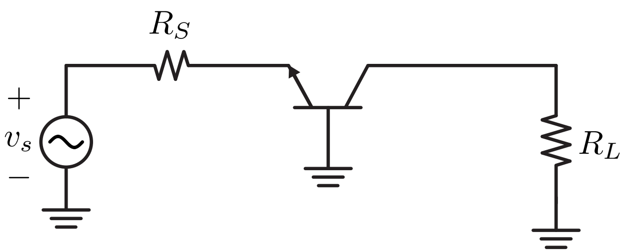
$$\omega_0 |A_v|^2 = 10^6 \omega_0 = \omega_T \times 2 \times 5$$

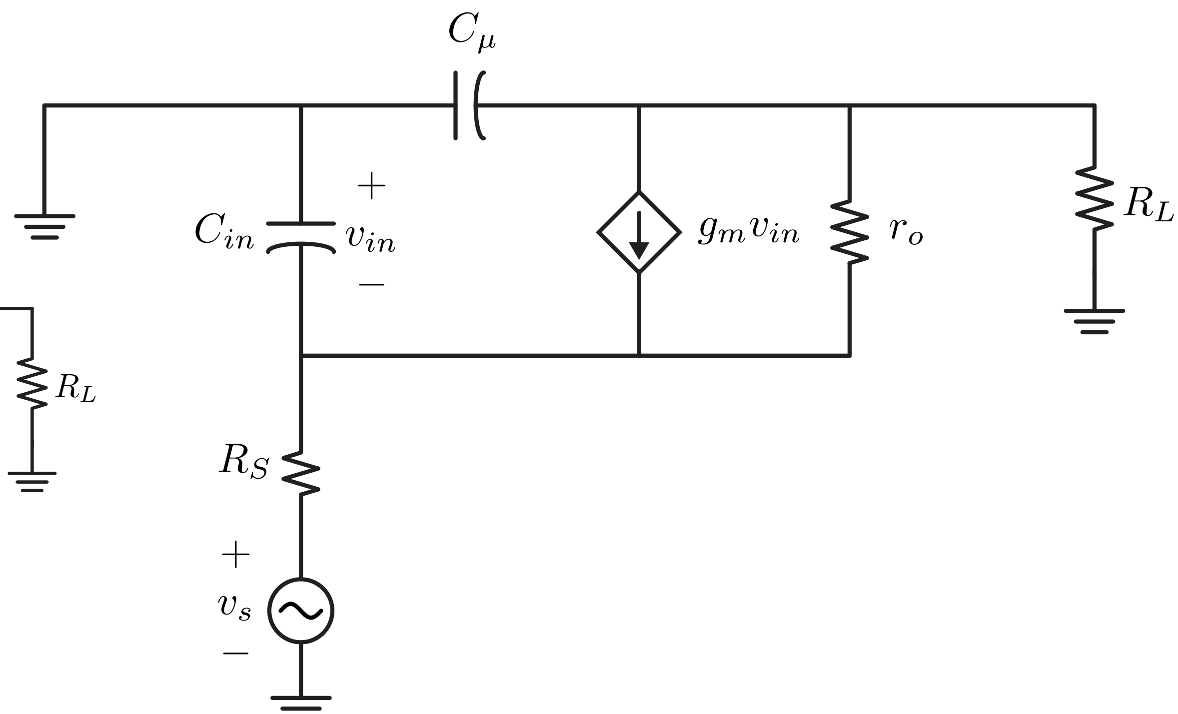
$$\omega_0 = \frac{\omega_T}{10^5}$$

- Compare this to a current mirror amplifier. When we follow the "normal" gain-bandwidth tradeoff, we have

$$\omega_0 = \frac{\omega_T}{A_i} = \frac{\omega_T}{1000}$$

Common Base Amplifier





- Write KCL at base node of circuit

$$\frac{v_s + v_{in}}{R_s} + g_m v_{in} + sC_{in} v_{in} = 0 \quad v_s = -v_{in}(1 + g_m R_s + sC_{in} R_s)$$

Common Base Amp (cont)

- And write KCL at the output node

$$(sC_o + \frac{1}{R_L})v_o + g_m v_{in} + sC_\mu v_o = 0$$

$$v_o \left(\frac{1}{R_L} + s(C_o + C_\mu) \right) = -g_m v_{in}$$

- The voltage gain is a product of two terms

$$A_v = \frac{v_o}{v_s} = \frac{-g_m R_L}{1 + s(C_o + C_\mu)R_L} \frac{v_x}{v_s}$$

$$A_v = \frac{G_m R_L}{(1 + s(C_o + C_\mu)R_L)(1 + sR_s \frac{C_{in}}{1+g_m R_s})}$$

Common Base Bandwidth

- Note the transconductance is degenerated, $G_m = g_m/(1 + g_m R_s)$. Note that the input capacitance is also degenerated by the action of series feedback.
- Unlike a CE/CS ampilifier, the poles do not interact (due to absence of feedback capacitor)
- First let's take the limit of high loop gain, $g_m R_s \gg 1$

$$A_v = \frac{\frac{R_L}{R_s}}{(1 + s/\omega_T)(1 + s/\omega_L)}$$

where $\omega_L = ((C_o + C_\mu)R_L)^{-1}$ is the pole at the output.

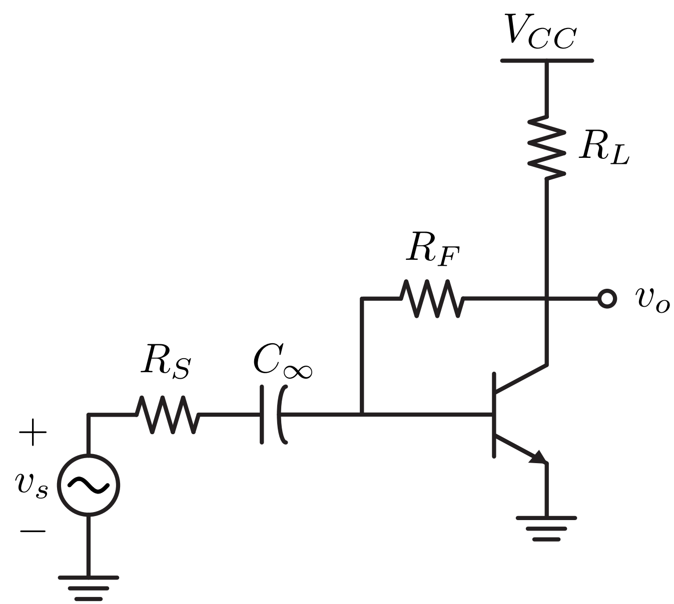
Matched Common Base Amp

- The common-base amplifier has the nice property that the input impedance is low (roughly $1/g_m$) and broadband, thus easily providing a termination to the driver (a filter, the antenna, or a previous stage). If we assume that $R_s = 1/g_m$, we have

$$A_v = \frac{\frac{1}{2} g_m R_L}{(1 + s/2\omega_T)(1 + s/\omega_L)}$$

- The 3dB bandwidth is thus most likely set by the time constant at the load.

Shunt Feedback Amp



- The shunt-feedback amplifier is a nice high-frequency broadband amplifier building block. The action of the shunt feedback is used to lower the input impedance and to set the gain.

Shunt Feedback Gain / Input Resis

- The in-band voltage gain and input impedance is given by (see PS 1)

$$A_v = \frac{-R_F}{R_s}$$

$$R_{in} = (1 + \frac{R_F}{R_L})\frac{1}{g_m}$$

- For an input match, $R_s = (1 + \frac{R_F}{R_L})\frac{1}{g_m}$, or $g_m R_s = (1 + \frac{R_F}{R_L})$
- Since the voltage gain sets R_F, the input impedance match determines the required transconductance g_m (and hence the power dissipation)
- A bipolar version will dissipate much less power due to the higher intrinsic g_m

Shunt Feedback Amp BW

- The amplifier is broadband and approximately obeys the classic gain-bandwidth tradeoff $A_v \omega_0 \approx \omega_T$
- A zero-value time constant analysis identifies the dominant pole

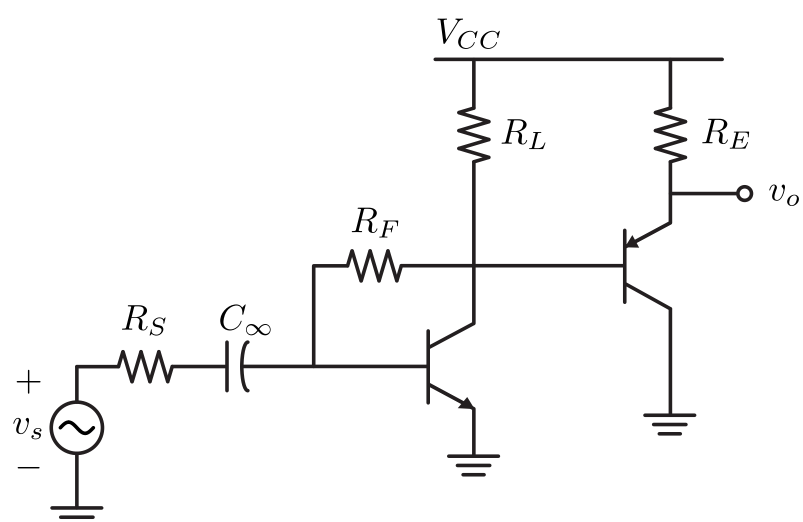
$$\tau_1 = C_{in} \left(R_s || r_\pi || \frac{R_F (1 + \frac{R_L}{R_F})}{1 + g_m R_L} \right)$$

$$\tau_2 = C_\mu \left(R_F || \frac{R_L (R_s || r_\pi)}{R_s || r_\pi || \frac{1}{g_m} || R_L} \right)$$

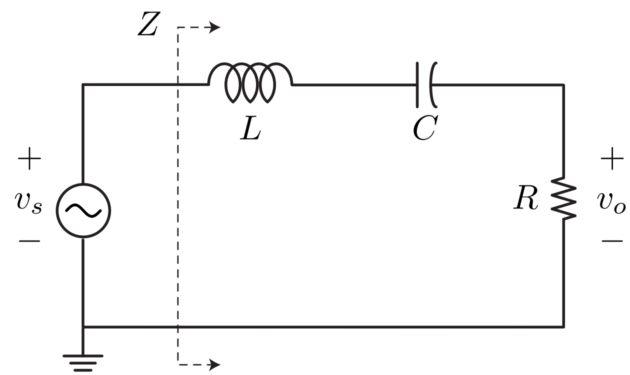
$$\omega_{-3\text{dB}} \approx \frac{1}{\tau_1 + \tau_2}$$

Shunt Feedback/CC Cascade

- If the shunt-FB amplifier needs to drive a low impedance load, a broadband voltage buffer is needed
- As shown below, an emitter follower (or source follower) provides the solution (note this is a fast pnp). Note the buffer is broadband (gain ≈ 1) and only loads the core amplifier by the degenerated input capacitance $C_{in2}/(1 + g_{m2}R_E)$



Series RLC Circuis

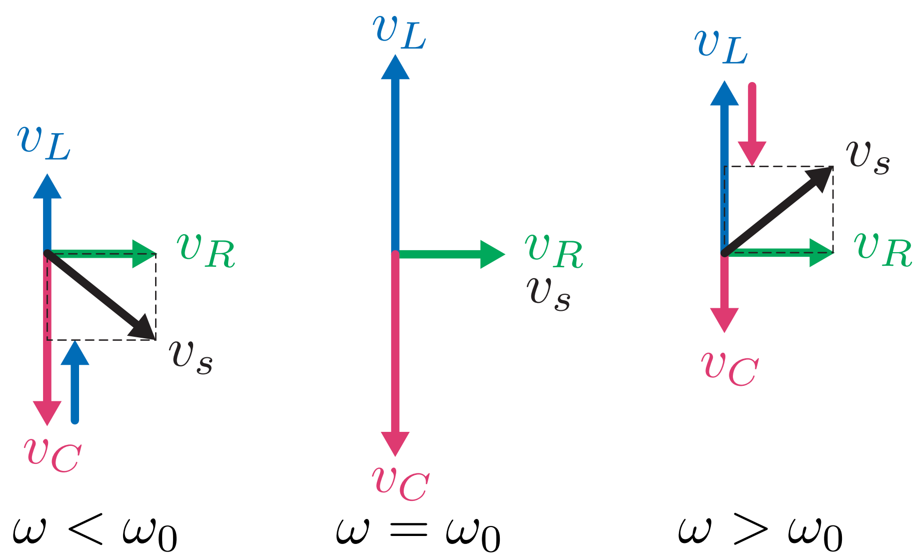


- The RLC circuit shown is deceptively simple. The impedance seen by the source is simply given by

$$Z = j\omega L + \frac{1}{j\omega C} + R = R + j\omega L \left(1 - \frac{1}{\omega^2 LC}\right)$$

- The impedance is purely real at at the *resonant frequency* when $\Im(Z) = 0$, or $\omega = \pm\frac{1}{\sqrt{LC}}$. At resonance the impedance takes on a minimal value.

Series Resonance



- It's worthwhile to investigate the cause of resonance, or the cancellation of the reactive components due to the inductor and capacitor. Since the inductor and capacitor voltages are always $180°$ out of phase, and one reactance is dropping while the other is increasing, there is clearly always a frequency when the magnitudes are equal.
- Resonance occurs when $\omega L = \frac{1}{\omega C}$.

Quality Factor

- So what's the magic about this circuit? The first observation is that at resonance, the voltage across the reactances can be larger, in fact much larger, than the voltage across the resistors R. In other words, this circuit has voltage gain. Of course it does not have power gain, for it is a passive circuit. The voltage across the inductor is given by

$$v_L = j\omega_0 L i = j\omega_0 L \frac{v_s}{Z(j\omega_0)} = j\omega_0 L \frac{v_s}{R} = jQ \times v_s$$

- where we have defined a circuit Q factor at resonance as

$$Q = \frac{\omega_0 L}{R}$$

Voltage Multiplication

- It's easy to show that the same voltage multiplication occurs across the capacitor

$$v_C = \frac{1}{j\omega_0 C} i = \frac{1}{j\omega_0 C} \frac{v_s}{Z(j\omega_0)} = \frac{1}{j\omega_0 RC} \frac{v_s}{R} = -jQ \times v_s$$

- This voltage multiplication property is the key feature of the circuit that allows it to be used as an impedance transformer.
- It's important to distinguish this Q factor from the intrinsic Q of the inductor and capacitor. For now, we assume the inductor and capacitor are ideal.

More of Q

- We can re-write the Q factor in several equivalent forms owing to the equality of the reactances at resonance

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C}\frac{1}{R} = \frac{\sqrt{LC}}{C}\frac{1}{R} = \sqrt{\frac{L}{C}}\frac{1}{R} = \frac{Z_0}{R}$$

- where we have defined the $Z_0 = \sqrt{\frac{L}{C}}$ as the characteristic impedance of the circuit.

Circuit Transfer Function

- Let's now examine the transfer function of the circuit

$$H(j\omega) = \frac{v_o}{v_s} = \frac{R}{j\omega L + \frac{1}{j\omega C} + R}$$

$$H(j\omega) = \frac{j\omega RC}{1 - \omega^2 LC + j\omega RC}$$

- Obviously, the circuit cannot conduct DC current, so there is a zero in the transfer function. The denominator is a quadratic polynomial. It's worthwhile to put it into a standard form that quickly reveals important circuit parameters

$$H(j\omega) = \frac{j\omega \frac{R}{L}}{\frac{1}{LC} + (j\omega)^2 + j\omega \frac{R}{L}}$$

Canonical Form

- Using the definition of Q and ω_0 for the circuit

$$H(j\omega) = \frac{j\omega \frac{\omega_0}{Q}}{\omega_0^2 + (j\omega)^2 + j\frac{\omega\omega_0}{Q}}$$

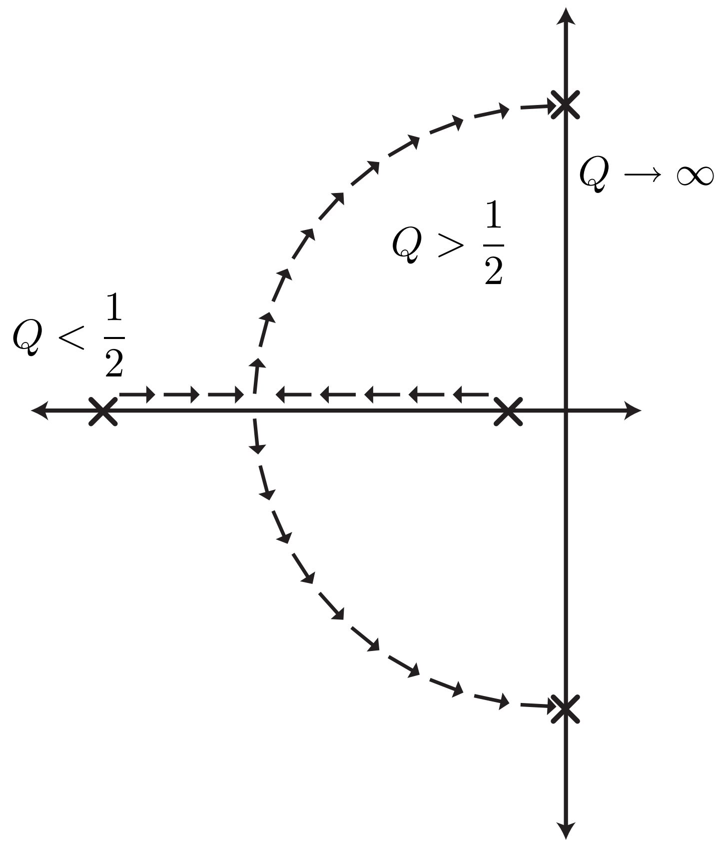
- Factoring the denominator with the assumption that $Q > \frac{1}{2}$ gives us the complex poles of the circuit

$$s^{\pm} = -\frac{\omega_0}{2Q} \pm j\omega_0 \sqrt{1 - \frac{1}{4Q^2}}$$

- The poles have a constant magnitude equal to the resonant frequency

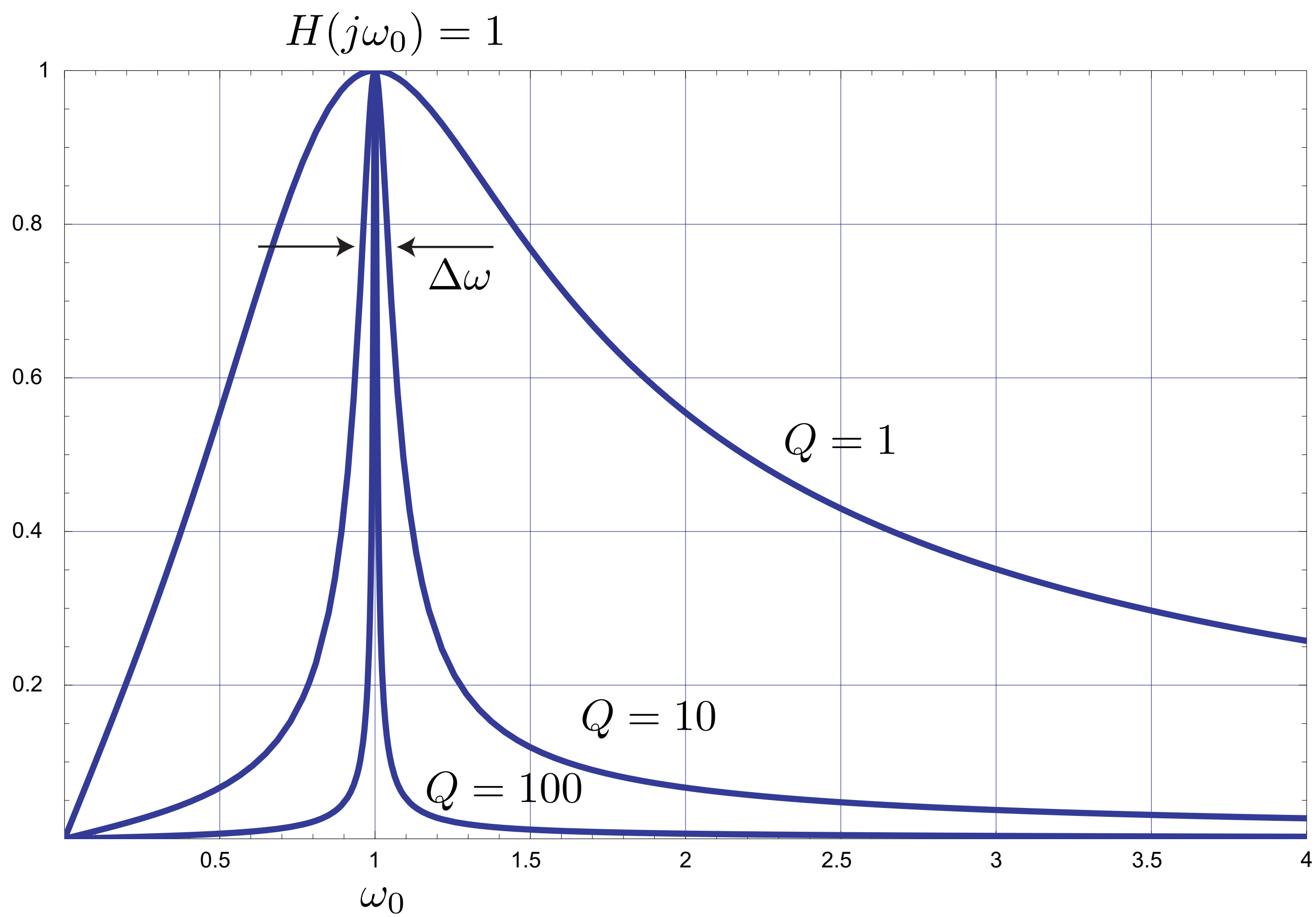
$$|s| = \sqrt{\cancel{\frac{\omega_0^2}{4Q^2}} + \omega_0^2 \left(1 - \cancel{\frac{1}{4Q^2}}\right)} = \omega_0$$

Root Locus



- A root-locus plot of the poles as a function of Q. As $Q \to \infty$, the poles move to the imaginary axis. In fact, the real part of the poles is inversely related to the Q factor.

Circuit Bandwidth



- As we plot the magnitude of the transfer function, we see that the selectivity of the circuit is also related inversely to the Q factor.

Selectivity

- In the limit that $Q \to \infty$, the circuit is infinitely selective and only allows signals at resonance ω_0 to travel to the load.
- Note that the peak gain in the circuit is always unity, regardless of Q, since at resonance the L and C together disappear and effectively all the source voltage appears across the load.
- The selectivity of the circuit lends itself well to filter applications. To characterize the peakiness, let's compute the frequency when the magnitude squared of the transfer function drops by half

$$|H(j\omega)|^2 = \frac{\left(\omega \frac{\omega_0}{Q}\right)^2}{\left(\omega_0^2 - \omega^2\right)^2 + \left(\omega \frac{\omega_0}{Q}\right)^2} = \frac{1}{2}$$

Selectivity Bandwidth

- This happens when

$$\left(\frac{\omega_0^2 - \omega^2}{\omega_0\omega/Q}\right)^2 = 1$$

- Solving the above equation yields four solutions, corresponding to two positive and two negative frequencies. The peakiness is characterized by the difference between these frequencies, or the bandwidth, given by

$$\Delta\omega = \omega_+ - \omega_- = \frac{\omega_0}{Q}$$

Selectivity Bandwidth (cont)

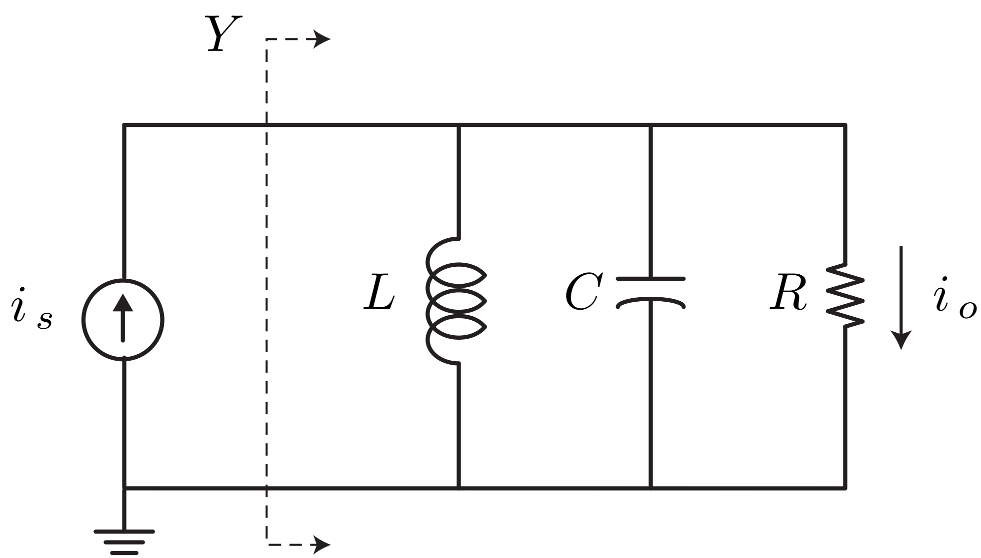
- The normalized bandwidth is inversely proportional to the circuit Q.

$$\frac{\Delta\omega}{\omega_0} = \frac{1}{Q}$$

- You can also show that the resonance frequency is the geometric mean frequency of the 3 dB frequencies

$$\omega_0 = \sqrt{\omega_+ \omega_-}$$

Parallel RLC



- The parallel RLC circuit is the dual of the series circuit. By "dual" we mean that the role of voltage and currents are interchanged.
- Hence the circuit is most naturally probed with a current source i_s. In other words, the circuit has current gain as opposed to voltage gain, and the admittance minimizes at resonance as opposed to the impedance.

Duality

- The role of capacitance and inductance are also interchanged. In principle, therefore, we don't have to repeat all the detailed calculations we just performed for the series case, but in practice it's worthwhile exercise.
- The admittance of the circuit is given by

$$Y = j\omega C + \frac{1}{j\omega L} + G = G + j\omega C\left(1 - \frac{1}{\omega^2 LC}\right)$$

which has the same form as before. The resonant frequency also occurs when $\Im(Y) = 0$, or when $\omega = \omega_0 = \pm\frac{1}{\sqrt{LC}}$.

Duality (cont)

- Likewise, at resonance the admittance takes on a minimal value. Equivalently, the impedance at resonance is maximum.
- This property makes the parallel RLC circuit an important element in tuned amplifier loads. It's also easy to show that at resonance the circuit has a current gain of Q

$$i_C = j\omega_0 C v_o = j\omega_0 C \frac{i_s}{Y(j\omega_0)} = j\omega_0 C \frac{i_s}{G} = jQ \times i_s$$

- where we have defined the circuit Q factor at resonance by

$$Q = \frac{\omega_0 C}{G}$$

Current Multiplication

- The current gain through the inductor is also easily derived

$$i_L = -jQ \times i_s$$

- The equivalent expressions for the circuit Q factor are given by the inverse of the previous relations

$$Q = \frac{\omega_0 C}{G} = \frac{R}{\omega_0 L} = \frac{R}{\frac{1}{\sqrt{LC}} L} = \frac{R}{\sqrt{\frac{L}{C}}} = \frac{R}{Z_0}$$

Phase Response

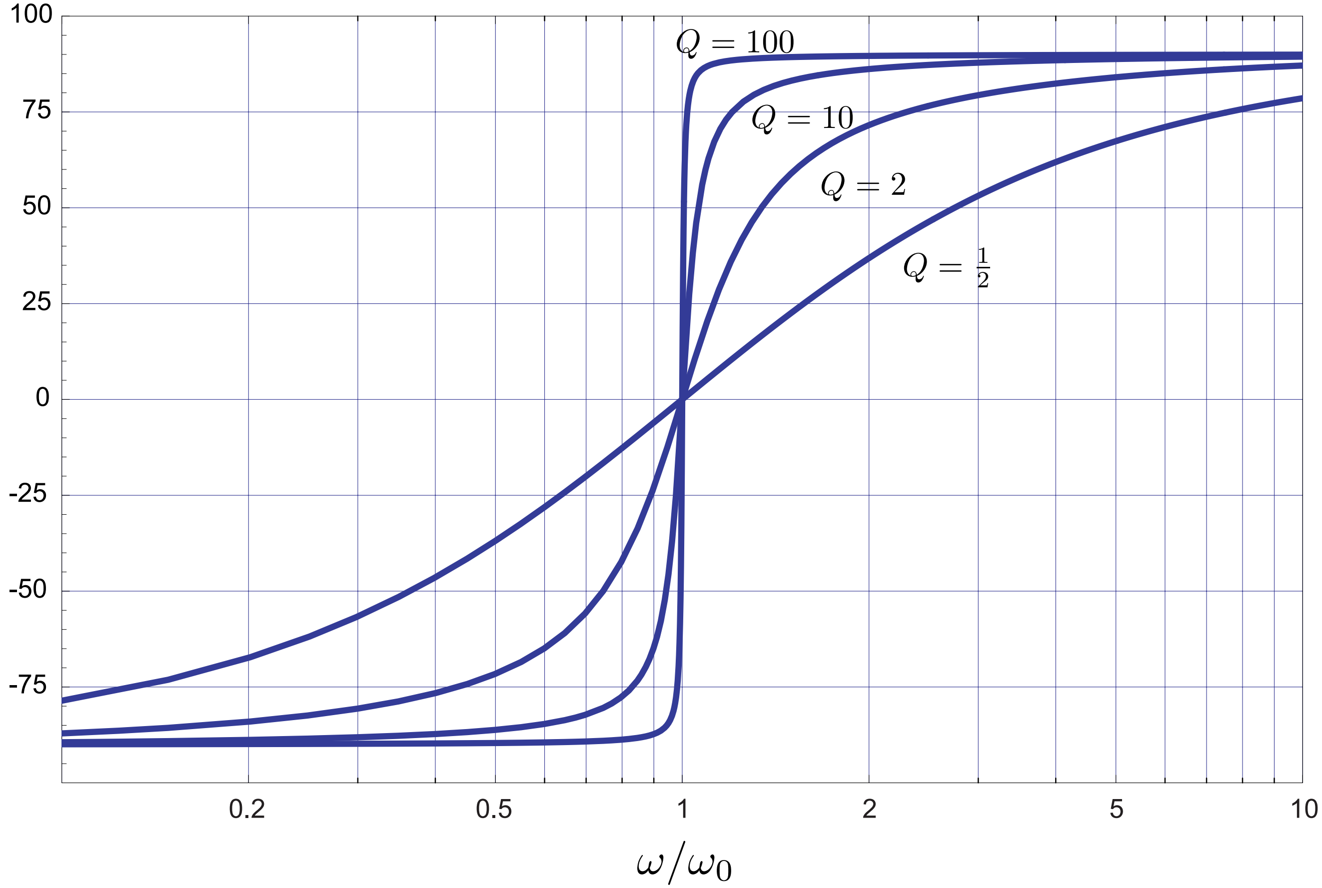
- The phase response of a resonant circuit is also related to the Q factor. For the parallel RLC circuit the phase of the admittance is given by

$$\angle Y(j\omega) = \tan^{-1}\left(\frac{\omega C\left(1 - \frac{1}{\omega^2 LC}\right)}{G}\right)$$

- The rate of change of phase at resonance is given by

$$\left.\frac{d\angle Y(j\omega)}{d\omega}\right|_{\omega_0} = \frac{2Q}{\omega_0}$$

Phase Response



- A plot of the admittance phase as a function of frequency and Q is shown. Higher Q circuits go through a more rapid transition.

Circuit Transfer Function

- Given the duality of the series and parallel RLC circuits, it's easy to deduce the behavior of the circuit. Whereas the series RLC circuit acted as a filter and was only sensitive to voltages near resonance ω_0, likewise the parallel RLC circuit is only sensitive to currents near resonance

$$H(j\omega) = \frac{i_o}{i_s} = \frac{v_o G}{v_o Y(j\omega)} = \frac{G}{j\omega C + \frac{1}{j\omega L} + G}$$

which can be put into the same canonical form as before

$$H(j\omega) = \frac{j\omega \frac{\omega_0}{Q}}{\omega_0^2 + (j\omega)^2 + j\frac{\omega\omega_0}{Q}}$$

Circuit Transfer Function (cont)

- We have appropriately re-defined the circuit Q to correspond the parallel RLC circuit. Notice that the impedance of the circuit takes on the same form

$$Z(j\omega) = \frac{1}{Y(j\omega)} = \frac{1}{j\omega C + \frac{1}{j\omega L} + G}$$

- which can be simplified to

$$Z(j\omega) = \frac{j\frac{\omega}{\omega_0}\frac{1}{GQ}}{1 + \left(\frac{j\omega}{\omega_0}\right)^2 + j\frac{\omega}{\omega_0 Q}}$$

Parallel Resonance

- At resonance, the real terms in the denominator cancel

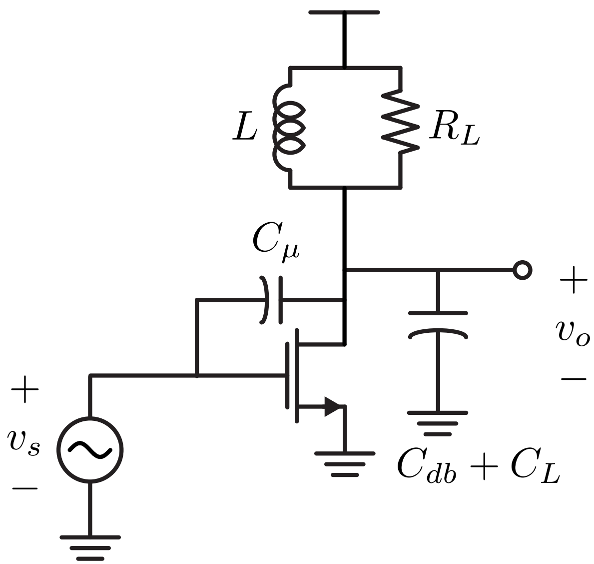
$$Z(j\omega_0) = \frac{j\frac{R}{Q}}{\underbrace{1 + \left(\frac{j\omega_0}{\omega_0}\right)^2}_{=0} + j\frac{1}{Q}} = R$$

- It's not hard to see that this circuit has the same half power bandwidth as the series RLC circuit, since the denominator has the same functional form

$$\frac{\Delta\omega}{\omega_0} = \frac{1}{Q}$$

- plot of this impedance versus frequency has the same form as before multiplied by the resistance R.

Tuned Amplifiers



The RLC loaded amplifier is a tuned amplifier. A transconductance device drives a shunt RLC load which results in a voltage gain.

$$A_v = -g_m Z(j\omega) = \frac{-g_m}{Y(j\omega)}$$

- The peak gain occurs at resonance

$$A_{v,\mathrm{max}} = -g_m R_{\mathrm{eff}}$$

Tuned Amp (cont)

- R_{eff} is the *loaded* resistance of the tank

$$R_{\text{eff}} = R_L || r_o || R_{x,L} || R_{x,C}$$

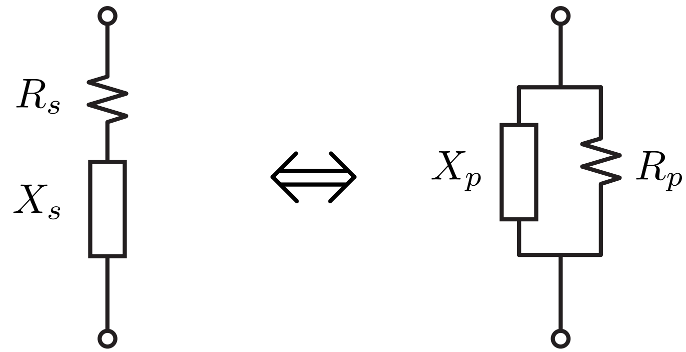
- In order to maximize the gain, we employ high-Q inductors and capacitors in the load and omit the explicit load resistance R_L. The peak gain is thus

$$A_{v,\text{max}} \approx -g_m (R_{x,L} || R_{x,C})$$

Assuming the Q factor is dominated by the inductor, a good assumption for monolithic IC inductors, we have

$$A_{v,\text{max}} \approx -g_m R_{x,L} = -g_m Q_L \omega L$$

Shunt-Series Transformation



- The key calculation aid is the series to parallel transformation. Consider the impedance shown above, which we wish to represent as a parallel impedance.
- We can do this at a single frequency as long as the impedance of the series network equals the impedance of the shunt network

$$R_s + jX_s = \frac{1}{\frac{1}{R_p} + \frac{1}{jX_p}}$$

Transformation (cont)

- Equating the real and imaginary parts

$$R_s = \frac{R_p X_p^2}{R_p^2 + X_p^2}$$

$$X_s = \frac{R_p^2 X_p}{R_p^2 + X_p^2}$$

- which can be simplified by using the definition of Q

$$Q_s = \frac{X_s}{R_s} = \frac{R_p^2 X_p}{R_p X_p^2} = \frac{R_p}{X_p} = Q_p = Q$$

Transformation (cont)

- Which shows that

$$R_p = R_s(1 + Q^2)$$

and

$$X_p = X_s(1 + Q^{-2}) \approx X_s$$

- where the approximation applies under high Q conditions.

Q Calculation

- We used the series to parallel transformation to calculate the effective shunt resistance due to the inductor

$$R_{\text{eff}} = (1 + Q_L^2) R_{x,L} \approx Q_L^2 R_{x,L} = Q_L^2 \frac{\omega L}{Q_L} = Q_L \times \omega L$$

The gain is maximized at a fixed bias current and frequency by maximizing the product $Q_L \times L$. So in theory, there is no limit to the voltage gain of the amplifier as long as we can maximize the quality factor Q_L.

Selection of L

- Note that the capacitance of the circuit is not detrimental since it is resonated away with the shunt inductance. In other words L is chosen such that

$$L = \frac{1}{\omega_0^2 C_{\text{eff}}}$$

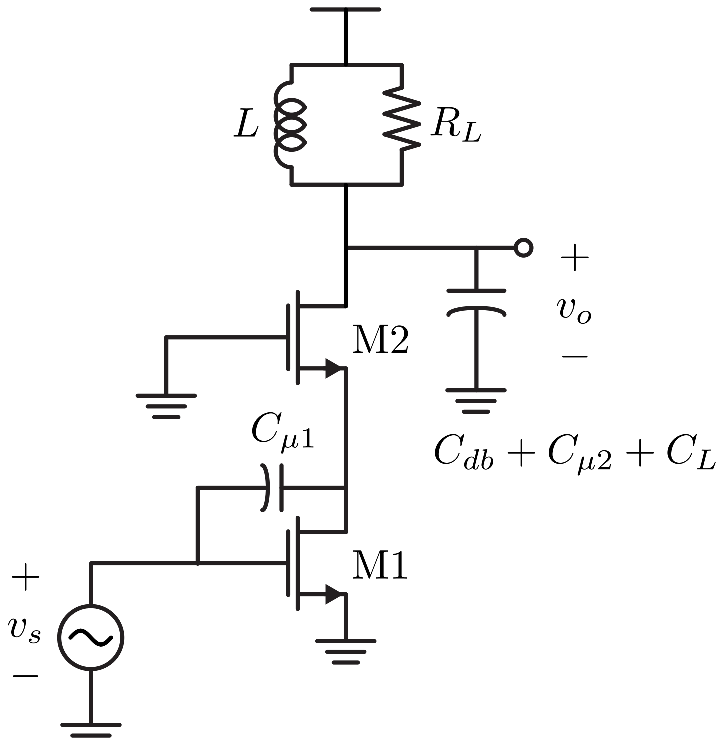
where $C_{\text{eff}} = C_{db} + (1 - |A_v^{-1}|)C_\mu + C_L$.

- The ability to tune out the parasitic capacitances in the circuit is a major advantage of the tuned amplifier. This is especially important as it allows low-power operation. Another important advantage of the circuit is that there is practically no DC voltage drop across the inductor, allowing very low supply voltage operation.

Tuned Amplifier Advantages

- Another less obvious advantage is the improved voltage swing at the output of the amplifier. Usually the voltage swing is limited by the supply voltage and the $V_{ds,sat}$ of the amplifier.
- In this case, though, the voltage can swing above the supply. Since the average DC voltage across an inductor is zero, the output voltage can swing around the DC operating point of V_{dd}.
- This is a major efficiency boost for the amplifier and is an indispensable tool in designing power amplifiers and buffers.

Cascode Tuned Amplifier



Besides the obvious advantage of boosting the output impedance, thus maximizing the Q of the load, the cascode device solves a major stability problem of the amplifier.

- We'll show that a feedback C_μ path can easily lead to unwanted oscillations in the amplifier. The cascode tuned amplifier effectively has zero feedback and thus is much more stable. The loss in voltage headroom is a small price to pay considering the improved headroom afforded by the inductor.

Bandwidth

- It's interesting to note that the bandwidth of the circuit is still determined by the RC time constant at the load.

$$BW = \frac{\omega_0}{Q} = \frac{\omega_0}{\omega_0 RC} = \frac{1}{RC}$$

- The ultimate sacrifice for high frequency operation in a tuned amplifier is that the amplifier is narrow-band with zero gain at DC. In fact, the larger the Q of the tank, the higher the gain and the lower the bandwidth.

How high can we go?

- To win some of the bandwidth back requires other techniques, such as shunt peaking and distributed amplifiers. Other techniques (some invented here at Berkeley) can also help out.
- But can we tune out parasitics and design amplifiers operating at arbitrarily high frequency?
- Based on the simple analysis thus far, it seems that for any given frequency, no matter how high, we can simply absorb the parasitic capacitance of the amplifier with an appropriately small inductor (say a short section of transmission line) and thus realize an amplifier at an arbitrary frequency. This is of course ludicrous and we'll re-examine this question in future lecture.