Last lecture we analyzed the small-signal behavior of the above circuit. We found that the closed-loop gain is given by

\[
H(s) = \frac{g_m R_s \frac{L}{R}}{1 + s \frac{L}{R} (1 - A_\ell) + s^2 LC}
\]
Review: Role of Loop Gain

The behavior of the circuit is determined largely by $A_\ell$, the loop gain at DC and resonance. When $A_\ell = 1$, the poles of the system are on the $j\omega$ axis, corresponding to constant amplitude oscillation.

When $A_\ell < 1$, the circuit oscillates but decays to a quiescent steady-state.

When $A_\ell > 1$, the circuit begins oscillating with an amplitude which grows exponentially. Eventually, we find that the steady state amplitude is fixed.
To find the steady-state behavior of the circuit, we will make several simplifying assumptions. The most important assumption is the high tank $Q$ assumption (say $Q > 10$), which implies the output waveform $v_o$ is sinusoidal.

Since the feedback network is linear, the input waveform $v_i = v_o/n$ is also sinusoidal.

We may therefore apply the large-signal periodic steady-state analysis of the BJT to the oscillator.
The collector current is not sinusoidal, due to the large signal drive.

The output voltage, though, is sinusoidal and given by

\[ v_o \approx I_{\omega_1} Z_T(\omega_1) = G_m Z_T v_i \]
Steady State Equations

- But the input waveform is a scaled version of the output

\[ v_o = G_m Z_T \frac{v_o}{n} = \frac{G_m Z_T}{n} v_o \]

- The above equation implies that

\[ \frac{G_m Z_T}{n} \equiv 1 \]

- Or that the loop gain in steady-state is unity and the phase of the loop gain is zero degrees (an exact multiple of \(2\pi\))

\[ \left| \frac{G_m Z_T}{n} \right| \equiv 1 \quad \angle \frac{G_m Z_T}{n} \equiv 0^\circ \]
Recall that the small-signal loop gain is given by

\[ |A_\ell| = \left| \frac{g_m Z_T}{n} \right| \]

Which implies a relation between the small-signal start-up transconductance and the steady-state large-signal transconductance

\[ \left| \frac{g_m}{G_m} \right| = A_\ell \]

Notice that \( g_m \) and \( A_\ell \) are design parameters under our control, set by the choice of bias current and tank \( Q \). The steady state \( G_m \) is therefore also fixed by initial start-up conditions.
To find the oscillation amplitude we need to find the input voltage drive to produce $G_m$.

For a BJT, we found that under the constraint that the bias current is fixed

$$I_{\omega_1} = \frac{2I_1(b)}{I_0(b)} I_Q = G_m v_i = G_m b \frac{kT}{q}$$
Thus the large-signal $G_m$ is given by

$$G_m = \frac{2I_1(b)}{bI_0(b)} \frac{qI_Q}{kT} = \frac{2I_1(b)}{bI_0(b)} g_m$$

$$\frac{G_m}{g_m} = \frac{2I_1(b)}{bI_0(b)}$$
Stability (Intuition)

Here’s an intuitive argument for how the oscillator reaches a stable oscillation amplitude. Assume that initially $A_l > 1$ and oscillations grow. As the amplitude of oscillation increases, though, the $G_m$ of the transistor drops, and so effectively the loop gain drops.

As the loop gain drops, the poles move closer to the $j\omega$ axis. This process continues until the poles hit the $j\omega$ axis, after which the oscillation ensues at a constant amplitude and $A_l = 1$. 
To see how this is a stable point, consider what happens if somehow the loop gain changes. If the loop gain changes to $A_\ell + |\epsilon|$, then we already see that the system will roll back. If the loop gain drops below unity, $A_\ell - |\epsilon|$, then the poles move into the LHP and amplitude of oscillation will begin to decay.

As the oscillation amplitude decays, the $G_m$ increases and this causes the loop gain to grow. Thus the system also rolls back to the point where $A_\ell = 1$. 
BJT Oscillator Design

- Say we desire an oscillation amplitude of $v_0 = 100\text{mV}$ at a certain oscillation frequency $\omega_0$.
- We begin by selecting a loop gain $A_\ell > 1$ with sufficient margin. Say $A_\ell = 3$.
- We also tune the $LC$ tank to $\omega_0$, being careful to include the loaded effects of the transistor ($r_o, C_o, C_{in}, R_{in}$).
- We can estimate the required first harmonic current from

$$I_{\omega_0} = \frac{v_o}{R'_T}$$
This is an estimate because the exact value of $R_T$ is not known until we specify the operating point of the transistor. But a good first order estimate is to neglect the loading and use $R_T'$.

We can now solve for the bias point from

$$I_{\omega_1} = \frac{2I_1(b)}{I_0(b)} I_Q$$

$b$ is known since it’s the oscillation amplitude normalized to $kT/q$ and divided by $n$. The above equation can be solved graphically or numerically.

Once $I_Q$ is known, we can now calculate $R_T''$ and iterate until the bias current converges to the final value.
Squegging is a parasitic oscillation in the bias circuitry of the amplifier.

It can be avoided by properly sizing the emitter bypass capacitance

\[ C_E \leq nC_T \]
Another BJT oscillator uses the common-base transistor. Since there is no phase inversion in the amplifier, the transformer feedback is in phase.

Since we don’t need phase inversion, we can use a simpler feedback consisting of a capacitor divider.
Colpitts Oscillator

The cap divider works at higher frequencies. Under the cap divider approximation

\[ f \approx \frac{C_1}{C_1 + C_2'} = \frac{1}{n} \]

\[ n = 1 + \frac{C_2'}{C_1} \]

\( C_2' \) includes the loading from the transistor and current source.
Since the bias current is held constant by a current source $I_Q$ or a large resistor, the analysis is identical to the BJT oscillator with transformer feedback. Note the output voltage is divided and applied across $v_{BE}$ just as before.
Colpitts Family

If we remove the explicit ground connection on the oscillator, we have the template for a generic oscillator.

It can be shown that the Colpitts family of oscillator never squegg.
If we ground the emitter, we have a new oscillator topology, called the Pierce Oscillator. Note that the amplifier is in CE mode, but we don’t need a transformer!

Likewise, if we ground the collector, we have an emitter follower oscillator.

A fraction of the tank resonant current flows through $C_{1,2}$. In fact, we can use $C_{1,2}$ as the tank capacitance.
If we assume that the current through $C_{1,2}$ is larger than the collector current (high Q), then we see that the same current flows through both capacitors. The voltage at the input and output is therefore

$$v_o = I_{\omega_1} \frac{1}{j\omega C_1}$$

$$v_i = -I_{\omega_1} \frac{1}{j\omega C_2}$$

or

$$\frac{v_o}{v_i} = n = \frac{C_1}{C_2}$$
Pierce Bias

A current source or large resistor can bias the Pierce oscillator.

Since the bias current is fixed, the same large signal oscillator analysis applies.
Common-Collector Oscillator

Note that the collector can be connected to a resistor without changing the oscillator characteristics. In fact, the transistor provides a buffered output for “free”.

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The common-collector oscillator shown above uses a large capacitor $C_T$ to block the DC signal at the base. $R_B$ is used to bias the transistor.

If the shunt capacitor $C_T$ is eliminated, then the capacitor $C_B$ can be used to resonate with $L$ and the series combination of $C_1$ and $C_2$. This is a series resonant circuit.