Lecture 16: I/Q Mixers; BJT Mixers

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An I/Q mixer implemented as shown above is known as a Hartley Mixer.

We shall show that such a mixer can be designed to select either the upper or lower sideband. For this reason, it is sometimes called a single-sideband mixer.

We will also show that such a mixer can perform image rejection.
Consider the action of a $90^\circ$ delay on an arbitrary signal. Clearley $\sin(x + 90^\circ) = \cos(x)$. Even though this is obvious, consider the effect on the complex exponentials

$$\sin(x - \frac{\pi}{2}) = \frac{e^{jx-j\pi/2} - e^{-jx+j\pi/2}}{2j}$$

$$= \frac{e^{jx}e^{-j\pi/2} - e^{-jx}e^{j\pi/2}}{2j} = \frac{e^{jx}(-j) - e^{-jx}(j)}{2j}$$

$$= \frac{e^{jx} + e^{-jx}}{2} = -\cos(x)$$

Notice that positive frequencies get multiplied by $-j$ and negative frequencies by $+j$. This is true for any waveform when it is delayed by $90^\circ$. 
Complex Modulation

Consider multiplying a waveform \( f(t) \) by \( e^{j\omega t} \) and taking the Fourier transform

\[
\mathcal{F} \{ e^{j\omega_0 t} f(t) \} = \int_{-\infty}^{\infty} f(t) e^{j\omega_0 t} e^{-j\omega t} dt
\]

Grouping terms we have

\[
= \int_{-\infty}^{\infty} f(t) e^{-j(\omega-\omega_0)t} dt = F(\omega - \omega_0)
\]

It is clear that the action of multiplication by the complex exponential is a frequency shift.
Now since $\cos(x) = (e^{jx} + e^{-jx})/2$, we see that the action of time domain multiplication is to produce two frequency shifts

$$\mathcal{F}\{\cos(\omega_0 t) f(t)\} = \frac{1}{2} F(\omega - \omega_0) + \frac{1}{2} F(\omega + \omega_0)$$

These are the sum and difference (beat) frequency components.
We see that the image problem is due to to multiplication by the sinusoid and not a complex exponential. If we could synthesize a complex exponential, we would not have the image problem.
Using the same approach, we can find the result of multiplying by \( \sin \) and \( \cos \) as shown above. If we delay the \( \sin \) portion, we have a very desirable situation! The image is inverted with respect to the \( \cos \) and can be cancelled.
Image Rejection

- The image rejection scheme just described is very sensitive to phase and gain match in the $I/Q$ paths. Any mismatch will produce only finite image rejection.

- The image rejection for a given gain/phase match is approximately given by

$$Irr dB = 10 \cdot \log \frac{1}{4} \left( \left( \frac{\delta A}{A} \right)^2 + (\delta \theta)^2 \right)$$

- For typical gain mismatch of $0.2 - 0.5 \text{ dB}$ and phase mismatch of $1° - 4°$, the image rejection is about $30 \text{ dB} - 40 \text{ dB}$. We usually need about $60 - 70 \text{ dB}$ of total image rejection.
The passive $R/C$ and $C/R$ lowpass and highpass filters are a nice way to implement the delay. Note that their relative phase difference is always $90^\circ$.

$$\angle H_{lp} = \frac{1}{1 + j\omega RC} = -\arctan \omega RC$$

$$\angle H_{hp} = \frac{j\omega RC}{1 + j\omega RC} = \frac{\pi}{2} - \arctan \omega RC$$
Gain Match / Quadrature LO Gen

- But to have equal gain, the circuit must operate at the \( \frac{1}{RC} \) frequency. This restricts the circuit to relatively narrowband systems. Multi-stage polyphase circuits remedy the situation but add insertion loss to the circuit.

- The I/Q LO signal is usually generated directly rather than through an high-pass and low-pass network.

- Two ways to generate the I/Q LO is through a divide-by-two circuit (requires \( 2 \times LO \)) or a quadrature oscillator (requires two tanks).
Consider a bipolar device driven with a large sine signal.

This occurs in many types of non-linear circuits, such as oscillators, frequency multipliers, mixers and class C amplifiers.
BJT Collector Current

The collector current can be factored into a DC bias term and a periodic signal

\[ I_C = I_S e^{V_a V_t} \hat{V}_i \cos \omega t \]

\[ I_C = I_S e^a e^b \cos \omega t \]

Where the normalized bias is \( a = V_a / V_t \) and the normalized drive signal is \( b = \hat{V}_i / V_t \).

Since \( I_C \) is a periodic function, we can expand it into a Fourier Series. Note that the Fourier coefficients of \( e^b \cos \omega t \) are modified Bessel functions \( I_n(b) \)

\[ e^b \cos \omega t = I_0(b) + 2I_1(b) \cos \omega t + 2I_2(b) \cos 2\omega t + \cdots \]
Assume that the bias current of the amplifier is stabilized. Then

\[ I_C = I_S e^{a I_0(b)} \left( 1 + \frac{2I_1(b)}{I_0(b)} \cos \omega t + \frac{2I_2(b)}{I_0(b)} \cos 2\omega t + \cdots \right) \]

\[ I_C = I_S e^{a} e^{b \cos \omega t} \]

\[ = \frac{I_Q}{I_0(b)} e^{b \cos \omega t} \]
With increasing input drive, the current waveform becomes “peaky”. The peak value can exceed the DC bias by a large factor.
Harmonic Current Amplitudes

The BJT output spectrum is rich in harmonics.
If we zoom in on the curves to small $b$ values, we enter the small-signal regime, and the weakly non-linear behavior is predicted by our power series analysis.
BJT with Stable Bias

Neglecting base current ($\beta \gg 1$), the voltage at the base is given by

$$V'_A = \frac{R_2}{R_1 + R_2} V_{CC}$$

$$V_E = V'_A - V_{BE}$$

$$I_Q = \frac{V_E}{R_E} = \frac{V'_A - V_{BE}}{R_E}$$

We see that the bias is fixed since $V_{BE}$ does not vary too much. Typically $V'_A$ is a few volts.

In this circuit $C_E$ is an emitter bypass capacitor used to short $R_E$ at high frequency.
Differential Pair with Sine Drive

The large signal equation for $I_{C1}$ is given by

$$I_{C1} + I_{C2} = I_{EE}$$

$$V_{BE1} - V_{BE2} = V_i = V_t \ln \left( \frac{I_{C1}}{I_{C2}} \right)$$

$$I_{C1} = \frac{I_{EE}}{1 + e^{-v_i/V_t}}$$

$$v_i = \hat{V}_i \cos \omega t$$

$$I_{C1} = \frac{I_{EE}}{1 + e^{-b \cos \omega t}}$$
For large $b$, the waveform approaches a square wave

$$\frac{I_C}{I_{EE}} = \frac{1}{2} + \frac{2}{\pi} \left( \cos \omega t - \frac{1}{3} \cos 3\omega t + \frac{1}{5} \cos 5\omega t + \cdots \right)$$
As expected, the ideal differential pair does not produce any even harmonics.
Mixer Analysis

- As we have seen, a mixer has three ports, the LO, RF, and IF port.
- Assume that a circuit is “pumped” with a periodic large signal at the LO port with frequency $\omega_0$.
- From the RF port, though, assume we apply a small signal at frequency $\omega_s$.
- Since the RF input is small, the circuit response should be linear (or weakly non-linear). But since the LO port changes the operating point of the circuit periodically, we expect the overall response to the RF port to be a linear time-varying response

$$i_o(t) = g(t)v_{in}$$
Mixer Assumptions

- The transconductance varies periodically and can be expanded in a Fourier series

\[ g(t) = g_0 + g_1 \cos \omega_0 t + g_2 \cos 2\omega_0 t + \cdots \]

- Applying the input \( v_{in} = \hat{V}_1 \cos \omega_s t \)

\[ i_o(t) = (g_0 + g_1 \cos \omega_0 t + g_2 \cos 2\omega_0 t + \cdots) \times \hat{V}_1 \cos \omega_s t \]
Mixer Output Signal

Expanding the product, we have

\[ i_o(t) = g_0 \hat{V}_1 \cos \omega_s t + \frac{g_1}{2} \hat{V}_1 \cos(\omega_0 \pm \omega_s) t + \frac{g_2}{2} \hat{V}_1 \cos(2 \omega_0 \pm \omega_s) t + \cdots \]

The first term is just the input amplified. The other terms are all due to the mixing action of the linear time-varying periodic circuit.

Let’s say the desired output is the IF at \( \omega_0 - \omega_s \). The conversion gain is therefore defined as

\[ g_{\text{conv}} = \frac{|\text{IF output current}|}{|\text{RF input signal voltage}|} = \frac{g_1}{2} \]