EECS 142

Integrated Circuits for Communication

Lecture 18: Introduction to Mixers

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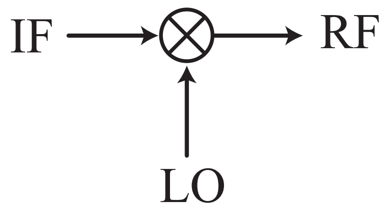
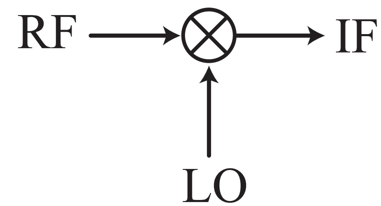
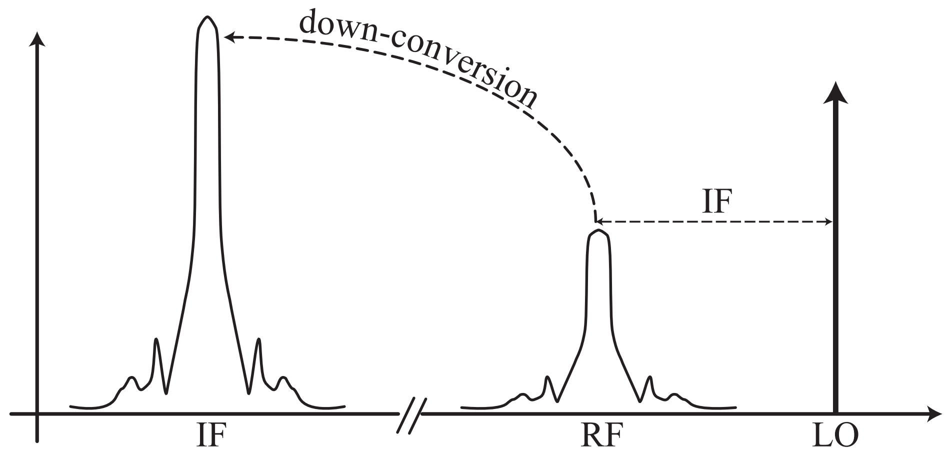
Mixers





- An ideal mixer is usually drawn with a multiplier symbol
- A real mixer cannot be driven by arbitrary inputs. Instead one port, the “LO” port, is driven by an *local oscillator* with a fixed amplitude sinusoid.
- In a *down-conversion* mixer, the other input port is driven by the “RF” signal, and the output is at a lower IF *intermediate frequency*
- In an *up-coversion* mixer, the other input is the IF signal and the output is the RF signal

Frequency Translation



- As shown above, an ideal mixer translates the modulation around one carrier to another. In a receiver, this is usually from a higher RF frequency to a lower IF frequency. In a transmitter, it's the inverse.
- We know that an LTI circuit cannot perform frequency translation. Mixers can be realized with either time-varying circuits or non-linear circuits

Ideal Multiplier

- Suppose that the input of the mixer is the RF and LO signal

$$v_{RF} = A(t)\cos\left(\omega_0 t + \phi(t)\right)$$

$$v_{LO} = A_{LO}\cos\left(\omega_{L0} t\right)$$

- Recall the trigonometric identity

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

- Applying the identity, we have

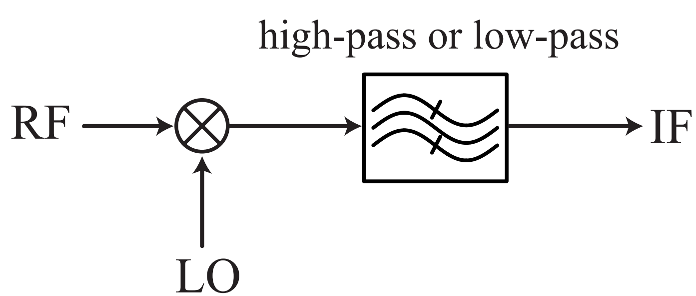
$$\begin{aligned} v_{out} &= v_{RF} \times v_{LO} \\ &= \frac{A(t)A_{LO}}{2}\left\{\cos\phi\left(\cos(\omega_{LO}+\omega_0)t + \cos(\omega_{LO}-\omega_0)t\right)\right. \\ &\quad \left. - \sin\phi\left(\sin(\omega_{LO}+\omega_0)t + \sin(\omega_{LO}-\omega_0)t\right)\right\} \end{aligned}$$

Ideal Multiplier (cont)

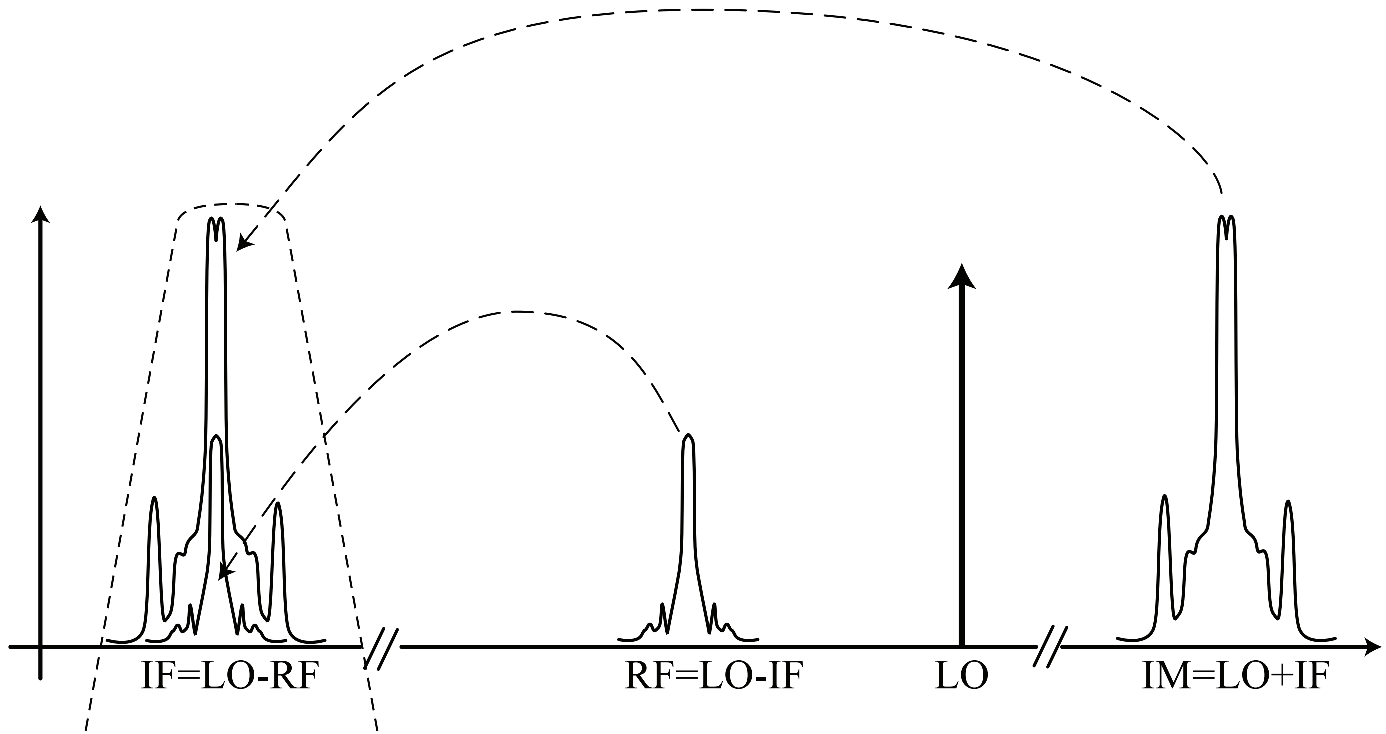
- Grouping terms we have

$$v_{out} = \frac{A(t)A_LO}{2} \quad \{\cos\left((\omega_{LO} + \omega_0)t + \phi(t)\right) + \cos\left((\omega_{LO} - \omega_0)t + \phi(t)\right)\}$$

- We see that the modulation is indeed translated to two new frequencies, $LO + RF$ and $LO - RF$. We usually select either the upper or lower "sideband" by filtering the output of the mixer



Mixer + Filter



- Note that the LO can be below the RF (lower side injection) or above the RF (high side injection)
- Also note that for a given LO, energy at $LO \pm IF$ is converted to the same IF frequency. This is a potential problem!

Upper/Lower Injection and Image

- Example: Downconversion Mixer

$$RF = 1\text{GHz} = 1000\text{MHz}$$

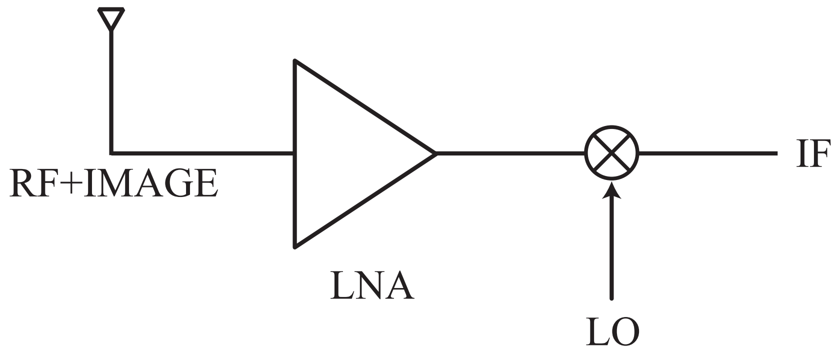
$$IF = 100\text{MHz}$$

Let's say we choose a low-side injection:

$$LO = 900\text{MHz}$$

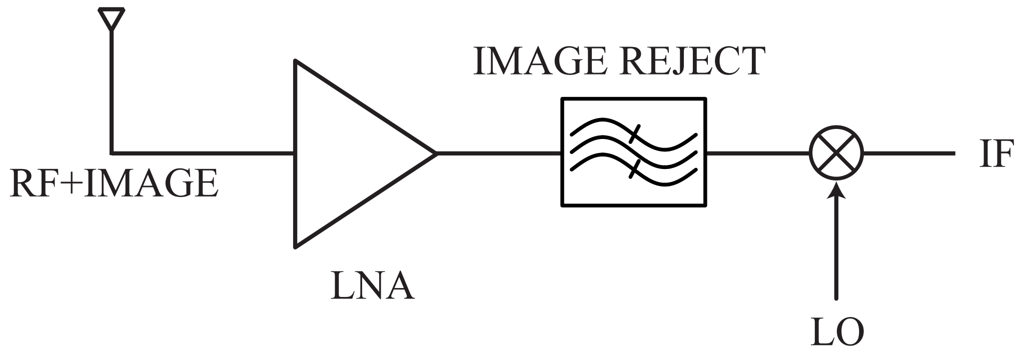
That means that any signals or noise at 800MHz will also be downconverted to the same IF

Receiver Application



- The image frequency is the second frequency that also down-converts to the same IF. This is undesirable becuase the noise and interferance at the image frequency can potentially overwhelm the receiver.
- One solution is to filter the image band. This places a restriction on the selection of the IF frequency due to the required filter Q

Image Rejection



- Suppose that $RF = 1000\mathrm{MHz}$, and $IF = 1\mathrm{MHz}$. Then the required filter bandwidth is much smaller than $2\mathrm{MHz}$ to knock down the image.
- In general, the filter Q is given by

$$Q = \frac{\omega_0}{BW} = \frac{RF}{BW}$$

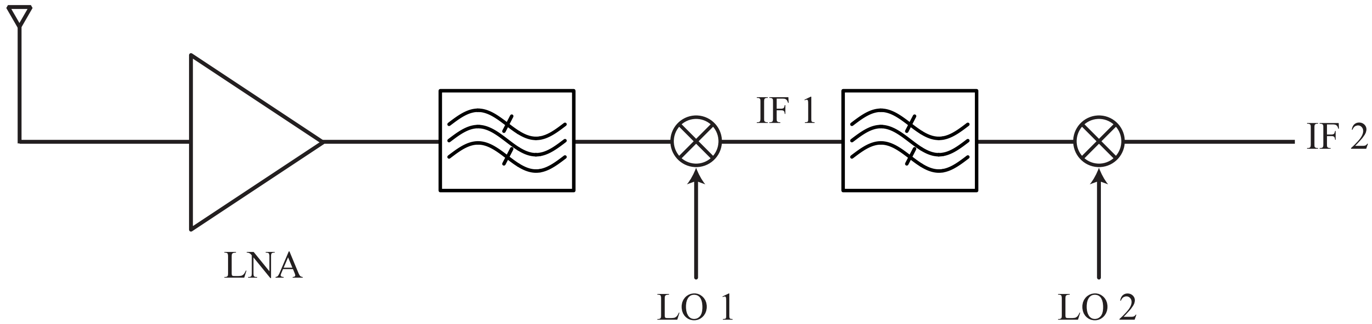
Image Reject Filter

- In our example, $RF = 1000\mathrm{MHz}$, and $IF = 1\mathrm{MHz}$. The Imagine is on $2IF = 2\mathrm{MHz}$ away.
- Let's design a filter with $f_0 = 1000\mathrm{MHz}$ and $f_1 = 1001\mathrm{MHz}$.
- A fifth-order Chebyshev filter with $0.2\,\mathrm{dB}$ ripple is down about $80\,\mathrm{dB}$ at the IF frequency.
- But the Q for such a filter is

$$Q = \frac{10^3\mathrm{MHz}}{1\mathrm{MHz}} = 10^3$$

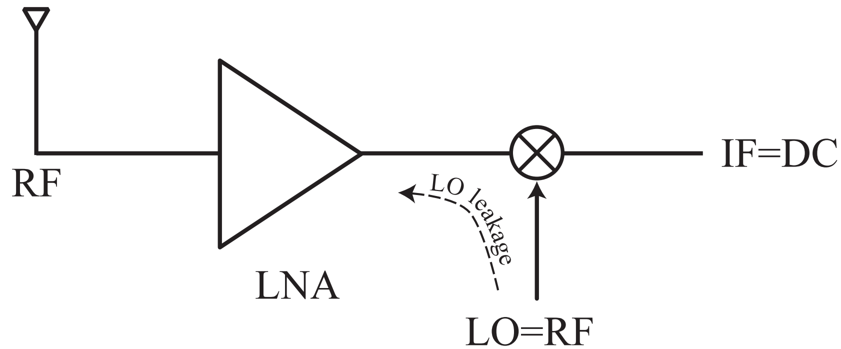
- Such a filter requires components with $Q > 10^3$!

RF Filtering



- The fact that the required filter Q is so high is related to the problem of filtering interferers. The very reason we choose the superheterodyne architecture is to simplify the filtering problem. It's much easier to filter a fixed IF than filter a variable RF.
- The image filtering problem can be relaxed by using multi-IF stages. Instead of moving to such a low IF where the image filtering is difficult (or expensive and bulky), we down-convert twice, using successively lower IF frequencies.

Direct Conversion Receiver



- A mixer will frequency translate two frequencies, $LO \pm IF$
- Why not simply down-convert directly to DC? In other words, why not pick a zero IF?
- This is the basis of the direct conversion architecture. There are some potential problems...

Direction Conversion

- First, note that we must down-convert the desired signal and all the interfering signals. In other words, the LNA and mixer must be extremely linear.
- Since IF is at DC, all *even* order distortion now plagues the system, because the distortion at DC can easily swamp the desired signal.
- Furthermore, CMOS circuits produce a lot of flicker noise. Before we ignored this source of noise becuase it occurs at low frequency. Now it also competes with our signal.
- Another issue is with LO leakage. If any of the LO leaks into the RF path, then it will self-mix and produce a DC offset. The DC offset can rail the IF amplifier stages.

Direct Conversion (cont)

- Example: If the IF amplifier has $80\,\mathrm{dB}$ of gain, and the mixer has $10\,\mathrm{dB}$ of gain, estimate the allowed LO leakage. Assume the ADC uses a $1\mathrm{V}$ reference.
- To rail the output, we require a DC offset less than $10^{-4}\mathrm{V}$. If the LO power is $0\,\mathrm{dBm}$ ($316\mathrm{mV}$), we require an input leakage voltage $< 10^{-5}\mathrm{V}$, or an isolation better than $90\,\mathrm{dB}$!

Phase of LO

- In a direction conversion system, the LO frequency is equal to the RF frequency.
- Consider an input voltage $v(t) = A(t)\cos(\omega_0 t)$. Since the LO is generated "locally", it's phase is random relative to the RF input:

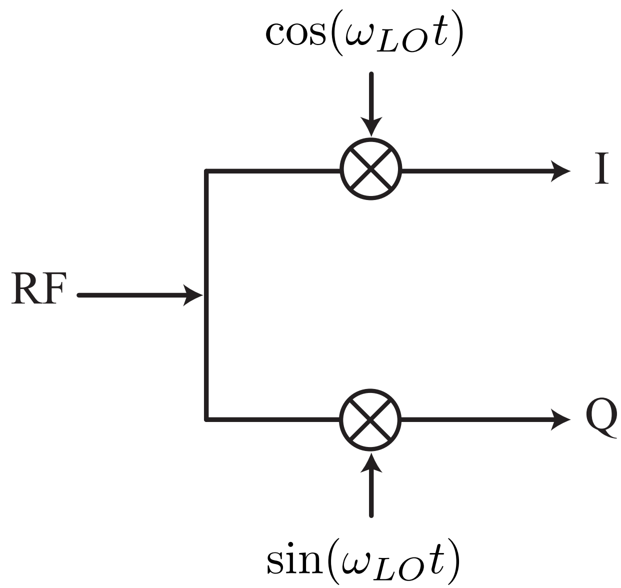
$$v_{LO} = A_{LO}\cos(\omega_0 t + \phi_0)$$

- If we are so unlucky that $\phi_0 = 90^\circ$, then the output of the mixer will be zero

$$\int_T A(t)A_{LO}\sin(\omega_0 t)\cos(\omega_0 t)dt$$

$$\approx A(t)A_{LO}\int_T \sin(\omega_0 t)\cos(\omega_0 t)dt = 0$$

IQ-Mixer



- To avoid this situation, we can *phase lock* the LO to the RF by transmitting a pilot tone. Alternatively, we can use two mixers
- As we shall see, there are other benefits to such a mixer.

AM Modulation

- We can see that an upconversion mixer is a natural amplitude modulator
- If the input to the mixer is a baseband signal $A(t)$, then the output is an AM carrier

$$v_o(t) = A(t)\cos(\omega_{LO} t)$$

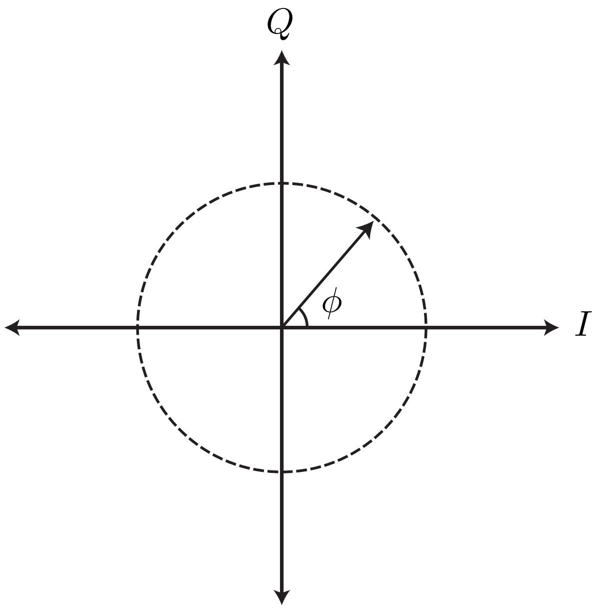
- How do we modulate the phase? A PLL is one way to do it. The IQ mixer is another way.
- Let's expand a sinusoid that has AM and PM

$$v_o(t) = A(t)\cos(\omega_0 t + \phi(t))$$

$$= A(t)\cos\omega_0 t\cos\phi(t) + A(t)\sin\omega_0 t\sin\phi(t)$$

$$= I(t)\cos\omega_0 t + Q(t)\sin\omega_0 t$$

I-Q Plane



$$I(t) = A(t)\cos\phi(t)$$

$$Q(t) = A(t)\sin\phi(t)$$

- We can draw a trajectory of points on the I-Q plane to represent different modulation schemes.
- The amplitude modulation is given by

$$I^2(t) + Q^2(t) = A^2(t)(\cos^2\phi(t) + \sin^2\phi(t)) = A^2(t)$$

General Modulator

- The phase modulation is given by

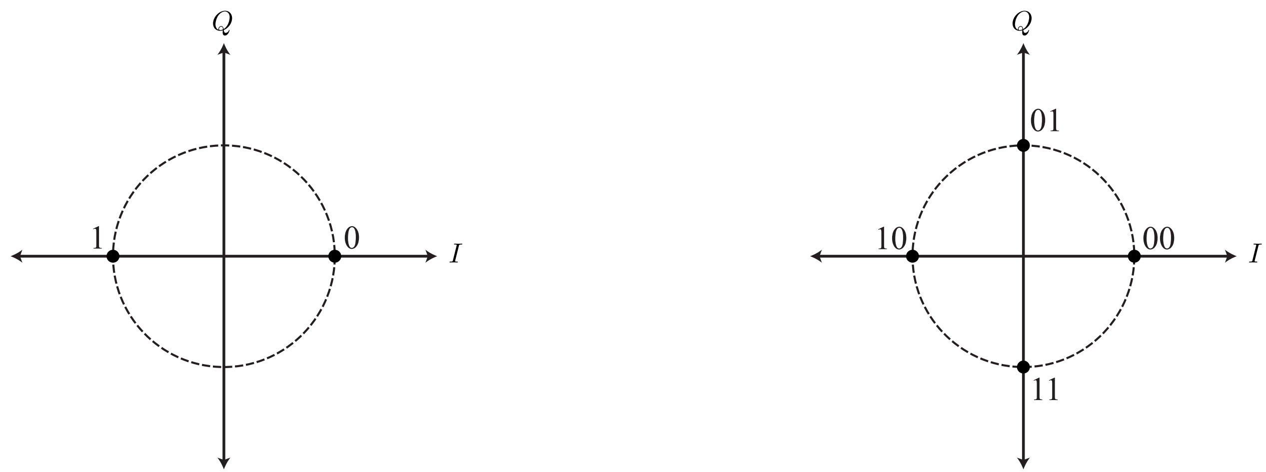
$$\frac{Q(t)}{I(t)} = \frac{\sin\phi(t)}{\cos\phi(t)} = \tan\phi(t)$$

or

$$\phi(t) = \tan^{-1}\frac{Q(t)}{I(t)}$$

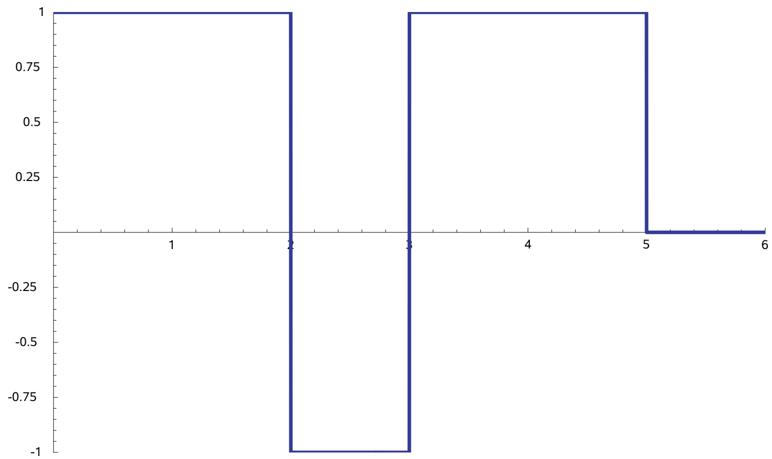
- The IQ modulator is a universal digital modulator. We can draw a set of points in the IQ plane that represent symbols to transmit. For instance, if we transmit $I = 0/A$ and $Q = 0$, then we have a simple ASK system (amplitude shift keying).

Digital Modulation: BPSK/QPSK

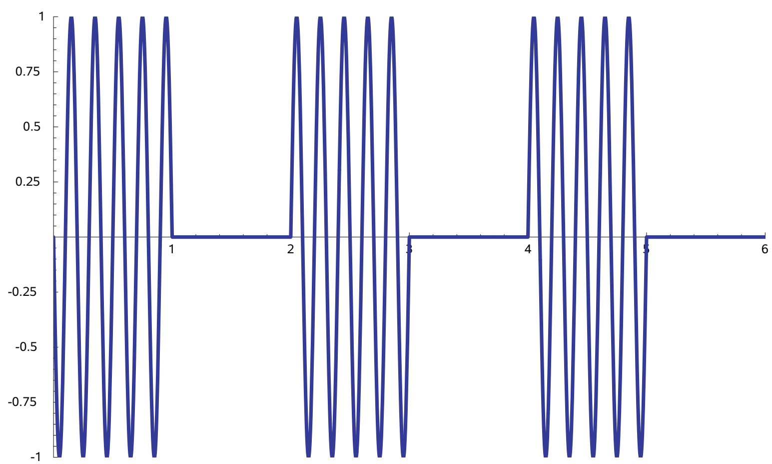


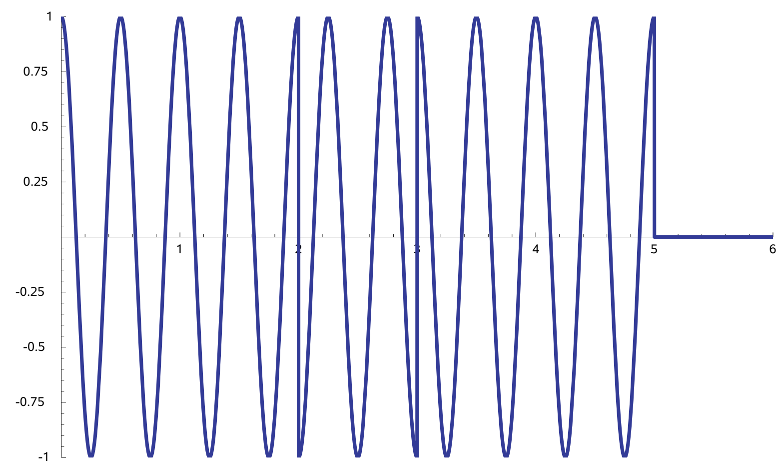
- For instance, if we transmit $I(t) = \pm 1$, this represents one bit transmission per cycle. But since the I and Q are orthogonal signals, we can improve the efficiency of transmission by also transmitting symbols on the Q axis.
- If we select four points on a circle to represent 2 bits of information, then we have a constant envelope modulation scheme.

Modulation Waveforms

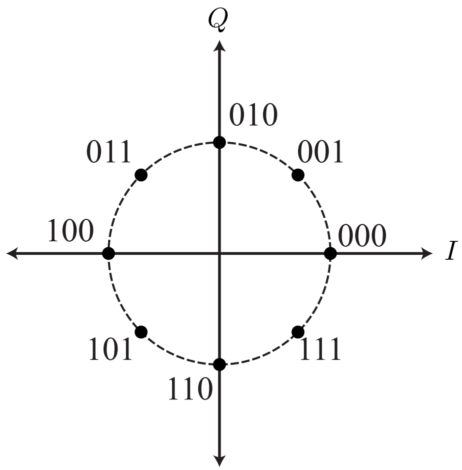


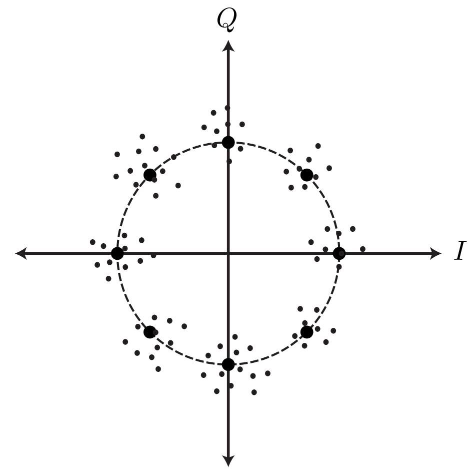
- The data wave form (left) is modulated onto a carrier (below). The first plot shows a simple on/off keying. The second plot shows binary phase shift keying on one channel (I).





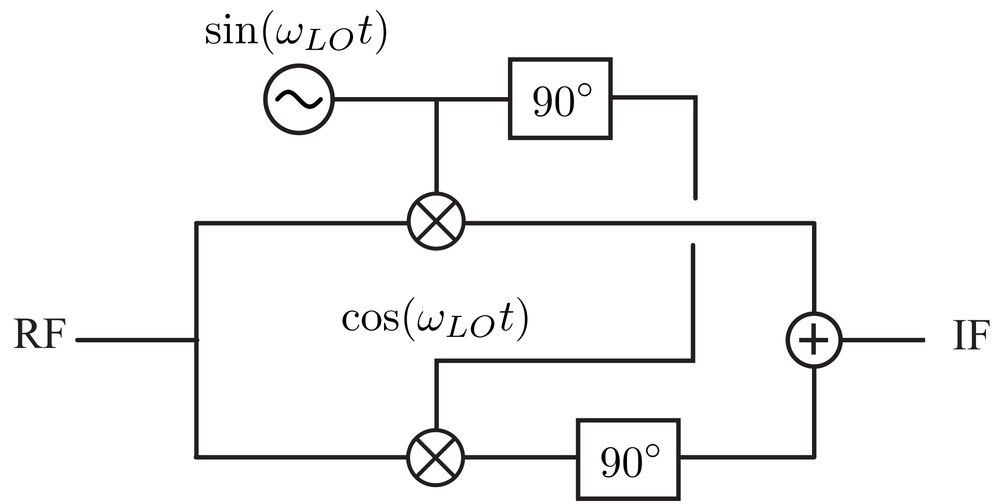
More Bits Per Cycle





- Eventually, the *constellation* points get very close together. Because of noise and distortion, the received spectrum will not lie exactly on the constellation points, but instead they will form a cluster around such points.
- If the clusters run into each other, errors will occur in the transmission.
- We can increase the radius but that requires more power.

I/Q Hartley Mixer



- An I/Q mixer implemented as shown above is known as a Hartley Mixer.
- We shall show that such a mixer can be designed to select either the upper or lower sideband. For this reason, it is sometimes called a single-sideband mixer.
- We will also show that such a mixer can perform image rejection.

Delay Operation

- Consider the action of a $90°$ delay on an arbitrary signal. Clearley $\sin(x + 90°) = \cos(x)$. Even though this is obvious, consider the effect on the complex exponentials

$$\sin(x - \frac{\pi}{2}) = \frac{e^{jx - j\pi/2} - e^{-jx + j\pi/2}}{2j}$$

$$= \frac{e^{jx}e^{-j\pi/2} - e^{-jx}e^{j\pi/2}}{2j} = \frac{e^{jx}(-j) - e^{-jx}(j)}{2j}$$

$$= -\frac{e^{jx} + e^{-jx}}{2} = -\cos(x)$$

- Positive frequencies get multiplied by $-j$ and negative frequencies by $+j$. This is true for a narrowband signal when it is delayed by $90°$.

Complex Modulation

- Consider multiplying a waveform $f(t)$ by $e^{j\omega t}$ and taking the Fourier transform

$$\mathcal{F}\left\{e^{j\omega_0 t}f(t)\right\} = \int_{-\infty}^{\infty} f(t)e^{j\omega_0 t}e^{-j\omega t}dt$$

- Grouping terms we have

$$= \int_{-\infty}^{\infty} f(t)e^{-j(\omega-\omega_0)t}dt = F(\omega - \omega_0)$$

- It is clear that the action of multiplication by the complex exponential is a frequency shift.

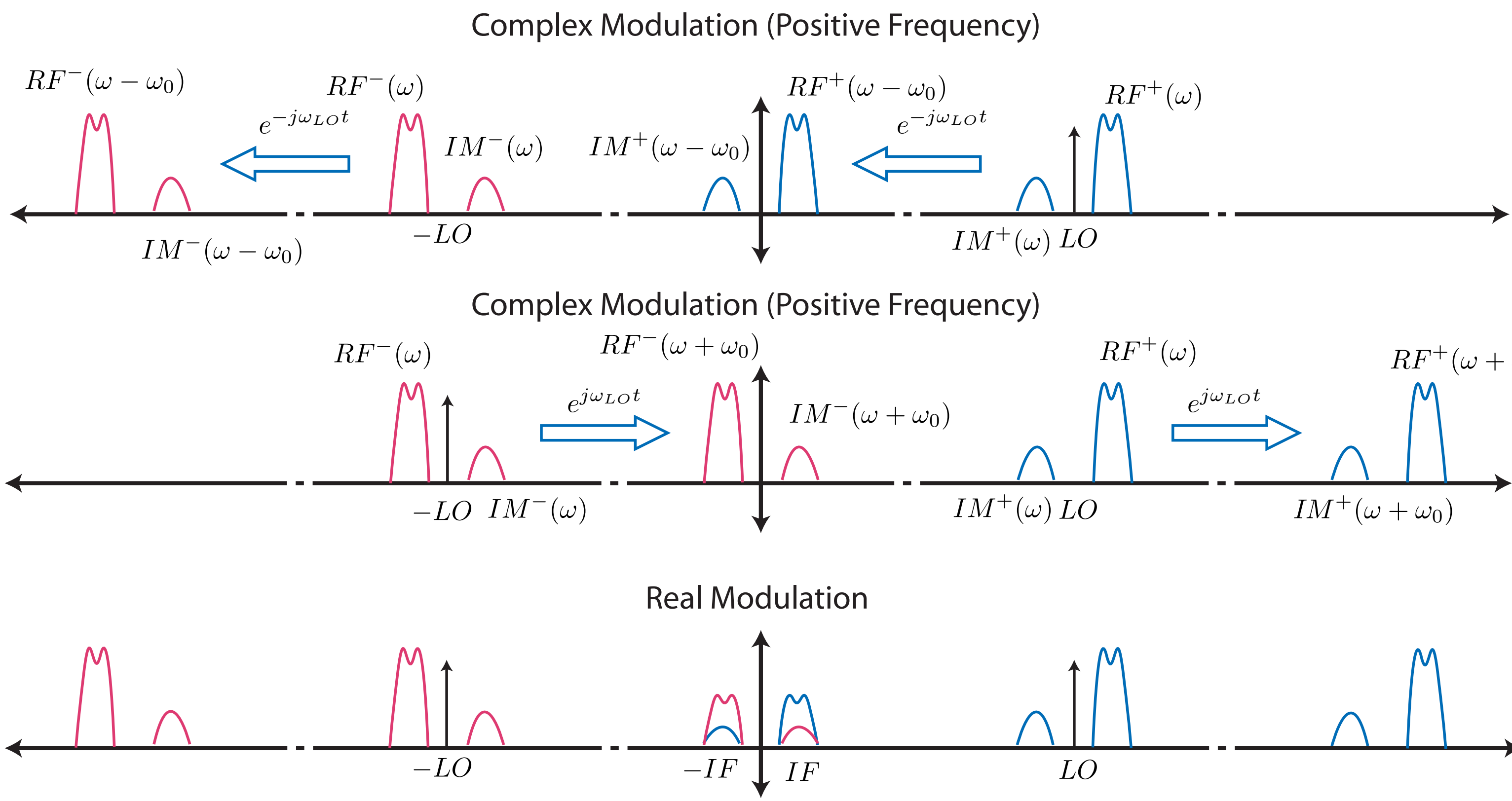
Real Modulation

- Now since $\cos(x) = (e^{jx} + e^{-jx})/2$, we see that the action of time domain multiplication is to produce two frequency shifts

$$\mathcal{F}\left\{\cos(\omega_0 t) f(t)\right\} = \frac{1}{2} F(\omega - \omega_0) + \frac{1}{2} F(\omega + \omega_0)$$

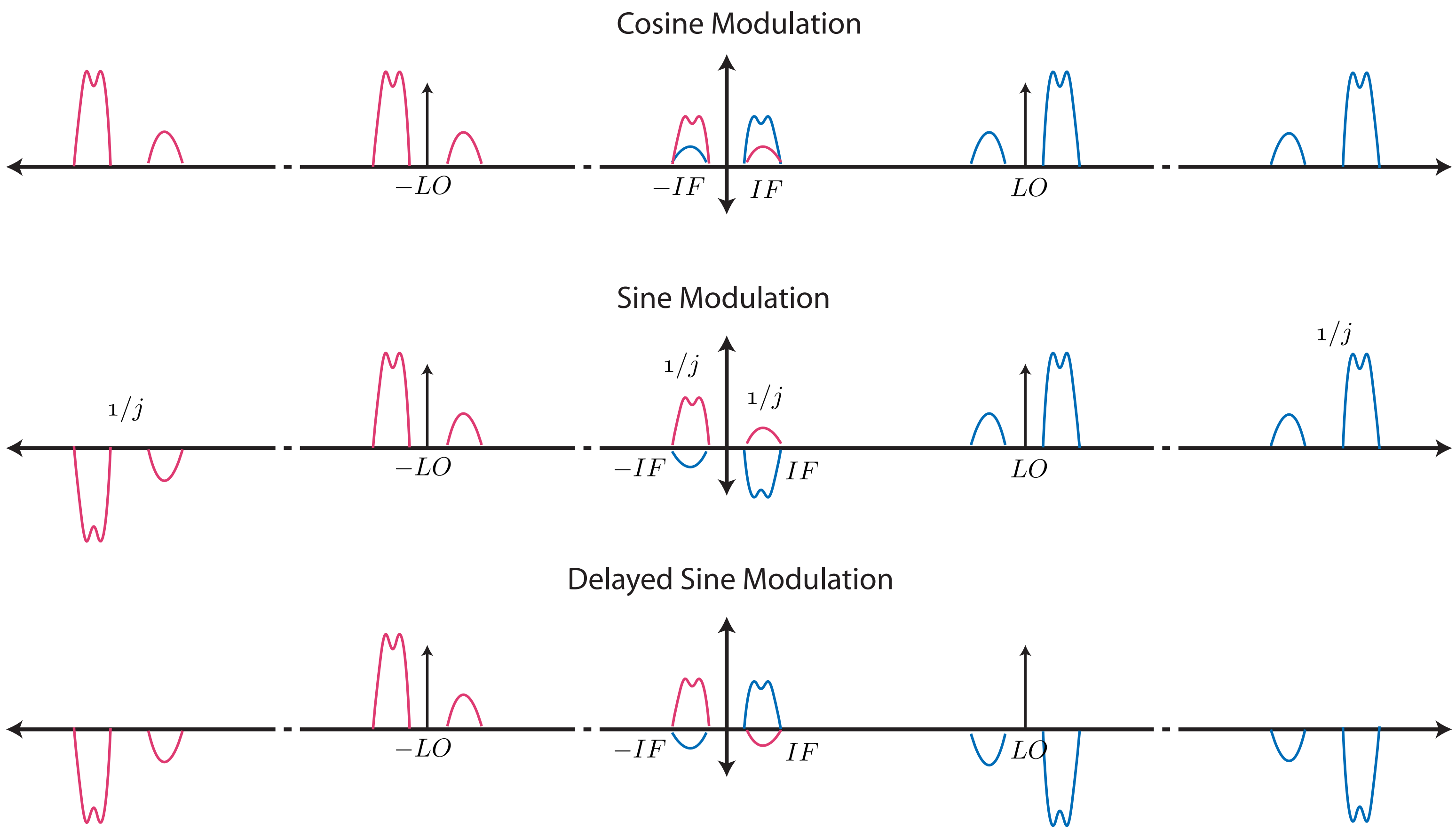
- These are the sum and difference (beat) frequency components.

Image Problem (Again)



- We see that the image problem is due to to multiplication by the sinusoid and not a complex exponential. If we could synthesize a complex exponential, we would not have the image problem.

Sine/Cosine Modulation



- Using the same approach, we can find the result of multipling by $\sin$ and $\cos$ as shown above. If we delay the $\sin$ portion, we have a very desirable situation! The image is inverted with respect to the $\cos$ and can be cancelled.

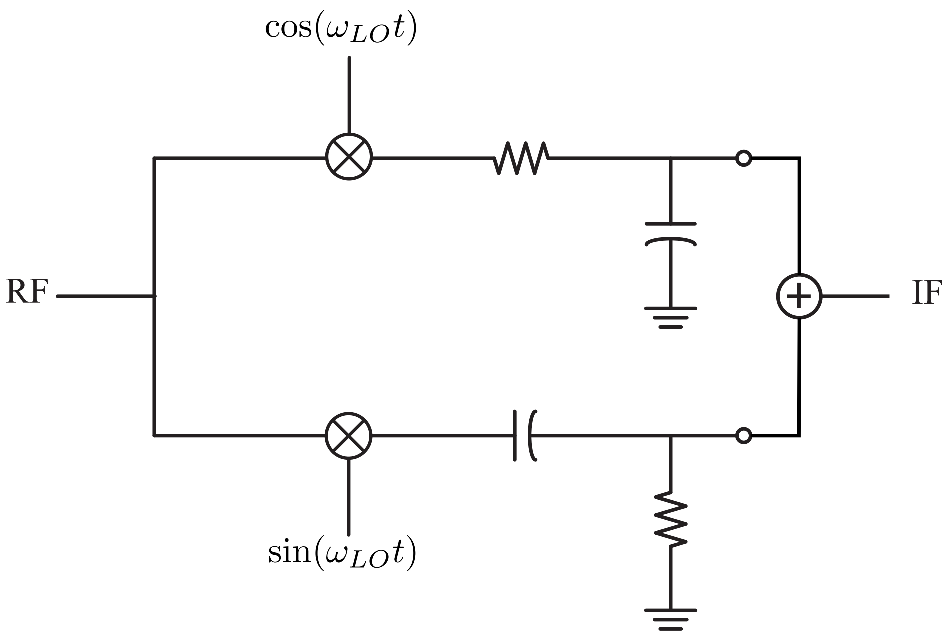
Image Rejection

- The image rejection scheme just described is very sensitive to phase and gain match in the I/Q paths. Any mismatch will produce only finite image rejection.
- The image rejection for a given gain/phase match is approximately given by

$$IRR(\,\mathrm{dB}) = 10 \cdot \log \frac{1}{4}\left(\left(\frac{\delta A}{A}\right)2 + (\delta\theta)^2\right)$$

- For typical gain mismatch of $0.2 - 0.5\,\mathrm{dB}$ and phase mismtach of $1^\circ - 4^\circ$, the image rejection is about $30\,\mathrm{dB}$ - $40\,\mathrm{dB}$. We usually need about $60 - 70\,\mathrm{dB}$ of total image rejection.

$\pm 45°$ Delay Element



- The passive R/C and C/R lowpass and highpass filters are a nice way to implement the delay. Note that their relative phase difference is always $90°$.

$$\angle H_{lp} = \angle \frac{1}{1 + j\omega RC} = -\arctan \omega RC$$

$$\angle H_{hp} = \angle \frac{j\omega RC}{1 + j\omega RC} = \frac{\pi}{2} - \arctan \omega RC$$

Gain Match / Quadrature LO Gen

- But to have equal gain, the circuit must operate at the $1/RC$ frequency. This restricts the circuit to relatively narrowband systems. Multi-stage polyphase circuits remedy the situation but add insertion loss to the circuit.
- The I/Q LO signal is usually generated directly rather than through an high-pass and low-pass network.
- Two ways to generate the I/Q LO is through a divide-by-two circuit (requires $2 \times LO$) or a quadrature oscillator (requires two tanks).