Lecture 15: Noise in Communication Systems

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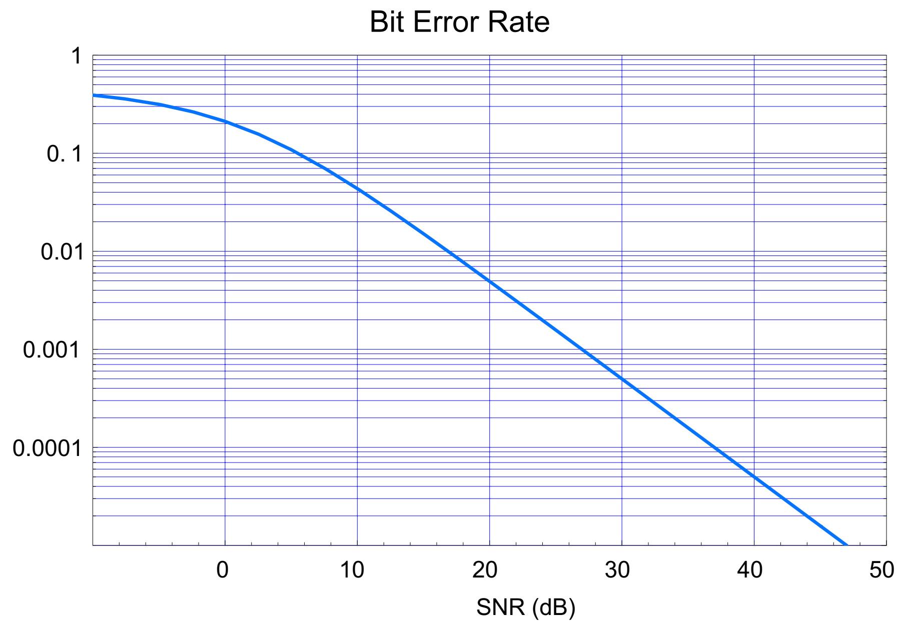
Degradation of Link Quality

- As we have seen, noise is an ever present part of all systems. Any receiver must contend with noise.
- In analog systems, noise deteriorates the quality of the received signal, e.g. the appearance of “snow” on the TV screen, or “static” sounds during an audio transmission.
- In digital communication systems, noise degrades the throughput because it requires retransmission of data packets or extra coding to recover the data in the presence of errors.

BER Plot



- It's typical to plot the Bit-Error-Rate (BER) in a digital communication system.
- This shows the average rate of errors for a given signal-to-noise-ratio (SNR)

SNR

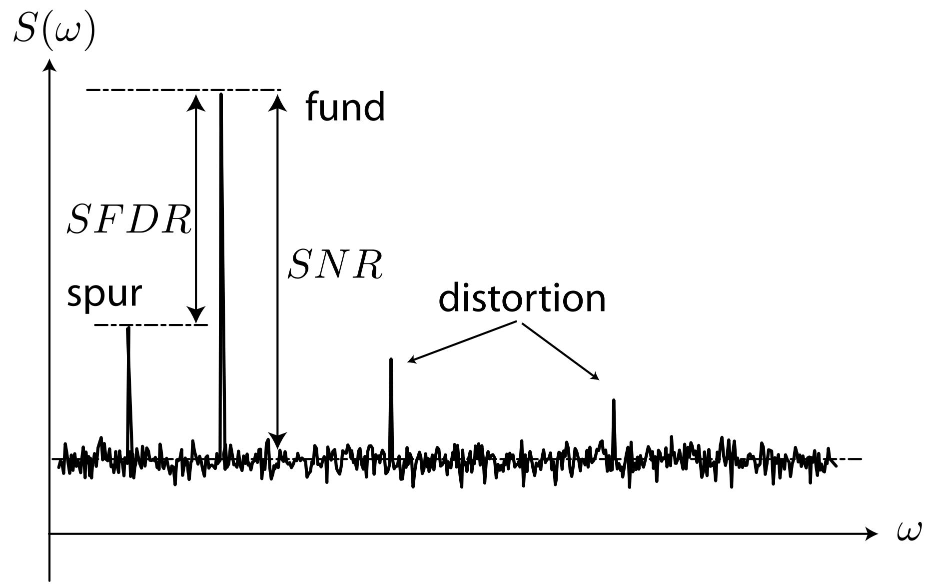
- In general, then, we strive to maximize the signal to noise ratio in a communication system. If we receive a signal with average power P_{sig}, and the average noise power level is P_{noise}, then the SNR is simply

$$SNR = \frac{S}{N}$$

$$SNR(\text{dB}) = 10 \cdot \log \frac{P_{sig}}{P_{noise}}$$

- We distinguish between random noise and “noise” due to interferers or distortion generated by the amplifier

Spurious Free Dynamic Range



- The spurious free dynamic range $SFDR$ measures the available dynamic range of a signal at a particular point in a system. For instance, in an amplifier the largest signal determines the distortion "noise" floor and the noise properties of the amplifier determine the "noise floor"

Noise Figure

- The *Noise Figure* (NF) of an amplifier is a block (e.g. an amplifier) is a measure of the degradation of the SNR

$$F = \frac{SNR_i}{SNR_o}$$

$$NF = 10 \cdot \log(F) \; (\mathrm{dB})$$

- The noise figure is measured (or calculated) by specifying a standard input noise level through the source resistance R_s and the temperature
- For RF communication systems, this is usually specified as $R_s = 50\Omega$ and $T = 293^\circ K$.

Noise Figure of an Amplifier

- Suppose an amplifier has a gain G and apply the definition of NF

$$SNR_i = \frac{P_{sig}}{N_s}$$

$$SNR_o = \frac{GP_{sig}}{GN_s + N_{amp,o}}$$

- The term $N_{amp,o}$ is the total output noise due to the amplifier in absence of any input noise.

$$SNR_o = \frac{P_{sig}}{N_s + \frac{N_{amp,o}}{G}}$$

Input Referred Noise (I)

- Let $N_{amp,i}$ denote the total input referred noise of the amplifier

$$SNR_o = \frac{P_{sig}}{N_s + N_{amp,i}}$$

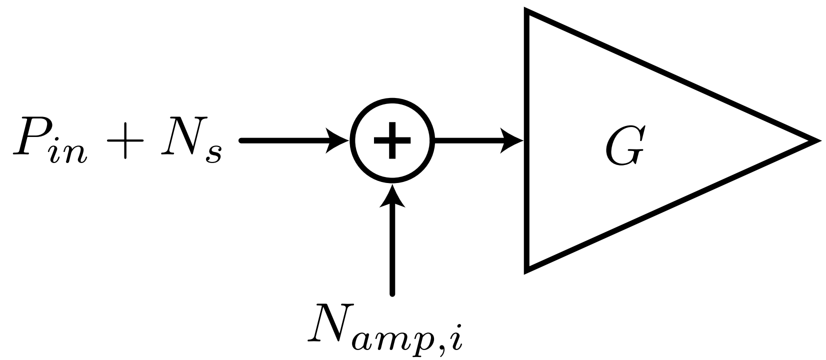
- The noise figure is therefore

$$F = \frac{SNR_i}{SNR_o} = \frac{P_{\not{sig}}}{N_s} \times \frac{N_s + N_{amp,i}}{P_{\not{sig}}}$$

$$F = 1 + \frac{N_{amp,i}}{N_s} \geq 1$$

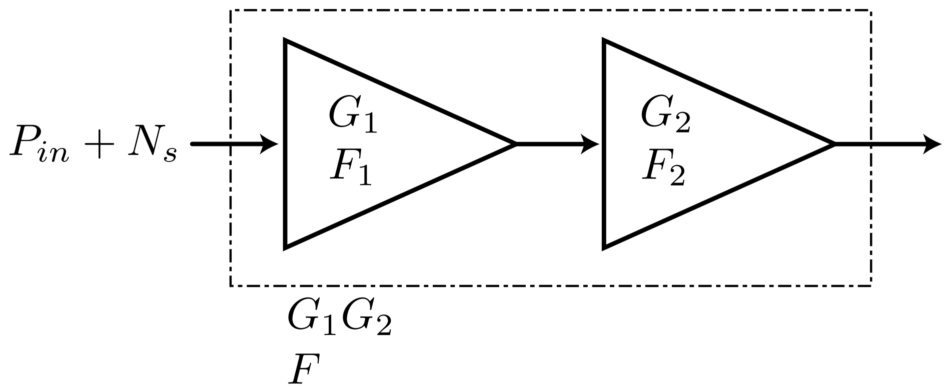
- All amplifiers have a noise figure ≥ 1. Any real system degrades the SNR since all circuit blocks add additional noise.

Input Referred Noise (II)



- The amount of noise added by the amplifier is normalized to the incoming noise N_s in the calculation of F. For RF systems, this is the noise of a 50Ω source at $293°\mathrm{K}$.
- Since any amplification degrades the SNR, why do any amplification at all? Because often the incoming signal is too weak to be detected without amplification.

Noise Figure of Cascaded Blocks



- If two blocks are cascaded, we would like to derive the noise figure of the total system.
- Assume the blocks are impedance matched properly to result in a gain $G = G_1 G_2$. For each amplifier in cascade, we have

$$F_i = 1 + \frac{N_{amp,i}}{N_s}$$

Total Input Noise for Cascade

- By definition, the noise added by each amplifier to the input is given by

$$N_{amp,i} = N_s(F-1)$$

- where N_s represents some standard input noise. If we now input refer all the noise in the system we have

$$N'_{amp,i} = N_s(F_1 - 1) + \frac{N_s(F_2 - 1)}{G_1}$$

- Which gives us the total noise figure of the amplifier

$$F = 1 + \frac{N'_{amp,i}}{N_s} = 1 + (F_1 - 1) + \frac{F_2 - 1}{G_1} = F_1 + \frac{F_2 - 1}{G_1}$$

General Cascade Formula

- Apply the formula to the last two blocks

$$F_{23} = F_2 + \frac{F_3 - 1}{G_2}$$

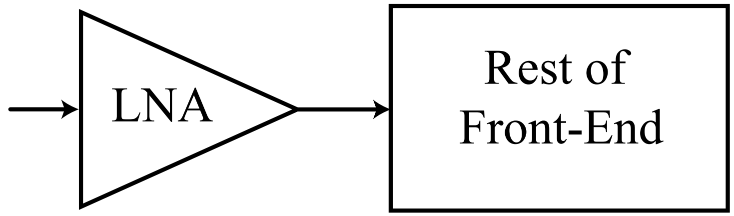
$$F = F_1 + \frac{F_{23} - 1}{G_1}$$

$$= F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2}$$

- The general equation is written by inspection

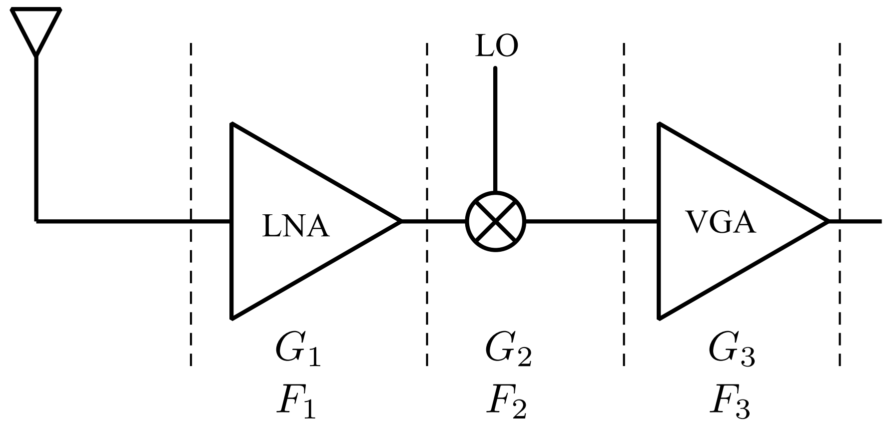
$$= F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \frac{F_4 - 1}{G_1 G_2 G_3} + \cdots$$

Cascade Formula Interpretation



- We see that in a cascade, the noise contribution of each successive stage is smaller and smaller.
- The noise of the *first* stage is the most important. Thus, every communication system employs a *low noise amplifier* (LNA) at the front to relax the noise requirements
- A typical LNA might have a $G = 20\,\mathrm{dB}$ of gain and a noise figure $NF < 1.5\,\mathrm{dB}$. The noise figure depends on the application.

NF Cascade Example



- The LNA has $G = 15\,\text{dB}$ and $NF = 1.5\,\text{dB}$. The mixer has a conversion gain of $G = 10\,\text{dB}$ and $NF = 10\,\text{dB}$. The IF amplifier has $G = 70\,\text{dB}$ and $NF = 20\,\text{dB}$.
- Even though the blocks operate at different frequencies, we can still apply the cascade formula if the blocks are impedance matched

$$F = 1.413 + \frac{10-1}{60} + \frac{100-1}{60 \cdot 10} = 2.4\,\text{dB}$$

Minimum Detectable Signal

- Say a system requires an SNR of $10\,\mathrm{dB}$ for proper detection with a minimum voltage amplitude of $1\mathrm{mV}$. If a front-end with sufficient gain has $NF = 10\,\mathrm{dB}$, let's compute the minimum input power that can support communication:

$$SNR_o = \frac{SNR_i}{F} = \frac{\frac{P_{min}}{N_s}}{F} > 10$$

or

$$P_{in} > 10 \cdot F \cdot N_s = 10 \cdot F \cdot kTB$$

- we see that the answer depends on the bandwidth B.

$$P_{in} = 10\,\mathrm{dB} + NF - 174\,\mathrm{dBm} + 10 \cdot \log B$$

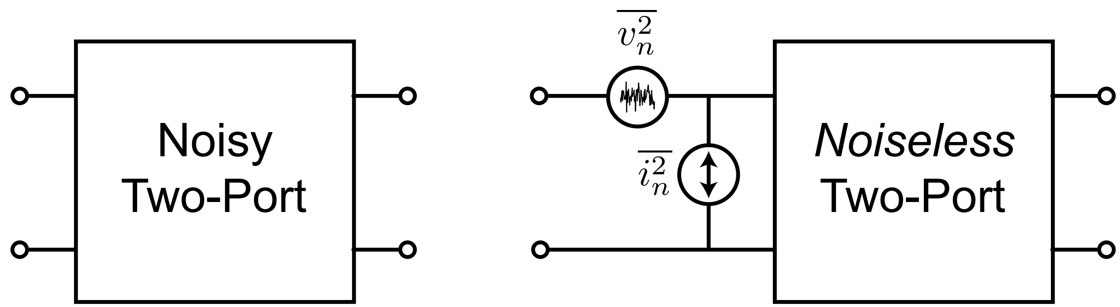
Minimum Signal (cont)

- For wireless data, $B \sim 10\mathrm{MHz}$:

$$P_{in} = 10\,\mathrm{dB} + 10\,\mathrm{dB} - 174\,\mathrm{dB} + 70\,\mathrm{dB} = -84\,\mathrm{dBm}$$

- We see that the noise figure has a dB for dB impact on the minimum detectable input signal. Since the received power drops $> 20\,\mathrm{dB}$ per decade of distance, a few dB improved NF may dramatically improve the coverage area of a communication link.
- Otherwise the transmitter has to boost the TX power, which requires excess power consumption due to the efficiency η of the transmitter.

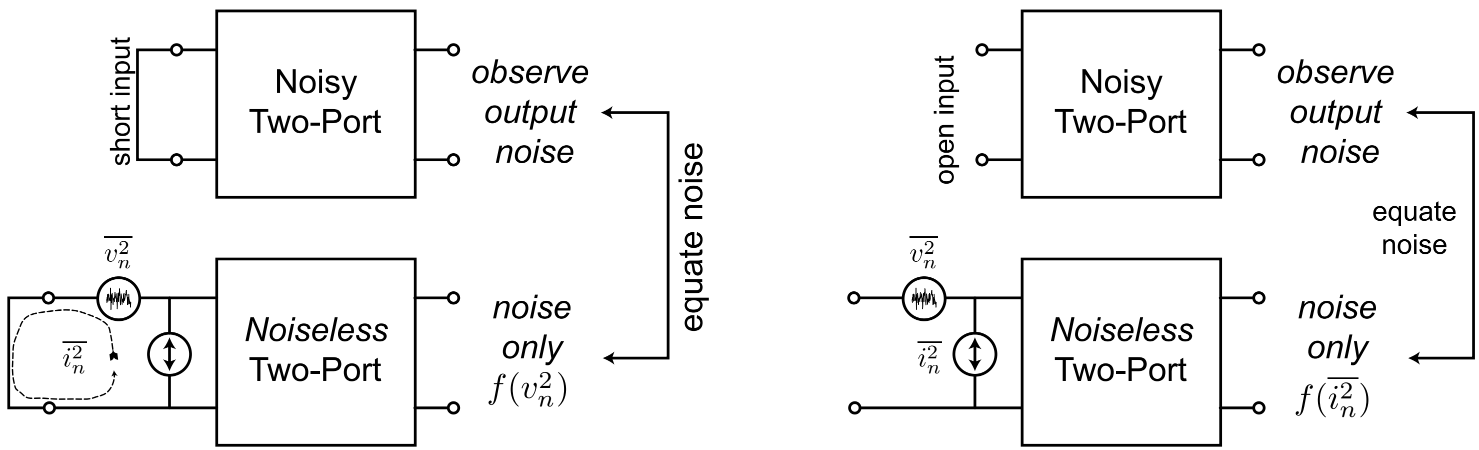
Equivalent Noise Generators



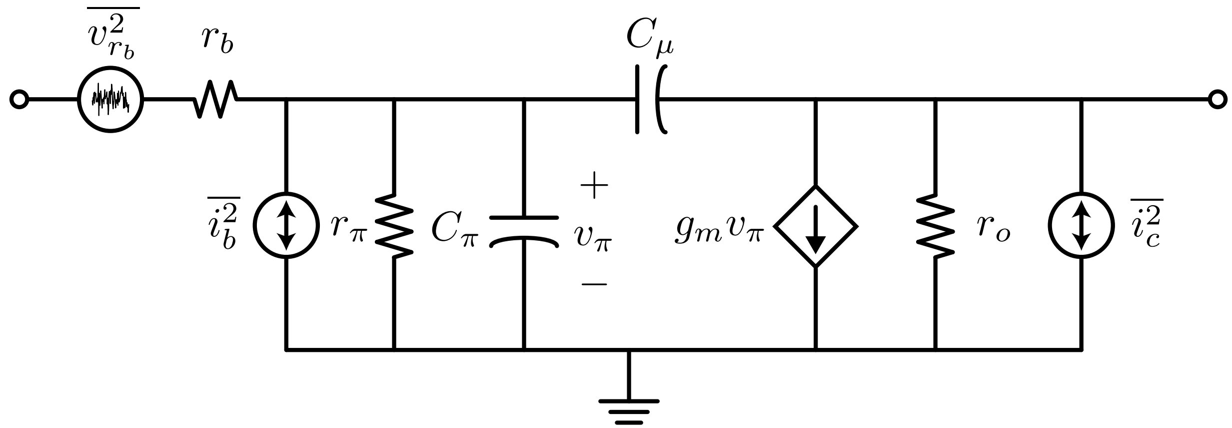
- Any noisy two port can be replaced with a *noiseless* two-port and equivalent input noise sources
- In general, these noise sources are correlated. For now let's neglect the correlation.

Equivalent Noise Generators (cont)

- The equivalent sources are found by opening and shorting the input.



Example: BJT Noise Sources



- If we leave the base of a BJT open, then the total output noise is given by

$$\overline{i_o^2} = \overline{i_c^2} + \beta^2 \overline{i_b^2} = \overline{i_n^2} \beta^2$$

or

$$\overline{i_n^2} = \frac{\overline{i_c^2}}{\beta^2} + \overline{i_b^2} \approx \overline{i_b^2}$$

BJT (cont)

- If we short the input of the BJT, we have

$$\overline{i_o^2} \approx g_m^2 \overline{v_n^2} \left(\frac{Z_\pi}{Z_\pi + r_b} \right)^2 = \beta^2 \frac{\overline{v_n^2}}{(Z_\pi + r_b)^2}$$

$$= \beta^2 \frac{\overline{v_{r_b}^2}}{(Z_\pi + r_b)^2} + \overline{i_c^2}$$

- Solving for the equivalent BJT noise voltage

$$\overline{v_n^2} = \overline{v_{r_b}^2} + \frac{\overline{i_c^2}(Z_\pi + r_b)^2}{\beta^2}$$

$$\overline{v_n^2} \approx \overline{v_{r_b}^2} + \frac{\overline{i_c^2} Z_\pi^2}{\beta^2}$$

BJT Generators at Low Freq

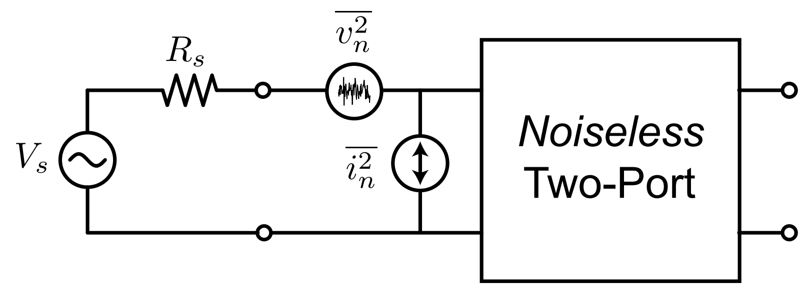
at low frequencies...

$$\overline{v_n^2} \approx \overline{v_{r_b}^2} + \frac{\overline{i_c^2}}{g_m^2}$$

$$\overline{v_n^2} = 4kTBr_b + \frac{2qI_C B}{g_m^2}$$

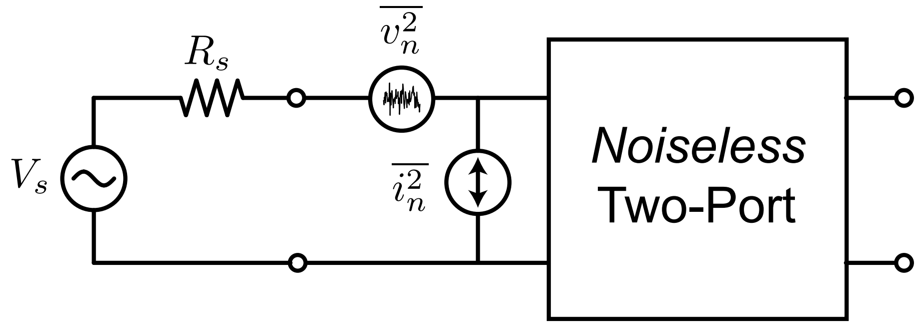
$$\overline{i_n^2} = \frac{2qI_c}{\beta}$$

Role of Source Resistance



- If $R_s = 0$, only the voltage noise $\overline{v_n^2}$ is important. Likewise, if $R_s = \infty$, only the current noise $\overline{i_n^2}$ is important.
- Amplifier Selection: If R_s is large, then select an amp with low $\overline{i_i^2}$ (MOS). If R_s is low, pick an amp with low $\overline{v_n^2}$ (BJT)
- For a given R_s, there is an optimal $\overline{v_n^2}/\overline{i_n^2}$ ratio. Alternatively, for a given amp, there is an optimal R_s

Equivalent Input Noise Voltage

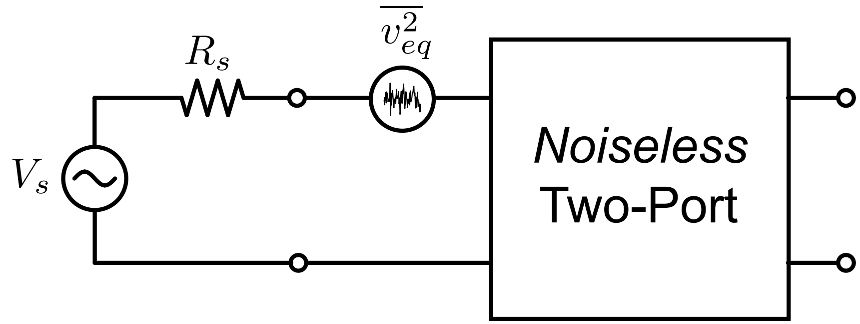


- Let's find the total output noise voltage

$$\overline{v_o^2} = (\overline{v_n^2} A_v^2 + \overline{v_{R_s}^2} A_v^2) \left(\frac{R_{in}}{R_{in} + R_s} \right)^2 + \left(\frac{R_{in}}{R_{in} + R_s} \right)^2 R_s^2 \overline{i_n^2} A_v^2$$

$$= (\overline{v_n^2} + \overline{i_n^2} R_s^2 + \overline{v_{R_s}^2}) \left(\frac{R_{in}}{R_{in} + R_s} \right)^2 A_v^2$$

Noise Figure



- We see that all the noise can be represented by a single equivalent source

$$\overline{v_{eq}^2} = \overline{v_n^2} + \overline{i_n^2} R_s^2$$

- Applying the definition of noise figure

$$F = 1 + \frac{N_{amp,i}}{N_s} = 1 + \frac{\overline{v_{eq}^2}}{\overline{v_s^2}}$$

Optimal Source Impedance

- Let $\overline{v_n^2} = 4kTR_nB$ and $\overline{i_n^2} = 4kTG_nB$. Then

$$F = 1 + \frac{R_n + G_nR_s}{R_s} = 1 + G_nR_s + \frac{R_n}{R_s}$$

- Let's find the optimum R_s

$$\frac{dF}{dR_s} = G_n - \frac{R_n}{R_s^2} = 0$$

- We see that the noise figure is minimized for

$$R_{opt} = \sqrt{\frac{R_n}{G_n}} = \sqrt{\frac{\overline{v_n^2}}{\overline{i_n^2}}}$$

Optimal Source Impedance (cont)

- The major assumption we made was that $\overline{v_n^2}$ and $\overline{i_n^2}$ are not correlated. The resulting minimum noise figure is thus

$$F_{min} = 1 + G_n R_s + \frac{R_n}{R_s}$$

$$= 1 + G_n \sqrt{\frac{R_n}{G_n}} + \sqrt{\frac{G_n}{R_n}} R_n$$

$$= 1 + 2\sqrt{R_n G_n}$$

F_{min}

- Consider the difference between F and F_{min}

$$F - F_{min} = G_n R_s + \frac{R_n}{R_s} - 2\sqrt{R_n G_n}$$

$$= \frac{R_n}{R_s}(1 + \frac{G_n R_s^2}{R_n} - 2\frac{R_s}{R_n}\sqrt{R_n G_n}$$

$$= \frac{R_n}{R_s}\left(1 + \left(\frac{R_s}{R_{opt}}\right)^2 - \frac{2R_s}{R_{opt}}\right)$$

$$= \frac{R_n}{R_s}\left|\frac{R_s}{R_{opt}} - 1\right|^2$$

$$= R_n R_s \left|G_{opt} - G_s\right|^2$$

Noise Sensitivity Parameter

- Sometimes R_n is called the noise sensitivity parameter since

$$F = F_{min} + R_n R_s \left| G_{opt} - G_s \right|^2$$

- This is clear since the rate of deviation from optimal noise figure is determined by R_n. If a two-port has a small value of R_n, then we can be sloppy and sacrifice the noise match for gain. If R_n is large, though, we have to pay careful attention to the noise match.
- Most software packages (Spectre, ADS) will plot Y_{opt} and F_{min} as a function of frequency, allowing the designer to choose the right match for a given bias point.

BJT Noise Figure

- We found the equivalent noise generators for a BJT

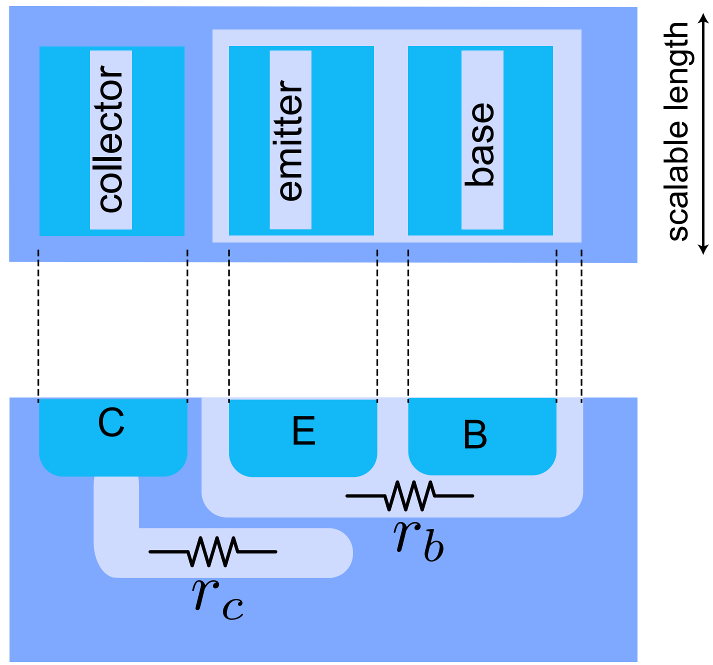
$$\overline{v_n^2} = \overline{v_{r_b}^2} + \frac{\overline{i_c^2}}{g_m^2} = 4kTBr_b + \frac{2qI_C B}{g_m^2} \qquad \overline{i_n^2} = \overline{i_b^2}$$

- The noise figure is

$$F = 1 + \frac{4kTr_b + \frac{2qI_C}{g_m^2}}{4kTR_s} + \frac{2qI_C R_s^2}{\beta 4kTR_s} = 1 + \frac{r_b}{R_s} + \frac{1}{2g_m R_s} + \frac{g_m R_s}{2\beta}$$

- According to the above expression, we can choose an optimal value of $g_m R_s$ to minimize the noise. But the second term r_b/R_s is fixed for a given transistor dimension

BJT Cross Section



- The device can be scaled to lower the net current density in order to delay the onset of the Kirk Effect
- The base resistance also drops when the device is made larger

BJT Device Sizing

- We can thus see that BJT transistor sizing involves a compromise:
  - The transconductance depends only on I_C and not the size (first order)
  - The charge storage effects and f_T only depend on the base transit time, a fixed vertical dimension.
  - A smaller device has smaller junction area but can only handle a given current density before Kirk effect reduces performance
  - A larger device has smaller base resistance r_b but larger junction capacitance